EVAPORATING, CONDENSING AND COOLING APPARATUS

EXPLANATIONS, FORMULÆ AND TABLES FOR USE IN PRACTICE

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PREFACE TO THE FIRST GERMAN EDITION

THE problems which are to be solved in the construction of apparatus for evaporating, condensing and cooling, are intimately connected with the laws of the transfer of heat. Although, generally speaking, these physical laws can be regarded as known, yet reliable knowledge of the practical coefficients, applicable in each of the many different cases, is often wanting. Without these coefficients the constructing engineer cannot work. Numberless experiments have been conducted by more or less competent observers to supply this want, but their results are scattered through the literature, were often obtained only for very special cases, and occasionally without regard to all the prevailing conditions. Many have been kept secret by their discoverers as valuable prizes.

The very excellent work published by Professor Molier at the instance of the *Verein deutscher Ingenieure* in the *Zeitschrift des Vereines deutscher Ingenieure*, 1897, Nos. 6 and 7, in which the present condition of our knowledge of these relations is very clearly displayed, does not give figures directly applicable in practice, which indeed was not its object.

For this purpose new experiments on the large scale are necessary, which shall take into consideration all the working conditions, and, in particular the absolute dimensions of the heating surfaces. Recently the *Verein deutscher Ingenieure* has turned its attention to this question. Its competence and ample funds permit us to anticipate the best success.

In the construction of evaporating and cooling apparatus other questions arise, which at present cannot be answered by a knowledge of the processes based on accurate and many-sided

researches—for example, as to the pressures exerted by rarefied and compressed gasses and vapours on floating drops, the resistance due to the friction of rarefied vapours in wide pipes, etc.

It is very desirable that these gaps should at once be filled by orderly and reliable researches available for the requirements of the whole industry.

But before these wishes can be fulfilled, all varieties of apparatus of this order must be built, and since to the author's knowledge there is no book in which, so far as it is possible, most of the questions and conditions relating to evaporation (in particular, the chief dimensions of the apparatus and the efficiency to be anticipated) are treated in a connected manner for practical purposes, an attempt to supply the deficiency has been made in the following pages.

In this task the generally available material, also very valuable communications from well-disposed friends, and, finally, the experience and experimental results of long practice, have been employed.

It lies in the nature of the circumstances indicated above that much of these explanations must have a hypothetical character, which the friendly reader must remember.

Lack of time will often prevent an engineer who is not quite at home in this branch from seeking, by a long study of the literature, the examples which are at once required, and from making long calculations. On this account, wherever it appeared advisable, tables have been introduced, which contain easily ascertained answers to certain definite questions arising from many cases. These tables also have the advantage of affording a clear insight into the alterations produced by variations in the data of the problem, which advantage constructors know well how to prize.

In view of the extreme variety of the apparatus and machines used in the industry, the constant and rapid changes of its requirements, and also its rapid progress, a complete treatment of *all* possible cases cannot well be attained.

The constant motive in writing this treatise has been the desire to provide as complete and reliable assistance as possible

for the solution of the problems of the construction and working of apparatus for evaporating, condensing and cooling. If this desire has not been quite fulfilled, the book will perhaps be regarded as a useful foundation for further endeavours.

There now remains the pleasant duty of expressing thanks to all the friends who have helped to enrich the contents of this work by communicating the results of experience, and to the publisher for the worthy appearance of the book.

THE AUTHOR.

BERLIN, *August*, 1899.

DIRECTOR OF INDUSTRIES
Gen.

PREFACE TO THE SECOND GERMAN EDITION

A SECOND edition of this work has become necessary in so short a time after the appearance of the first, that there has been no opportunity for extensive alterations.

Apart from small corrections, which arise in part from friendly criticisms, the present edition is an unaltered reprint of the first. May this also participate in the favourable reception offered to the former.

THE AUTHOR.

BERLIN, *April*, 1900.

TRANSLATOR'S PREFACE.

The need for a book of this nature, which is sufficiently indicated in the author's preface, is perhaps not less in England than in Germany. It may therefore be permissible to hope that the translation will approach the success of the original. A number of misprints contained in the German edition have been removed and the proof-sheets have been submitted to the author, who has made certain additions and corrections. I trust therefore that the book may be found reliable and accurate.

A. C. WRIGHT.

December, 1902.

PREFACE TO THE SECOND ENGLISH EDITION.

A number of arithmetical and printers' errors have been corrected and conversion diagrams have been appended by means of which the quantities in metric units may be readily converted into British units. In using the tables given in this book for practical problems, it should be remembered that in many of the tables a larger number of significant figures is given than the formulæ upon which they are based can justify; in most practical calculations three significant figures are all that can be relied upon and that should be employed.

October, 1916.

[stamp: DIRECTOR OF INDUSTRIES]

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Metres.	Deci-metres.	Centi-metres.	Milli-metres.	Inches.	Metres.	Deci-metres.	Centi-metres.	Milli-metres.	Inches.
·001	·01	·1	1	·039	·06	·6	6	60	2·362
·002	·02	·2	2	·079	·07	·7	7	70	2·756
·003	·03	·3	3	·118	·08	·8	8	80	3·150
·004	·04	·4	4	·157	·09	·9	9	90	3·543
·005	·05	·5	5	·197	·1	1	10	100	3·94
·006	·06	·6	6	·236	·2	2	20	200	7·87
·007	·07	·7	7	·276	·3	3	30	300	11·81
·008	·08	·8	8	·315	·4	4	40	400	15·75
·009	·09	·9	9	·354	·5	5	50	500	19·69
·01	·1	1	10	·394	·6	6	60	600	23·62
·02	·2	2	20	·787	·7	7	70	700	27·56
·03	·3	3	30	1·181	·8	8	80	800	31·50
·04	·4	4	40	1·575	·9	9	90	900	35·43
·05	·5	5	50	1·968	1	10	100	1,000	39·37

WEIGHT.

1 gramme = 15·44 grains.
28⅜ grammes = 1 oz. avoird.
kilogramme = 1,000 „ = 2·20 lb. avoird.

LENGTH.

1 metre = 100 centimetres = 39·37 inches. Roughly speaking, 1 metre = a yard and a tenth. 1 centimetre = two-fifths of an inch. 1 kilometre = 1,000 metres = five-eighths of a mile.

VOLUME.

1 cubic metre = 1,000 litres = 35·32 cubic feet. 1 litre = 1,000 cubic centimetres = ·2202 gall.

HEAT.

1 calorie = 3·96 British thermal units.

COMPARISON BETWEEN FAHRENHEIT AND CENTIGRADE THERMOMETERS.

C.	F.	C.	F.	C.	F.	C.	F.	C.	F.
−25	−13	5	41	25	77	65	149	105	221
−20	−4	8	46·4	30	86	70	158	110	230
−17	1·4	10	50	35	95	75	167	115	239
−15	5	12	53·6	40	104	80	176	120	248
−10	14	15	59	45	113	85	185	125	257
−5	23	17	62·6	50	122	90	194	130	266
0	32	18	64·4	55	131	95	203	135	275
1	33·8	20	68	60	140	100	212	140	284

To Convert:—

Degrees C. to Degrees F., multiply by 9, divide by 5, then add 32.

Degrees F. to Degrees C., first subtract 32, then multiply by 5 and divide by 9.

[See also Appendix.]

SYMBOLS AND CONTRACTIONS.

Atmos. = atmospheres.

a_l = volume, in litres, of 1 kilo. of air.

α = coefficient of expansion of air.

B = height of the barometer in metres of water.

b = height of the barometer in mm. of mercury.

β = the ratio $\frac{J}{V_g}$

$= \frac{\text{useful volume of the air-pump}}{\text{volume of vessel}}$.

C = calories.

C_c = " in condensing.

C_e = " " heating.

C_k = " " cooling.

C_l = $C_e + C_v$ calories removed by air.

C_v = calories in evaporating.

C_I, C_{II}, C_{III}, C_{IV} = losses of heat, in calories, by the elements of the quadruple-effect evaporator.

c = total heat in 1 kilo. of water vapour.

c_1, c_2, c_3, c_4 = heat in 1 kilo. of steam in the elements of the quadruple evaporator.

Dia. = diameter.

D = weight of steam, in kilos.

D_e = total weight of extra steam in the multiple evaporator.

d = diameter in metres.

Δ = diameter of the condenser.

δ = thickness of a plate of metal film, jet or drop of water, in mm.

ϵ = the ratio $\frac{V_s}{J} = \frac{\text{dead space}}{\text{useful volume}}$ of the air-pump.

e = weight of extra steam, in kilos., withdrawn from the elements of the multiple-effect evaporator.

E = weight of ice in kilos.

η = depth, in mm., to which heat penetrates into a body of water.

F = weight of a liquid, in kilos.

F_k = " of the cold liquid.

F_w = " of the warm liquid.

G = " of a drop in kilos.

g = acceleration due to gravity.

γ_d = weight, in kilos., of 1 cubic metre of steam.

γ_l = weight, in kilos., of 1 c. metre of air.

H = heating or cooling surface in sq. metres.

H = height of the water-barometer.

H_c = cooling surface for condensing.

H_e = heating surface for warming.

H_k = cooling surface for cooling.

H_v = heating surface for evaporating.

h = vertical height (fall) in metres.

h = head of water.

h_s = height of splash of evaporating liquids.

J = space traversed by the piston of the air-pump.

i = volume of a mass of water, in cub. mm.

k = coefficient of transmission of heat, for 1 sq. m., 1 hour, 1° C.

k_c = coefficient of transmission of heat in condensing.

k_h = coefficient of transmission of heat in heating.

k_k = coefficient of transmission of heat in cooling.

k_v = coefficient of transmission of heat in evaporating.

k_l = coefficient of transmission of heat between air and steam or water.

kilo. = kilogram.

L = weight of air in kilos.

l = length in metres.
l = „ of fall-pipe in metres.
λ = coefficient of conduction of heat.
λ = coefficient of friction in tubes.
m. = metre.
mm. = millimetre.
n = number of holes in the perforated plate.
O = surface in sq. metres.
o = „ of a mass of water in sq. mm.
P = pressure in kilos.
p = „ „ „ per sq. cm.
p_a = „ of the atmosphere.
p_e = final pressure in the vessel.
p_n = pressure in the air-pump after n half strokes.
p_0 = the lowest pressure which the air-pump can create.
p_s = pressure in the air-pump after equalisation of pressure.
p_∞ = pressure in the air-pump after an infinite number of strokes.
Q = section or plane surface in sq. m.
q = section of a pipe in sq. cms.
r = percentage of solids in a liquid.
r_1, r_2, r_3, r_4 = percentage strengths of the liquor in the elements of the quadruple effect.
r_a = percentage strength of the evaporated liquor.
sq. cm. = square centimetre.
sq. dcm. = „ decimetre.
sq. m. = „ metre.
s = space traversed by a falling body in m.
s_d = specific gravity of steam at constant pressure.
s_f = specific gravity of the liquid.
s_w = space traversed by a drop under the action of a force.
s_p = space traversed by a drop under the action of the force P.
σ_d = specific heat of steam.
σ_e = „ „ „ ice.
σ_{f1} = „ „ „ a liquid.
σ_{f2} = „ „ „ a second liquid.
σ = „ „ „ air at constant pressure.
σ_k = specific heat of the cold liquid.
σ_w = „ „ „ „ hot „
σ_v = „ „ „ air at constant volume.
T = absolute temperature.
t = temperature in °C.
t_a = temperature at commencement.
t_e = „ „ end.
t_d = „ of steam.
t_f = „ „ liquid.
t_{fa} = „ „ „ at the commencement.
t_{fe} = temperature of liquid at the end.
t_{fk} = „ „ the cold liquid.
t_{fw} = „ „ „ hot „
t_{la} = „ „ „ air at the commencement.
t_{le} = temperature of air at the end.
t_m = mean temperature.
t_{ka} = temperature of the cold liquid at the commencement.
t_{ke} = temperature of the cold liquid at the end.
t_u = temperature at the bottom of the evaporating apparatus.
t_0, t_1, t_2, t_3, t_4, = temperatures of the steam in the elements of the quadruple effect.
t_{em} = mean increase in temperature.
t_{ec} = mean increase in temperature of a jet of water.
t_{ek} = mean increase in temperature of a drop of water.
t_{ep} = mean increase in temperature of a water surface (sheet).
θ = temperature difference.
θ_a = „ „ at the commencement.
θ_e = temperature difference at the end.
θ_m = mean temperature differsnce.
θ_{mc} = „ „ „ in condensing.
θ_{mk} = mean temperature difference in cooling.
$\theta_{m1}, \theta_{m2}, \theta_{m3}, \theta_{m4}$ = mean temperature differences in the elements of the quadruple effect.
U = the residual weight of an evaporated liquid.
V_a = volume of the "equaliser" channel of the air-pump.
V_d = volumes of the steam in litres.
V_f = „ „ „ liquid „ „
V_{df} = „ „ „ steam and liquid in litres.
V_q = volume of a vessel in litres.
V_l = „ „ the air.
V_s. = „ „ „ dead spaces of the pump.
V_w = volume of water in litres.
v = velocity in metres.
v_d = „ of the steam.

v_{fl} = velocity of a liquid.
v_{f2} = " " " second liquid.
v_l = " " the air.
v_t = " " a drop.
v_w = " " the water.
W = weight of water in kilos.
w = the weight of water evaporated by 1 sq. m. of heating surface.

z_d = loss of pressure of steam in pipes.
z_l = " " " " air "
z^h = time in hours.
z_s = " " seconds.
χ_{va} = volumetric efficiency of the air-pump (adiabatic).
χ_{vio} = volumetric efficiency of the air-pump (isothermal).

CHAPTER I.

THE COEFFICIENT OF TRANSMISSION OF HEAT, k; AND THE MEAN TEMPERATURE DIFFERENCE, θ_m.

THE unit of heat, the calorie, is the quantity of heat required to heat 1 kilo. of water through 1° C. The necessary number of units of heat, or calories, in each case will be represented in what follows by the symbol C.

The coefficient of transmission of heat is the figure which gives the number of units of heat (calories) which pass in one hour from a warmer to a colder fluid through 1 sq. m. of the partition (or of surface, in case of direct contact) when the difference in temperature between the warmer and colder fluids is 1° C. This coefficient is represented by k. Without a knowledge of this quantity the calculation of the necessary heating and cooling surface in any case is impossible. Its magnitude varies greatly in different cases, but unfortunately it has not been found for every case by exact experiment. It will be a part of our task to fix it for various conditions, according to known and reliable data or on the ground of the author's own observations, so far as the present state of knowledge permits.

It is generally assumed that the transmission of heat through metal divisions between steam, gases and liquids, is proportional to the difference in temperature between the substances on each side of the division or surface. However, the temperature of the substances themselves is not always the same at all parts of a surface, for high pressure steam loses a portion of its pressure and temperature towards the end of the surface; gases or liquids in motion, heating or being heated, enter cold and leave hot.

In calculations only *one* temperature can be used and that is the mean; hence it is necessary to ascertain what is the mean difference in temperature in each case between the heating and the heated substance. The mean temperature difference is not perhaps always the arithmetic mean of the least and greatest temperature difference, that

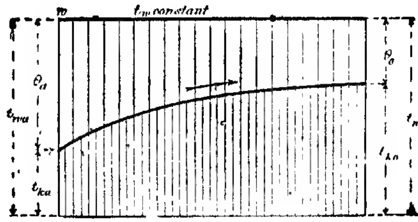

FIG. 1.

is only approximately correct when the least temperature difference is at least half as large as the largest. Thus, in general, the arithmetic mean between the smallest and largest temperature differences cannot be taken as the correct mean temperature difference.

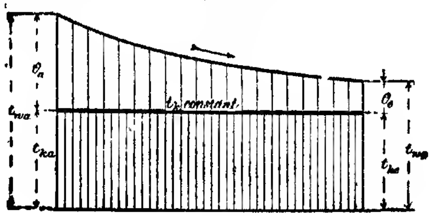

FIG. 2.

Let t_{wa} denote the initial temperature, t_{we} the final temperature of the warmer liquid; and t_{ka} the initial, t_{ke} the final temperature of the colder liquid. Then four separate cases may occur:—

1. The warmer liquid has a constant temperature $t_{wa} = t_{we} = t_w$ and the colder liquid changes from t_{ka} to t_{ke} (Fig. 1).

2. The colder liquid has a constant temperature $t_{ka} = t_{ke} = t_k$ and the hotter liquid changes from t_{wa} to t_{we} (Fig. 2).

3. Both liquids change in temperature; they flow parallel to one another over the two sides of the hot surface (parallel currents); t_{wa} changes to t_{we}, and t_{ka} to t_{ke} (Fig. 3).

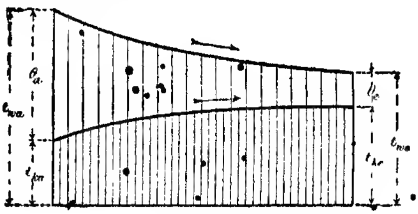

FIG. 3.

4. Both liquids change in temperature; they flow in opposite directions over the hot surface (opposite currents); the temperatures change as in 3 (Fig. 4).

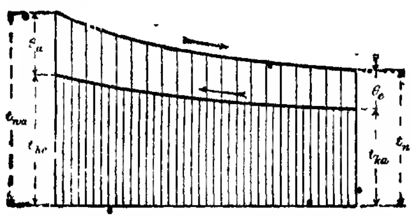

FIG. 4.

The mean difference in temperature between the liquids is then, according to Grashof, *Theoretische Maschinenlehre I.*:—

$$1.\ \theta_m = \frac{t_{ke} - t_{ka}}{\log \frac{t_{wa} - t_{ka}}{t_{we} - t_{ke}}} \quad \ldots\ldots\ldots \quad (1)$$

$$2.\ \theta_m = \frac{t_{wa} - t_{we}}{\log \frac{t_{wa} - t_k}{t_{we} - t_k}} \quad \ldots\ldots\ldots \quad (2)$$

$$3.\ \theta_m = \frac{(t_{wa} - t_{ka}) - (t_{we} - t_{ke})}{\log \frac{t_{wa} - t_{ka}}{t_{we} - t_{ke}}} \quad \ldots\ldots\ldots \quad (3)$$

$$4.\ \theta = \frac{(t_{wa} - t_{ke}) - (t_{we} - t_{ka})}{\log \frac{t_{wa} - t_{ke}}{t - t}} \quad \ldots\ldots\ldots \quad (4)$$

If θ_a = the difference in temperature between the two liquids at the commencement, and

θ_e = the difference in temperature between the two liquids at the end,

then it may at once be seen, by a glance at the four diagrams (Figs. 1-4), that the four equations may be written:—

$$\theta_m = \frac{\theta_a - \theta_e}{\log \frac{\theta_a}{\theta_e}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\theta_m = \frac{\theta_a - \theta_e}{\log \frac{\theta_a}{\theta_e}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\theta_m = \frac{\theta_a - \theta_e}{\log \frac{\theta_a}{\theta_e}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\theta_m = \frac{\theta_a - \theta_e}{\log \frac{\theta_a}{\theta_e}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The equations thus all reduce to the same form, so that the determination of the mean temperature difference for all cases is considerably facilitated.

Now we may evidently express the smaller difference in temperature as a fraction or percentage of the larger. If we suppose the larger temperature difference to be θ_a, which is manifestly permissible, and the smaller θ_e, then

$$\theta_e = \frac{p}{100} \theta_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and the equation applicable in all cases then reads

$$_m = \frac{\theta_a \left(1 - \frac{p}{100}\right)}{\log \frac{100}{p}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

By means of equation (10) we can obtain the mean difference in temperature θ_m between two fluids, each of which is occupied in modifying the temperature of the other, if the largest difference in temperature at their first contact, θ_a, and the smallest difference in temperature at the end of contact, θ_e, are known, by first determining what percentage of θ_a is the difference θ_e.

Example.—In an opposite current condenser the cold liquid enters at $t_{ka} = 10°$ C. and leaves at $t_{ke} = 80°$ C. The hot liquid enters at $t_{wa} = 100°$ C. and leaves at $t_{we} = 50°$ C. what is the mean difference in temperature θ_m?

The largest difference in temperature is $\theta_a = 50° - 10° = 40°$; the smallest difference in temperature is $\theta_e = 100° - 80° = 20°$; thus

$$\theta_e \text{ is } \frac{100 \times 20}{40} = 50 \text{ per cent. of } \theta_a, \text{ or } p = 50.$$

$$\text{Then } \theta_m = \frac{40\left(1 - \frac{50}{100}\right)}{\log \frac{100}{50}} = \frac{20}{0.6931} = 28.85° \text{ C.}$$

In Table 1 are given the values of the mean difference in temperature θ_m for the case that the largest difference in temperature $\theta_a = 1$ and the smallest $\theta_e = 0.01\theta_a$ to $1.00\theta_a$. In any individual case, in order to find the correct mean temperature difference, it is only necessary to multiply the proper figure of column 4 by the greatest temperature difference θ_a of the particular case.

The mean difference in temperature of two fluids in motion, engaged in an exchange of heat, may also be obtained in the following manner:—

If we consider the whole heating or cooling surface (surface of separation) divided into n parts, in such a manner that the moving fluids are in contact with each part during an equal time (the nth part of the whole duration of contact z), then the increase in temperature of the colder fluid is directly proportional to the difference in temperature in each division.

If, in the first division, during the time $\frac{z}{n}$ at the temperature difference θ_a, this difference is diminished by the part $x\theta_a$, then in the second division the diminution of the difference in temperature will be

$$\theta_1 = (\theta_a - x\theta_a)x = x\theta_a(1 - x) \quad . \quad . \quad . \quad . \quad . \quad (11)$$

In the third division the decrease in the temperature difference will be

$$\theta_2 = \theta_a - x\theta_a - x\theta_a(1 - x) = x\theta_a(1 - x)^2 \quad . \quad . \quad . \quad (12)$$

Similarly, in the fourth,

$$\theta_3 = x\theta_a(1 - x)^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and in the last or nth layer

$$\theta_{n-1} = x\theta_a(1 - x)^{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14$$

Since in each division the increase or decrease of temperature is always only a fraction of the total difference, it follows that in the last division only a part of the still remaining difference in temperature will be removed, so that complete equalisation of the temperatures of the two fluids cannot occur according to this finite conception.

If we suppose that the final difference in temperature between the liquids is θ_e, then $\theta_a - \theta_e$ is the *sum* of the diminutions of the temperature difference produced in the n divisions. Thus

$$\theta_a - \theta_e = x\theta_a\{1 + (1-x) + (1-x)^2 + (1-x)^3 + \ldots + (1-x)^{n-1}\} \quad (15)$$

or, summing the geometrical progression,

$$\frac{\theta_a - \theta_e}{\theta_a} = \frac{x\{(1-x)^n - 1\}}{(1-x) - 1} = \frac{x\{(1-x)^n - 1\}}{-x} = \frac{(1-x)^n - 1}{-1} \quad (16)$$

therefore

$$\frac{\theta_e}{\theta_a} = (1-x)^n \quad \ldots \ldots \quad (17)$$

$$(1-x) = \sqrt[n]{\frac{\theta_e}{\theta_a}} \quad \ldots \ldots \quad (18)$$

$$x = 1 - \sqrt[n]{\frac{\theta_e}{\theta_a}} \quad \ldots \ldots \quad (19)$$

The figure x (always a proper fraction) gives the fraction of θ_a by which the temperature difference has been diminished at the end of the first layer.

As will be seen later, there is a reason for ascertaining the value of $(1-x)$ and for knowing the temperature difference even at the end of the first layer. These values are accordingly given in Table 1, columns 2 and 3.

The value of θ_e may be expressed as a percentage of θ_a, thus in Table 1 the figures are given for $\frac{\theta}{\theta_a}$ under the assumption of $n = 100$ layers, which affords a very close approximation to reality.

After finding in this manner the diminution in the difference of temperature in the first layer, $x\theta_a$, it is necessary to find the *average* temperature difference between the fluids during the *whole* period of the transference of heat.

At the commencement of the uppermost layer the temperature difference $= \theta_a \quad \ldots \quad (20)$

" " " " " next lower layer the temperature difference $= \theta_1 = \theta_a - \theta_a x = \theta_a(1-x) \quad \ldots \quad (21)$

TABLE 1.

The Mean Temperature Difference, θ_m, between two liquids (or between steam or air and liquid), which alter their temperatures during the exchange of heat.

1	2	3	4	1	2	3	4
$\frac{\theta_e}{\theta_a}$	$1-x=\sqrt[n]{\frac{\theta_e}{\theta_a}}$	$x=1-\sqrt[n]{\frac{\theta_e}{\theta_a}}$	Mean temp. diff., θ_m, for $\theta_a=1$	$\frac{\theta_e}{\theta_a}$	$1-x=\sqrt[n]{\frac{\theta_e}{\theta_a}}$	$x=1-\sqrt[n]{\frac{\theta_e}{\theta_a}}$	Mean temp. diff., θ_m, for $\theta_a=1$
0·0025	0·9400	0·0600	0·166	0·20	0·98404	0·01596	0·500
0·005	0·9482	0·0518	0·188	0·21	0·98452	0·01548	0·509
0·01	0·9550	0·0450	0·215	0·22	0·98497	0·01503	0·518
0·02	0·9615	0·03845	0·251	0·23	0·98541	0·01459	0·526
0·03	0·96554	0·03446	0·277	0·24	0·98583	0·01417	0·535
0·04	0·96833	0·03167	0·298	0·25	0·98623	0·01377	0·544
0·05	0·97048	0·02952	0·317	0·30	0·98802	0·01198	0·583
0·06	0·97226	0·02773	0·335	0·35	0·98957	0·01043	0·624
0·07	0·97376	0·02624	0·352	0·40	0·99088	0·00912	0·658
0·08	0·97506	0·02494	0·368	0·45	0·99205	0·00795	0·693
0·09	0·97621	0·02379	0·378	0·50	0·99309	0·00691	0·724
0·10	0·97724	0·02276	0·391	0·55	0·99404	0·00596	0·756
0·11	0·97817	0·02183	0·405	0·60	0·99491	0·00509	0·786
0·12	0·97902	0·02098	0·418	0·65	0·99570	0·00430	0·815
0·13	0·97980	0·02020	0·430	0·70	0·99644	0·00356	0·843
0·14	0·98053	0·01947	0·440	0·75	0·99713	0·00287	0·872
0·15	0·98132	0·01868	0·451	0·80	0·99777	0·00223	0·897
0·16	0·98184	0·01816	0·461	0·85	0·99837	0·00162	0·921
0·17	0·98244	0·01756	0·466	0·90	0·99895	0·00105	0·953
0·18	0·98300	0·01701	0·478	0·95	0·99949	0·00051	0·982
0·19	0·98353	0·01647	0·489	1·00	1·00000	0·00000	1·000

At the commencement of the third layer the temperature difference $= \theta_2 = \theta_a(1-x)^2$. (22)

" " " " " last layer the temperature difference $= \theta_n = \theta_{n-1}(1-x)^{n-1}$ (23)

The sum of the temperature differences is thus

$$S = \theta_a\{1 + (1-x) + (1-x)^2 + (1-x)^3 \ldots + (1-x)^{n-1}\} \quad (24)$$

and the *mean* temperature difference is the *n*th part of this sum.

$$\theta_m = \frac{\theta_a\{(1-x)^n - 1\}}{n\{(1-x) - 1\}} \quad . \; . \; . \; . \; . \quad (25)$$

Inserting for $(1 - x)^n$ the value from equation (17), we obtain

$$\theta_m = \frac{\theta_a\left(\frac{\theta_e}{\theta_a} - 1\right)}{n\left(\sqrt[n]{\frac{\theta_e}{\theta_a}} - 1\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Since $\frac{\theta_e}{\theta_a}$ is always a proper fraction, the right hand side may be multiplied by -1, thus giving

$$\theta_m = \frac{\theta_a\left(1 - \frac{\theta_r}{\theta_a}\right)}{n\left(1 - \sqrt[n]{\frac{\theta_r}{\theta_a}}\right)} = \frac{\theta_a - \theta_r}{n\left(1 - \sqrt[n]{\frac{\theta_r}{\theta_a}}\right)} \quad . \quad . \quad . \quad . \quad (27)$$

The results obtained by calculating the mean temperature difference by means of equation (27) are given in Table 1, column 4, and differ very little from those given by equation (10).

OR OF INDU

CHAPTER II.

PARALLEL AND OPPOSITE CURRENTS.

Two liquids, gases, or vapours, one of which is to transfer heat to the other, may be conducted either in the same or in opposite directions over the surface of separation. If the two fluids move parallel to one another in the same direction, the condition is known as that of "parallel currents".

If, however, they move in opposite directions, the condition is that of "opposite currents".

In the case of parallel currents, the fluid to be cooled has its highest temperature at the commencement, the liquid to be heated its lowest temperature; at the end the reverse is the case.

In the case of opposite currents the fluid to be cooled and *also* that to be heated have their highest temperatures at one end, and their lowest temperatures at the other.

In all cases the quantity of heat lost by one fluid is exactly the same as that gained by the other.

If F_w is the weight and σ_w the specific heat of the originally hot fluid, F_k the weight and σ_k the specific heat of the originally cold fluid, and, further, if t_{wh} and t_{wn} be the highest and lowest temperatures of the originally hot fluid and t_{kh} and t_{kn} the highest and lowest temperatures of the originally cold fluid, then, always,

$$F_w\sigma_w(t_{wh} - t_{wn}) = F_k\sigma_k(t_{kh} - t_{kn}) \quad . \quad . \quad . \quad . \quad (28)$$

Thus the weight of cooling liquid, F_k, necessary to cool the weight F_w of the hot fluid from t_{wh} to t_{wn} is

$$F_k = \frac{F_w\sigma_w(t_{wh} - t_{wn})}{\sigma_k(t_{kh} - t_{kn})} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

In every definite case F_w, σ_w, σ_k, t_{wh}, t_{wn}, t_{kn}, are known; the outflow temperature t_{kh} of the cooling liquid varies with its quantity, and this quantity is greater the lower t_{kh} is.

In the case of opposite currents, the cooling medium may flow away at a temperature only slightly lower than the *highest* temperature of the hot fluid. In the case of parallel currents the cooling medium must always run off at a temperature lower than the *lowest* temperature of the hot fluid. Thus t_{kh} is always lower with parallel than with opposite currents, accordingly it follows that, with parallel currents, much more cooling liquid (generally water) must be used than with opposite currents.

Similarly, in order to heat a cold fluid F_k by means of a hot fluid F_w, much more hot fluid must be used with parallel than with opposite currents.

In the case of parallel currents the greatest difference in temperature occurs between the highest temperature of the hot and the lowest temperature of the cold fluid, the smallest difference in temperature between the lowest temperature of the warm and the highest temperature of the cold fluid. The first-named difference is the greatest which arises under any conditions, the second is always very much less, which is also the case with opposite currents. Since with opposite currents the highest possible temperature difference can never occur, it follows at once, in general, that the mean difference in temperature is greater with parallel than with opposite currents, and, consequently, that in the former case the necessary heating or cooling surface may almost always be smaller than in the latter case. An opposite current apparatus is thus always larger than a parallel current apparatus, but is cheaper to work, and in particular, with similar materials, permits the attainment of higher temperatures in heating apparatus and lower temperatures in cooling than is possible to obtain with parallel currents.

Heating and cooling apparatus should always be constructed for opposite currents.

The following table (2) gives the dimensions of the hot surfaces necessary for cooling 100 kilos. of an aqueous liquid from 100° C. to 50°, 40°, 30°, 20°, and 15° C. by means of water at 10° C. The water is supposed to leave the parallel currents apparatus 5° below the temperature of the *cooled* liquid, and the opposite current apparatus at 80° C. (*i.e.*, 20° below the temperature of the *hot* liquid).

Let us now consider an opposite current apparatus, upon one side of which a liquid is cooled from 100° to 10°, whilst on the other side a larger quantity of another liquid of equal specific heat is heated

TABLE 2.

Dimensions of the heating surfaces with parallel and opposite currents.

Final temp. of the cooled liquid.	Parallel Currents.				Opposite Currents.			
	Final temp. of the cooling water.	Quantity of cooling water.	Mean temp. diff.	Cooling surface.	Final temp. of the cooling water.	Quantity of cooling water.	Mean temp. diff.	Cooling surface.
°C.	°C.	Litres.	θ_m.	Sq. m.	°C.	Litres.	θ_m.	Sq. m.
50	45	140	29·7	0·7	80	72	29	0·70
40	35	240	,,	0·8	,,	86	24·6	0·95
30	25	465	,,	0·9	,,	100	20	1·35
20	15	1600	,,	1·05	,,	115	14·5	2·20
15	12	4250	,,	1·15	,,	122	10·88	3·10

from 5° to 50°, the rates of flow of the two liquids being constant but unequal. Fig. 5 gives a representation of the proportion of

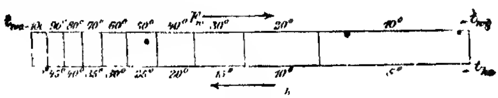

FIG. 5.

the sections of the cooling surface. In order to carry over equal quantities of heat in each section, those sections, which lie between small differences in temperature, must be much larger than those which lie between large differences in temperature.

CHAPTER III.

APPARATUS FOR HEATING WITH DIRECT FIRE.

INSTALLATIONS for heating with a direct fire are described in detail in many excellent works; in this place only a few important remarks will be briefly recapitulated.

The weight of fuel burnt upon a certain grate in a definite time, the quantity of useful heat obtained therefrom, and that which passes through 1 sq. metre of the hot surface to be heated, the temperatures of the gases produced—in fact all the conditions, actions and results of a heating apparatus—are very variable, depending on the demands made upon it, the skill with which it is tended, and the quality of the materials. This is the more true, the smaller the apparatus.

Since there is no intention to treat of firing in detail, the data collected in Table 3 must be regarded merely as useful landmarks.

The quantity of heat passing in one hour through 1 sq. m. of boiler surface increases in direct proportion with the difference in temperature between the liquid and the flue gases, and also probably with the square and cube root of the velocity with which the liquid and flue gases respectively pass along the wall. It diminishes, however, with the growth of the coating of soot and dust on the outside of the heating surface and of boiler-scale on the inside.

The mean difference in temperature is naturally less, and the transmission of heat per hour through 1 sq. m. correspondingly less, the colder the flue gases leave the boiler, but the economy in fuel is then proportionately greater.

The true coefficients of transmission for this case are not yet known with sufficient accuracy; many and varied experiments (which are still lacking) would be required to determine them. But a knowledge of these figures would not be of very great service, since the

conditions which hinder the transmission of heat are very numerous and variable, and cannot be accurately taken into account either before or after construction. Thus it is necessary to be satisfied with applying the results of practical observations.

If k be the coefficient of transmission of heat, which gives the number of units of heat (calories) passing through 1 sq. m. in one hour with the total difference in temperature, then we may reckon that with steam boilers $k = 8{,}000$ to 12,000 calories; in the mean, $k = 9{,}000$ calories.

For heating surfaces, on which the liquid is not boiled, surrounded by the gases of combustion, $k = 6{,}000$ to 10,000 calories; in the mean, $k = 7{,}000$ calories.

In the case of very small boiler surfaces, transmission of 18,000-20,000 calories may occur, yet this high efficiency causes wet steam, and does not generally result in economy of fuel.

Researches on the transmission of heat from flue gases and air to water which does not boil have been performed by Joule and Ser; they show that the transmission is probably proportional to the square root of the velocity of the gases or air, v_{l}, and that the coefficient k_l for *clean* wrought iron pipes is approximately

$$k_l = 16\sqrt{v_l} \text{ to } k_l = 19\sqrt{v_l} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Having regard to the coating of the heating surface with substances which hinder the transmission of heat, which always occurs in practice, we shall assume for this case the coefficient of transmission

$$k_l = 2 + 10\sqrt{v_l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

in so far as it refers to pure air. If the liquid is heated by flue gases, on account of the greater amount of coating in unfavourable cases, it is necessary to take

$$k_l = 2 + 5\sqrt{v_l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

In the mean, for this case, k_l may be taken as about 13.

By means of this figure the following small table (4) has been calculated; it shows how large the heating surface must be in order to heat in the boiler-flue, in one hour, 100 litres of water from 10° or 15° to 80° or 130° C., when the flue gases reach the economiser at a temperature of 300°-400° C. and are there cooled to 150° or 300°, by giving out heat.

TABLE 3. **The Properties of**

	Wood, air-dried.	Peat.	Earthy Lignite.	Coal, long flame.	Coal, bituminous.
Weight of 1 cub. m. - - kilos.	370-465	260-380	610-700	740	—
Temperature of the flame °C.	1969	2149	2357	2595	2664
Temperature with a double quantity of air - - °C.	800-1000	900-1200	900-1200	1000-1300	1000-1300
1 kilo. of fuel theoretically evolves calories	2820	3550	4450	6600	7500
Useful heat from 1 kilo. calories	60-80 per cent. of the theoretical				
Theoretical quantity of air for 1 kilo. of fuel cub. m.	3·46	4·04	4·88	6·97	7·78
kilos.	4·65	5·30	6·34	9·5	10·8
Quantity of air required for 1 kilo. in practice cub.m.	6·92	8·08	9·76	13·95	15·56
kilos.	9·3	10 60	12·68	15	21·6
Theoretical volume of gas from 1 kilo. cub. m. at 0° C.	4·20	4·759	5·44	7·42	8·20
„ at 300° C.	8·82	9·928	11·44	15·69	17·24
Carbonic acid in flue gas - -	10-14 per cent.				
Quantity burnt upon 1 sq. m. of grate kilos. per hour	70-120	80-120	100-200	50-120	50-120
average.	100	100	150	75	75
Ratio of openings to total grate surface - - - - - - - -	$\frac{1}{3}$-$\frac{1}{6}$	$\frac{1}{4}$-$\frac{1}{6}$	$\frac{1}{4}$-$\frac{1}{5}$	$\frac{1}{2}$-$\frac{1}{4}$	$\frac{1}{2}$-$\frac{1}{4}$
Thickness of the burning layer m. m.	250	200	150	100	100
Resistance to the draught caused by the fuel m. m.	1-4	1-4	1-4	5-12	5-12
Ash - - - - - - - per cent.	1-1·5	1-5	5-10	3-4	3-4
1 sq. m. of heating surface requires a grate of sq. m.	$\frac{1}{10}$-$\frac{1}{20}$	$\frac{1}{15}$-$\frac{1}{30}$	$\frac{1}{15}$-$\frac{1}{30}$	$\frac{1}{30}$-$\frac{1}{50}$	$\frac{1}{30}$-$\frac{1}{50}$
1 sq. m. of heating surface evaporates kilos. of water per hour	15-20 kilos.; average,				
1 kilo. of fuel evaporates kilos. of water - - - - - - -	2·5-3·5	1·5-3	2-4·5	5·5-10	5·5-10
Speed of gases in flue m. per sec.	3-4 metres per sec.—				
Section of flue - - - - sq. m.	decreasing from 0·375-				
Section of chimney - - sq. m.	$\frac{1}{6}$ of the grate			$\frac{1}{4}$ of the grate	
Height of the chimney - -m.	at least 16 metres.				
Temperature of the flue gases ° C.	250°-				

Certain Fuels. TABLE 3.

Coal, short flame.	Anthracite.	Coke.	Charcoal.	Alcohol.	Petroleum.	Masut.	Coal Gas.	Water Gas.
960	—	520-570	194	793	785	928	0·34-0·45	—
2688	2734	2774	2104	—	—	—	2390	—
1000-1300	1000-1300	—	—	—	—	—	—	—
7760	8110	7430	7750	7184	10000	—	13745	—
60-80 p.c. of the theoretical				—	—	10700	4500-7000 1 c. m. = 5500	3500
8·04	8·49	7·441	8·01	—	—	—	12	—
11·5	12·5	9·7	10·30	—	—	—	16	—
16·09	16·98	14·88	16·08	—	—	20 per cent. less than by coal	5-8 per cub. m.	—
23	25	19·4	20·6	—	—			—
8·43	8·74	8·04	8·42	—	—	—	13·6	—
17·71	18·38	16·89	17·70	—	—	—	27·5	—
10-14 per cent.				—	—	—	—	—
50-120	25-60	35-80	—	—	—	—	—	—
75	35-40	60	—	—	—	—	—	—
$\frac{1}{2}$-$\frac{1}{4}$	$\frac{1}{2}$-$\frac{1}{3}$	$\frac{1}{4}$-$\frac{1}{6}$	—	—	—	—	—	—
100	100	250	—	—	—	—	—	—
5-12	—	—	—	—	—	—	—	—
3-4	2	5·6	2·5	—	—	—	—	—
$\frac{1}{30}$-$\frac{1}{50}$	$\frac{1}{30}$-$\frac{1}{50}$	$\frac{1}{30}$-$\frac{1}{50}$	—	Straw	Tan bark	—	—	—
18 kilos.			—	—	—	—	30-35 litres heat 1 litre of water from 0°-100° C.	—
5·5-10	5·5-10	4·5-8	—	1·5-2	1-1·1	—	—	—
6 metres permissible—3-4 metres at the top of the chimney								
0·43 of the grate at the beginning to 0·25 at the end								
$\frac{1}{4}$ of the grate			—	—	—	—	—	—
otherwise 25 times the diameter of the top								
450°			—	—	—	—	—	—

TABLE 4.

Heating surface, H, required to heat 100 kilos. of water in one hour in the boiler-flue from 10° to 80°-130° C.

Water heated from	to	Temperatures of the flue gases.				
		At entry - - -	300°	250°	400°	450°
		At exit - - -	150°	200	250°	300°
10°	80°	Temp. difference, θ_m	176°	226°	268°	329°
		Heating surface, H -	3·08	2·39	2·0	1·7 sq. m.
10°	100°	Temp. difference, θ_m	170°	217°	267°	315°
		Heating surface, H -	4·07	3·2	2·65	2·0 sq. m.
10°	110°	Temp. difference, θ_m	164°	213°	261°	312
		Heating surface, H -	4·7	3·6	2·89	2·43 sq. m.
10°	120°	Temp. difference, θ_m	160°	207°	257°	311°
		Heating surface, H -	5·29	4·12	3·3	2·70 sq. m.
10°	130°	Temp. difference, θ_m	153°	206°	254°	307
		Heating surface, H -	6·03	4·48	3·7	3·0 sq. m.

Example.—In order to heat 100 litres of water from 10° to 100° C., 100 (100 - 10) = 9,000 units of heat are required. The flue gases enter the economiser at 300 and leave at 150° C., so that the temperature difference is at first 300 - 100 = 200, and at the end 150 - 10 = 140°; thus, in the mean, since $\frac{140}{200} = 0·7$, $\theta_m = 168·6$ (Table 1). The necessary heating surface is therefore

$$H = \frac{9000}{\theta_m k} = \frac{9000}{168·6 \times 13} = 4·07 \text{ sq. m.}$$

Observation (Zeits. d. V. d. I., 1888, 438).—5,197 litres of water per hour were forced with a velocity of 0·118 m. through six parallel iron pipes of 51 mm. internal diameter, which had a total heating surface of 315 sq. m. The water was heated from 48·5° to 180° C. by means of the flue gases from a marine boiler, which were thereby cooled from 338° to 149° C.

There were transmitted

$$C = 5,179\,(180 - 48·5) = 683,405 \text{ calories.}$$

The initial difference in temperature was

$$\theta_a = 338^\circ - 180^\circ = 158^\circ.$$

The final difference in temperature was

$$\theta_e = 149^\circ - 48{\cdot}5^\circ = 100{\cdot}5^\circ.$$

Thus the mean difference in temperature, $\theta_m = 126^\circ$.

The coefficient of transmission of heat was

$$k_t = \frac{C}{H\,\theta_m} = \frac{683{,}405}{315 \times 126} = 17{\cdot}2.$$

The velocity of the gases over the pipes was about 1·2 m., thus the *calculated* coefficient of transmission was

$$k_t = 2 + 10\,\sqrt{1{\cdot}2} = 13{\cdot}0.$$

CHAPTER IV.

THE INJECTION OF SATURATED STEAM.

SATURATED steam, directly injected, is used for heating water, for distilling low-boiling liquids (alcohol, methyl alcohol, etc.) and for carrying over high-boiling liquids.

If saturated steam be conducted into cold water, it liquefies and gives up its heat to the water. The previous pressure of the steam is immaterial, since it is lost in condensing. An almost complete vacuum would be produced throughout the steam pipe, owing to the sudden disappearance of the steam at the end where it enters the water, did not the steam always contain air; since, however, this is always the case, only a fall in pressure in the pipe results. The water is gradually heated by the steam and may reach 100° C., if it is under atmospheric pressure. If the water be under a higher pressure, as that of a column of water, it can reach that temperature which steam of this pressure would have.

Example.—The water in a closed vessel in the cellar of a house 20 m. high, from which rises a pipe, 20 m. long (2 atmospheres) and filled with water, may reach at the bottom the temperature of steam at a pressure of 2 atmospheres, *i.e.*, 120·6° C. The temperature of the water in the full pipe diminishes from below upwards, a circulation takes place, the warm water rising and the colder flowing down. The rising warm water, as it gradually comes under less pressure, gives off its excessive heat by forming steam.

Thus steam gives up its heat to water which is not boiling, liquefying and increasing the weight of water by its own weight. However, if the water boils, it evolves as much steam as is led into it, and its weight remains constant.

1 kilo. of steam at atmospheric pressure has 637 calories. If the temperature of the water is t, each kilo. of steam brings to it $(637 - t)$ calories.

To heat 100 kilos. of water at 0° C. (taking its specific heat as constant)

through	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°C.
there must be injected	1·60	3·24	4·87	6·71	8·53	10·4	12·4	14·4	16·5	18·6

kilos. of steam.

If steam is blown into a boiling liquid (not water), with which water *mixes*, and the boiling point of which lies below that of water, vapours are formed composed of a mixture of steam and the vapour of the liquid. The composition of these vapours depends, according to certain laws, upon the composition of the boiling mixture of liquids, but, unfortunately, is not accurately known for most mixtures of liquids, although this property is utilised on the largest scale in the industries for the distillation of such liquids. The heat of evaporation of the mixture of vapours is the sum of the heats of evaporation of the water and the liquid. The temperature of the mixture lies between those of the single vapours.

Example.—1 kilo. of a mixture of vapours, containing 0·5 kilo. of water vapour and 0·5 kilo. of alcohol vapour, is at the boiling temperature of 92° C.; 0·5 kilo. of steam at 92° contains 271 calories of heat of evaporation, and 0·5 kilo. of alcohol vapour at 92° contains 103 calories. Thus, 1 kilo. of the mixture contains 271 + 103 = 374 calories.

This question has been treated in a previous work (*Wirkungsweise der Rektifizier- und Destillir-Apparate*, Julius Springer, Berlin), which should be mentioned here.

When saturated steam is blown into a hot liquid, which *does not mix* with water, part of the liquid is mechanically taken away along with the steam, even when its boiling point is considerably above that of water. This process of carrying over small particles of liquid is not evaporation, and, according to the author's observations, the heat of evaporation of the vapours evolved is but little greater than that of the water alone.

The quantities of different liquids carried over by 1 kilo. of saturated steam are very different; they depend essentially upon the nature of the liquid, the dryness and the temperature of the steam. In almost all cases, if not exactly necessary, it is still very desirable to heat the liquid under distillation in some other manner, since by this means the work to be performed by the steam is made

considerably easier. Experience has shown that 1 kilo. of steam carries over more liquid *in vacuo* than at atmospheric pressure.

As approximate data it may be stated that to carry over

100 kilos.	of toluene	there are	required	13-15	kilos. of steam.
100 ,,	benzene	,,	,,	25-28	,,
100 ,,	fatty acids	,,	,,	100	,,
100 ,,	tar	,,	,,	150	,,
100 ,,	glycerin	,,	,,	250	,,
100 ,,	nitrobenzene	,,	,,	250-300	,,
100 ,,	nitrotoluene	,,	,,	400-450	,,

CHAPTER V.

SUPERHEATED STEAM.

THE steam superheater consists of metal pipes, through which saturated steam is led, and which are generally surrounded outside by fire. But the superheating of steam is not of necessity done by direct fire; a sand or oil-bath, or even high pressure steam, may be used. When saturated high pressure steam is allowed to expand, its temperature and pressure sink. If this expanded or low pressure steam at a low temperature is passed through pipes heated outside by hotter high pressure steam, the low pressure steam is brought up to the temperature of the high pressure steam, *i.e.*, it is superheated. It is a matter of indifference by what means the superheating is accomplished.

The specific heat of superheated steam at constant pressure, which comes into consideration here, is $\sigma_d = 0{\cdot}4805$. Thus, in order to superheat 1 kilo. of steam at 100° C. through 100° C., *i.e.*, to heat it to 200° C., there are required $100 \times 0{\cdot}4805 = 48{\cdot}05$ units of heat. Since saturated steam always contains water, the heat required to vapourise the latter and then superheat it to the same degree must also be calculated. It is important and useful to keep as low as possible the amount of water in the steam to be superheated, since the evaporation of the water requires much heat and seriously diminishes the efficiency of the superheater. But in spite of all separating arrangements, which are always used in conjunction with superheaters, the saturated steam always carries a certain quantity of water (3 - 5 - 10 per cent.) into the superheater. The heat required to vapourise this water must be calculated.

If the whole weight of steam to be superheated is D, its original temperature t, the temperature to which it is to be superheated t_u,

and the percentage of water w, then the amount of heat required for superheating is

$$C = \frac{Dw}{100}537 + D(t_u - t)0{\cdot}4805$$

and, when $t = 100°$,

$$C = D\{5{\cdot}37w + 0{\cdot}4805(t_u - 100)\} \quad . \quad . \quad . \quad . \quad (33)$$

Thus, in order to superheat 100 kilos. of steam, more or less heat is required according to the percentage of water.

Table 5 gives the number of units of heat required to superheat steam at 100° C. through 100°, 200°, 300°, 400°, 500° and 600° C., when it contains 0, 3, 5 or 10 per cent. of water.

TABLE 5.

Expenditure of heat, in calories, in order to superheat 100 kilos. of steam from 100° C. through 100° to 600° C., when it contains 0-10 per cent of water.

Water-content of the steam.	Superheating through					
	100°	200°	300°	400°	500°	600
Per cent.	Calories.	Calories.	Calories.	Calories.	Calories.	Calories.
0	4,750	9,500	14,250	19,000	23,750	28,500
3	6,361	11,111	15,861	20,611	25,361	30,111
5	7,435	12,185	16,935	21,685	26,435	31,185
10	10,120	14,870	19,620	24,370	29,120	33,870

The volume of superheated steam is, according to Zeuner,

$$pV_a = 50{\cdot}9T - 192{\cdot}5\sqrt[4]{p} \quad . \quad . \quad . \quad . \quad . \quad (34)$$

where p denotes the pressure in kilos. per sq. m., V_a the volume in cub. m. and T the absolute temperature.

In Table 6 are given the volumes, V_a, of 1 kilo. of superheated steam, in cub. m., for pressures of 0·1, 0·2, 0·5, 1, 2, 3 and 4 atmospheres and temperatures from 200° to 500° C.

The quantity of heat, which is carried to the steam through 1 sq. m. of heating surface, depends, as we may readily imagine, on the velocity with which the steam to be superheated moves along the

inner face, and the heating gases or liquids pass along the outer face of the superheater. Exact figures are, however, wanting for this transference of heat, owing to lack of accurate experiments. But if these figures were known, the coating of the surfaces with ash and rust, and also the variable and generally unknown proportion of water in the steam, would make the theoretical figures useless for practical purposes, without large corrections.

TABLE 6.

Absolute pressure.	Absolute pressure. p.	Temperature of the superheated steam, t				
		200°	250°	300°	400°	500°
		Absolute temperature of the superheated steam, T.				
Atmos.	Kilos. per sq. m.	473°	523°	573°	673°	773°
		Volumes of 1 kilo. of superheated steam, V_s in cub. m.				
0·1	1,000	23·000	25·540	27·987	33·176	38·260
0·2	2,000	11·390	12·670	13·890	16·483	19·027
0·5	5,000	4·496	5·005	5·494	6·530	7·549
1	10,000	2·215	2·469	2·714	3·233	3·741
2	20,000	1·089	1·217	1·339	1·598	1·853
3	30,000	0·718	0·803	0·884	1·057	1·227
4	40,000	0·534	0·597	0·659	0·788	0·909

Experience shows that, by means of 1 sq. m. of superheater surface in one hour, 25-45 kilos. of high pressure steam may be superheated through 100°, 150° or 200° C., when the temperature of the hot gases is 450°-550° C., the speed of the steam in the superheater being 15-40 m. per second.

This is true for those cases in which the steam is superheated by means of waste gases; when, however, the superheater lies immediately after the fire, so that the flames directly impinge on its tubes, the efficiency is considerably greater, especially with steam a little above

the atmospheric pressure. Under these circumstances, in one hour by means of 1 sq. m. of surface, as much as 300 kilos. of steam may be superheated through 200°-300° C. The velocity of the steam may then reach 60-70 m.

If the steam is expanded, *i.e.*, if it has a lower pressure than that of the atmosphere, for example, $\frac{1}{4}$ atmos. (absolute), the velocity in the pipes may attain 150, or even 400 m.; an average would be 250 m.

According to Hirn, the coefficient of transmission between hot gases and steam with cast-iron heating surfaces, $k = 10$ to 15. Assuming it to be $k = 10$, a number which must be regarded as extremely low, the heating surfaces necessary to superheat 100 kilos. of steam, containing 0-10 per cent. of water, through 50°, 100°, 200° and 300° C., with a mean difference in temperature between steam and hot gases of 100° and 150° C., have been calculated and arranged in the following table:—

TABLE 7.

Water content of the steam.	For superheating through									
	50°		75°		100°		200°		300°	
	with mean differences in temperature of									
Per cent.	100°	150°	100°	150°	100°	150°	100°	150°	100°	150°
	the necessary heating surface, in sq. m., for 100 kilos. of steam per hour.									
0	2·38	1·65	3·60	2·40	4·75	3·3	9·5	6·6	14·2	9·9
3	3·18	2·15	5·21	3·48	6·36	4·3	13·76	8·6	19·0	12·9
5	3·72	2·5	6·29	4·20	7·43	5·0	14·86	10·0	22·2	15·0
10	5·07	3·35	8·97	5·98	10·12	6·7	20·24	13·4	30·2	20·1

With the same assumption, it may be found that 1 sq. m. of the heating surface of the superheater superheats the following weights of steam in one hour:—

TABLE 8.

Water-content of the steam.	Superheating through									
	50°		75°		100°		200°		300°	
	with mean differences in temperature of									
Per cent.	100°	150°	100°	150°	100°	150°	100°	150°	100°	150°
	1 sq. m. of heating surfaces superheats kilog. of steam per hour.									
0	42·0	63·0	28·0	42·0	21·0	31·5	10·5	16	7·0	10·5
3	31·4	47·4	19·0	28·5	15·7	23·6	7·85	12	5·3	8·0
5	26·8	40·2	16·0	24·0	13·4	20·1	6·7	10	4·5	6·8
10	20	30·0	11·0	16·6	10·0	15·0	5·0	7·5	3·3	5·0

CHAPTER VI.

EVAPORATION BY MEANS OF HOT LIQUIDS.

OCCASIONALLY liquids are evaporated by means of heating coils, through which steam is not conducted, but a strongly heated liquid of high boiling point (400°-500° C.) is pumped. The rate at which this hot liquid is forced through the coil can rarely be very large, since the considerable length of the coiled pipe and its small internal diameter would otherwise largely increase the friction, and thus the necessary pressure. We may regard a velocity, v_f, of 1 m. per second as suitable, though often this is not attained.

In estimating the quantity of heat given up in this case from the hot coil to the *boiling* liquid, the coefficient of transmission may be assumed, according to the author's observations, to be

$$k_v = 700\sqrt{v_f} \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

The heating surface H in sq. m., required to transfer C calories per hour, is, with the mean temperature difference θ_m,

$$H = \frac{C}{\theta_m 700\sqrt{v_f}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

Accordingly, 1 sq. m. of heating surface in one hour, with a velocity of the heating liquid in the coil of $v_f = 1$ m., and with mean differences in temperature of

$\theta_m =$	5°	10°	15°	20°	50° C.	
would transfer	3,500	7,000	10,000	14,000	35,000	calories

to the boiling liquid.

The necessary weight of the hot liquid, F_w, which must be forced in one hour through the heating coil is, if C represents the quantity of heat to be transferred in one hour,

$$F_w = \frac{C}{\sigma_f(t_{w_1} - t_{w_2})} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36a)$$

The diameter of the coiled pipe in metres (d) is obtained from the equation

$$\frac{d^2\pi}{4}100 \times v_f \times 10 \times 3600 = \frac{F_w}{s_f}$$

or

$$d = \frac{1}{1679}\sqrt{\frac{F_w}{s_f v_f}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36b)$$

The length of the heating coil is

$$l = \frac{H}{\pi d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36c)$$

For the hot liquids considered here the specific heat, σ_f, is generally 0·5 and the specific gravity, s_f = 07.

CHAPTER VII.

THE TRANSFERENCE OF HEAT IN GENERAL AND TRANSFERENCE BY MEANS OF SATURATED STEAM IN PARTICULAR.

THE physical properties of saturated steam are the basis of many of the following considerations; a compilation of these properties, according to Zeuner, is given in Table 9.

Water and many other liquids are evaporated by means of saturated steam. The hot steam employed has usually a *pressure* of 3-5 atmospheres, but, frequently, for liquids of high boiling point, steam of 12-15 atmospheres must be used. It is often advantageous to heat with steam at a pressure of 1-2 atmospheres (absolute).

The *temperature* of the hot steam must always be some degrees higher than the boiling point of the liquid to be evaporated. The transfer of heat is greater, the larger the difference in temperature between the steam and the boiling liquid, and it may be properly assumed that the action of the heating surface increases in direct proportion with the difference in temperature, θ_m. In order to make this difference large, a vacuum is frequently maintained over the boiling liquid, *i.e.*, the liquid is brought into a closed vessel provided with heating surfaces in contact with steam, from which the vapours are conducted through a pipe into a condenser, where they liquefy and are cooled, and then either flow away spontaneously (by a barometer column), or are drawn off by means of a pump or other apparatus.

The *pressure* of the hot steam is without influence on the efficiency of the heating surface. But the *temperature*, which is in a definite connection with the pressure of saturated steam, has considerable influence, since, other things being the same, with increasing pressure the temperature of the steam also rises to an extent which is perfectly well known, and thus proportionately increases the difference in tem-

perature between steam and liquid. In this sense the capacity of the heating surface rises with the pressure of the steam.

By many researches it has been shown that with *increasing temperature of the steam,* or, in general, with an increase in the temperature at which the transference of heat takes place, there is a certain increase in the efficiency; this effect is, however, not proportional to the increase in temperature, and appears again to decrease when certain limits of temperature are exceeded. The cause of this behaviour is to be found in the increasingly rapid movement of the particles of liquid over the heated surface at the higher temperatures. The effect is more noticeable in heating non-boiling liquids by means of saturated steam, than in evaporating.

The hot steam always carries air with it (Zeits. d. V. d. Ing., 1887, 284), which considerably hinders the transference of heat. It appears as if the air attached itself to the hot surface, forming a net-like layer upon it, thus hindering the action of the steam. The removal of the air from the tubes or spaces, in which the steam is to give out its heat, is extremely important for effective working. Every care must be taken to remove, as quickly and completely as possible, the air which the steam brings to the hot spaces. It naturally collects where it is driven by the moving steam, that is, at the end of the heating surface. At that place there must be provided a continuous outlet, and since diffusion between air and steam is tolerably slow, the outlet should be placed rather towards the bottom than the top of the hot space.

The pressure in the hot space is *the sum of the pressures of air and steam.* The total pressure in the steam space is, therefore, always rather greater than the pressure of the steam alone, and since the temperature (the most important condition) in the hot space depends upon the pressure of the steam and not on the sum of the pressures, the temperature in a steam space is always somewhat lower than would be supposed from the total pressure as indicated by a gauge. In heating experiments it is, therefore, necessary to observe the *temperature* of the hot steam and not its pressure, since the latter, on account of the varying amount of air, cannot give a reliable indication of the temperature.

The pressure and temperature of the steam are not equal in all parts of the steam space; they are always somewhat, often much, lower at the end of the heating surface than at the beginning. When

TABLE 9. Saturated Water Vapour—Pressure; Total Evaporation; Specific Volume

Pressure.			Vacuum.		Temperature.
Atmospheres, absolute.	Mercury.	Water.	Mercury.	Water.	
	mm.	m.	cm.	m.	° C.
0·0061	4·60	0·063	75·540	10·273	0
0·0086	6·53	0·089	75·347	10·247	5
0·012	9·17	0·124	75·038	10·212	10
0·017	12·70	0·176	74·730	10·160	15
0·023	17·39	0·238	74·261	10·098	20
0·031	23·55	0·320	73·645	10·016	25
0·042	31·55	0·434	72·845	9·902	30
0·055	41·83	0·568	71·817	9·768	35
0·072	54·91	0·744	70·509	9·592	40
0·094	71·39	0·972	69·861	9·364	45
0·121	91·98	1·251	66·802	9·085	50
0·155	117·48	1·602	64·252	8·734	55
0·196	148·79	2·026	61·121	8·310	60
0·246	186·95	2·543	57·305	7·793	65
0·257	195·50	2·656	56·450	7·680	66
0·303	233·09	3·163	52·601	7·173	70
0·380	288·55	3·928	47·148	6·408	75
0·466	354·64	4·817	40·536	5·519	80
0·506	384·44	5·230	37·556	5·106	82
0·570	433·04	5·892	32·696	4·441	85
0·691	525·45	7·142	23·455	3·194	90
0·746	566·76	7·711	19·342	2·625	92
0·834	633·78	8·602	12·622	1·706	95
1·000	760·00	10·336	0	0	100
1·25	950	12·920			106·38
1·50	1140	15·50			111·74
1·75	1330	18·09			116·42
2·00	1520	20·67			120·60
2·25	1710	23·26			124·35
2·50	1900	25·84			127·80
2·75	2090	28·42			130·96
3·00	2280	31·00			133·91
3·50	2660	36·18			139·24
4·00	3040	41·34			144·00
4·50	3420	46·51			148·29
5·00	3800	51·68			152·22
6·00	4560	62·02			159·22
7·00	5320	72·35			165·34
8·00	6080	82·69			170·81
9·00	6840	93·02			175·77
10·00	7600	103·36			180·31
11·00	8360	113·70			184·50
12·00	9120	124·03			188·41
13·00	9880	134·37			192·08
14·00	10640	144·70			195·53
15·00	11400	155·04			198·98

Heat; Heat of the Water, of the Liquid and of and Weight (after Zeuner). TABLE 9.

Latent heat of the vapour, $606{\cdot}5 - 0{\cdot}595t - 0{\cdot}00002t^2 - 0{\cdot}0000003t^3$. Calories.	Heat of the liquid, $t + 0{\cdot}00002t^2 + 0{\cdot}0000003t^3$. Calories.	Total heat, $606{\cdot}5 + 0{\cdot}305t$. Calories.	Specific volume. 1 vol. water gives vols. of vapour.	Specific weight. Weight of the vapour in kilos. per cub. m.
606·5	0	606·5	198567	0·00504
603·030	5	608·03	143811	0·00696
599·548	10·02	609·55	105170	0·00951
596·074	15·006	611·08	75824	0·01319
592·590	20·010	612·60	57087	0·01753
589·113	25·047	614·13	43126	0·02320
585·623	30·026	615·65	32423	0·03086
582·143	35·037	617·18	25168	0·03975
577·649	40·051	618·70	19542	0·05119
575·162	45·068	620·23	15213	0·06576
571·662	50·088	621·75	12001	0·08336
568·170	55·110	623·28	9510	0·10519
564·763	60·137	624·80	7629	0·13114
561·463	65·167	626·33	6163	0·16234
560·458	66·172	626·63	5915	0·16915
557·649	70·201	627·85	5020	0·19928
554·144	75·239	629·38	4096	0·24423
550·618	80·282	630·90	3382	0·29582
549·210	82·300	631·51	3139	0·31961
547·101	[illegible]3·329	632·43	2799	0·35744
543·569	90·381	633·95	2336	0·42829
542·157	92·403	634·56	2177	0·45966
540·037	95·443	635·48	1958	0·51105
536·500	100·500	637·00	1650·5	0·60590
531·983	106·967	638·95	1338·6	0·71738
528·173	112·408	640·58	1126·9	0·88740
524·670	117·340	642·01	975·9	1·0252
521·863	121·447	643·28	859·9	1·1631
519·193	125·237	644·43	776·7	1·2981
516·727	128·753	645·48	697·2	1·4345
515·379	131·061	646·44	638·3	1·5674
512·351	134·989	647·34	587·5	1·7024
508·532	140·438	648·97	508·2	1·9676
505·110	145·310	650·42	448·4	2·2303
502·022	149·708	651·73	401·4	2·4911
499·189	153·741	652·93	363·6	2·7500
494·122	160·938	655·02	306·4	3·2632
489·687	167·243	656·93	265·2	3·771[illegible]
485·712	172·888	658·60	233·9	4·2745
482·093	178·017	660·11	209·5	4·7741
478·791	182·719	661·50	189·7	5·2704
475·705	187·065	662·77	173·5	5·7636
472·844	191·126	663·97	159·9	6·2543
470·136	194·944	665·08	148·4	6·7424
467·603	198·537	666·14	138·4	7·2283
465·120	202·041	667·16	127·7	7·6270

hot steam is conducted into a double bottom, or a coil in contact with cold water, the pressure at the end of the heating surface is generally *nil* in the first moments of the entry of the steam, it gradually increases as the water becomes heated, until, finally, when boiling commences, it reaches the permanent highest point.

The following may serve as an *example* :—

A copper pan of 1,000 mm. diameter, with a double bottom of 1·4 sq. m., contained 720 litres of water at 13° C. Steam entry valve, 25 mm.; pressure of steam in the boiler, 3·5 atmos.; at its entry into the double bottom, about 3 atmos.

Time. Hrs.	Mins.	Temperature of the water in the double bottomed pan. ° C.	Pressure of the steam at the side opposite to the steam entrance. Atmos. excess pressure.	Calories transferred per 1 sq. m. in 1 hour with 1° C. difference in temperature.
9	20	13	0·0	1224
9	25	30	0·4	1530
9	30	47	0·7	1690
9	35	64	1·2	1950
9	40	80	1·75	2090
9	45	93	1·85	2045
9	48	100	1·95	
to 10	18	100	2-2·3-2·5-2·6	80 litres of water evaporated in 30 mins.

The more rapidly the liquid moves over the heating surface, the more rapid is also the transference of heat. The larger the number of particles of liquid brought to the heating surface in a definite time, the more heat will the liquid take up in this time. The example just quoted shows this clearly: as the water becomes hotter and hotter, its circulation or movement over the heating surface increases, and so does the number of units of heat conveyed across 1 sq. m. in a definite time *per* 1° difference in temperature. Also when the liquid to be heated or evaporated is moved by artificial means rapidly and frequently over the hot surface, the amount of heat transferred in a definite time is increased. This increase is, however, not directly proportional to the increase in velocity, but in a lower ratio (Chapter XXI.).

The conclusions to be drawn from the observations of Joule, Ser, and others, lead to the belief that the increase in the transference

of heat between steam and a non-boiling liquid is proportional to the cube root of the velocity of the liquid.

The *rate of movement of the steam* over the heating surfaces also exerts a considerable influence on the transference of heat. There is always observed close to the entry of the steam, where it first comes in contact with the heating surface, a much more lively motion of the particles of a non-boiling liquid, and a very much more rapid evaporation of a boiling liquid, than at places more distant from the entry. It is evident that the more heat will be imparted by the steam, the more of its particles rapidly touch the surface of separation.

Around coils, pipes, over double bottoms and tubular heaters, filled with steam, a very lively movement of non-boiling liquids, and an extremely energetic ebullition of boiling liquids, takes place at the entrance of the steam; towards the end the action decreases considerably, until it appears almost entirely to cease. If the hot space be opened at the end, so that steam escapes, whilst the pressure in the hot space remains constant, the transference of heat is increased; a larger portion of the heating surface takes part in the violent action. In practice this opening of the hot space cannot always be effected, since it generally results in a costly loss of steam, yet there are cases in which it is the regular condition, *e.g.*, with several heating bodies placed one after the other, in the condensers of rectifying apparatus, etc.

In all these cases the largest transmission of heat is observed where the most steam passes over the hot surface, and the heating surface as a whole is the more efficient, the more steam passes over its total extent, although this steam is not quite condensed. It is believed that the average evaporative efficiency of a unit of surface decreases with its size, and, in fact, approximately in proportion to the square root of the surface. Thus, if k_v denotes the quantity of heat transferred through unit surface in unit time with 1° difference in temperature, then, through the surface, H, the quantity of heat, $C = k_v \sqrt{H}$, is transferred. In the case of tubes, inside which is steam, it is probable, as observation has shown, that this relation always holds good; in the case of double bottoms, perhaps in default of accurate experiments, the connection is more uncertain, which is also true of tubular heating apparatus with the steam outside the tubes.

When the space containing the hot steam is very large, so that only slight movement takes place in it, almost a stagnation occurs, and the influence of the absolute size of the surface is diminished.

The *condensed water* formed from the steam precipitated on the heating surface, considerably hinders the transference of heat, since the conductivity of water is very low. The more rapidly and completely this condensed water is removed from the heating surface, the more efficient the latter will be. To a certain extent the condensed water drops more readily from a horizontal tube, heated externally, than from a vertical pipe, down the whole length of which the water would have to run.

The *nature of the metal*, of which the heating surface is composed, appears to effect the amount of heat transferred only through differences in conductivity. On the other hand, the nature of the surface, whether rough or smooth, seems to be almost entirely without action on the movement of heat.

The heat, which a heating medium (steam, water, air) is to transmit through a metallic diaphragm to the heated medium (water, air), has three resistances to overcome, *viz.*:—

1. The entry through the surface of the metal plate.
2. The passage through the metal.
3. The exit from the metal into the heated fluid.

These resistances may be expressed by Péclet's method, taking for each a coefficient, which gives the number of calories passing through a surface of 1 sq. m. in one hour with a temperature difference of 1°. Let the entering coefficient be ϵ, the exit coefficient be a, the conductivity through a wall 1 mm. thick be λ, the thickness in millimetres be δ. Then if k be the total quantity of heat which passes through 1 sq. m. in one hour, with a temperature difference of 1° C., and a thickness of 1 mm. these coefficients are related according to the general equation (Péclet):—

$$\frac{1}{k} = \frac{1}{\epsilon} + \frac{\delta}{\lambda} + \frac{1}{a} \quad \ldots \ldots \ldots \quad (37)$$

or

$$k = \frac{1}{\frac{1}{\epsilon} + \frac{\delta}{\lambda} + \frac{1}{a}} \quad \ldots \ldots \ldots \quad (38)$$

The coefficients of entry and exit, ϵ and a, are practically unknown, since they are hardly capable of measurement by direct experiment.

However, for the cases dealt with here, the so-called coefficient of transmission, k, alone comes into consideration; we may thus omit the researches designed to determine the values of ϵ and a.

The conductivity coefficient, λ, of the metals has been determined by several observers; the values found are, however, somewhat different. It is probable that slight variations in the composition of the metals (impurities) exert considerable influence on the conductivity for heat. The following values for λ may be taken as the mean of many experiments, they give the number of calories which pass in one hour through a metal block of 1 sq. m. section, 1,000 mm. thick, with a temperature difference of 1° C. (Zeits. d. V. d. Ing., 1896, 46) :—

Copper, 330.	Tin, 54.
Iron, 56·1.	Zinc, 105.
Steel, 22·3-40.	Lead, 28·4.

If we put $\frac{1}{k_o}$ for the sum of the reciprocals of a and ϵ, then

$$\frac{1}{k_o} = \frac{1}{\epsilon} + \frac{1}{a}$$

and

$$k = \frac{1}{\frac{1}{k_o} + \frac{\delta}{\lambda}} \quad \ldots \ldots \ldots \quad (39)$$

or

$$k = \frac{k_o}{1 + k_o \frac{\delta}{\lambda}} \quad \ldots \ldots \ldots \quad (40)$$

If we now insert for k_o those values which are to be regarded as most nearly correct, we may form an idea of the influence exerted by the greater or less conductivity, and the greater or less thickness of the walls of the heating surface, upon the coefficient of transmission, k.

According to Molier (and others) k_o lies between 3,500 and 7,000.

In order to obtain an idea of the retarding effect of the increasing thickness of the material of the heating surface, the Tables 10 and 11 have been calculated.

Table 10 gives, for the metals, copper, zinc, iron and lead, the values of the coefficient of transmission for thicknesses of 2-10 mm.,

when that coefficient is 100 for a thickness of 1 mm. The values are given on two assumptions :—

1. The coefficient $k_o = 3,500$.
2. $k_o = 7,000$.

In practice k_o would rarely be greater than 3,500.

TABLE 10.

If the coefficient of transmission of heat, k, is 100 for a thickness in wall of 1 mm., then for greater thickness of 2-10 mm. it has the values given in the columns.

Thickness of wall. mm.	Copper.		Zinc.		Iron.		Lead.	
	k_o = 7000.	k_o = 3500.	k_o = 7000.	k_o = 3500.	k_o = 7000.	k_o = 3500.	k_o = 7000.	k_o = 3500.
1	100	100	100	100	100	100	100	100
2	98	99	94	97	87	93	83	90
3	96	98	89	94	77	86	71	82
4	94	97	84	91	69	80	63	75
5	92	96	80	89	63	76	55	69
6	90	95	76	86	57	71	50	64
7	89	94	73	83	53	68	45	60
8	87	93	69	82	49	64	42	56
9	86	92	66	79	46	61	38	53
10	84	91	64	77	43	58	36	50

From this table it is seen that the coefficient of transmission, k, decreases the more, with increasing thickness of wall, the worse conductor is the metal.

For copper, which is rarely used in thicknesses exceeding 1·4 mm., the decrease in k with increasing thickness of wall is unimportant, and may almost be neglected.

With wrought iron, which is generally thicker, the thickness at once exerts an unfavourable influence, and in the case of cast-iron heating surfaces, which are made 10 mm. thick and more, the efficiency is very considerably diminished at these thicknesses.

In the case of lead, which is used in thick-walled pipes, and has a low conductivity, the efficiency of the heating surface diminishes very rapidly with increasing thickness.

The next, Table 11, shows the values of the coefficient of transmission for iron and lead heating surfaces, when they are of equal thickness with copper, the coefficient of transmission for the latter being taken as 100. It will be seen that heating surfaces of iron and lead, of the same thickness of wall, have considerably lower efficiencies than those of copper; the former metals are also generally used in greater thicknesses than copper.

TABLE 11.

When the coefficient of transmission of heat for copper in thicknesses of 1-10 mm. is taken at 100, the coefficient for iron and lead of equal thickness has the values given

Thickness of wall. mm.	Copper.	Iron.		Lead.	
		$k_o = 7000.$	$k_o = 3500.$	$k_o = 7000.$	$k_o = 3500.$
1	100	89	93	82	90
2	100	77	87	69	82
3	100	70	82	60	75
4	100	64	77	54	70
5	100	58	73	49	63
6	100	55	70	45	60
7	100	51	67	42	57
8	100	48	63	39	54
9	100	46	61	37	51
10	100	44	60	35	49

Thick viscous liquids, which move slowly, acquire heat with more difficulty than water or dilute solutions, alcohol, etc., consequently the coefficient of transmission, k, is much lower, so that it may often be only 0·5, or even 0·2, of the coefficient for water, according to the consistency and nature of the liquid.

Finally, there is still another hindrance to the transference of heat, which arises more or less in all cases—*the incrustation or coating of the heating surface* with more or less solid, pasty or crystalline formations, corresponding to boiler scale. All these precipitates adhere firmly to the hot surface, they conduct heat very badly, and thus diminish the efficiency to a great extent. Since

these hindrances are different in each single case, can never be exactly estimated beforehand, and afterwards can practically never be controlled, the figures obtained in practice for the transference of heat are appreciably smaller than those found by careful researches; frequently the difference is so great that even the agreement of the action with the laws cannot be recognised.

The conditions of the exchange of heat through metallic diaphragms between gases, vapours and liquids, have not yet been elucidated with the desirablo certainty by means of careful experiments conducted with large apparatus on a practical scale. A theoretical consideration of all the different practical cases is also wanting. Theoretical results, however, would not be directly applicable to the large scale practico owing to the varying difficulties which occur there. Thus, in the present condition of our knowledge, there is no other course than to consider the results and observations of the author and others, obtained from large apparatus in industrial use, whilst giving due regard to the rules, coefficients and laws obtained by experiment, unfortunately, as a rule, from very small apparatus.

We shall at once endeavour to state such rules for the estimation of the necessary heating and cooling surfaces for the different cases which occur in practice.

In all cases it is an advantage to make *the passage of the gases, vapours and liquids over the hot surface* as rapid as possible. Thus, vortices and alterations in the direction of flow favour the transference of heat; the more rapidly the liquids and gases flow through the pipes, and are driven over the heating surfaces, the more rapid is the transference of heat. A current of steam or gas, flowing rapidly through a pipe or flue of regular section, gives out heat more quickly than a current of stoam, which, when led to a flat wide heating surface, spreads out over it to all sides as soon as it reaches it. The greatest loss of heat takes place at the spot where the hot current first touches the heating surface.

Towards the end of long heating pipes and flues the temperature and pressure of vapours and gases sink, so that the end itself is almost inoperative. *The shorter and narrower is a steam heating pipe, the more efficient is its surface.*

The hot space should always be kept free from air, and the water should be rapidly and completely removed.

DIRECTOR OF INDUSTRIES
Bengal

CHAPTER VIII.

THE TRANSFERENCE OF HEAT FROM SATURATED STEAM IN PIPES (COILS) AND DOUBLE BOTTOMS.

A. Evaporation and Heating by Means of Steam Pipes (Coils).

PROFESSOR R. MOLIER in a fine compilation published by request of the Vereins deutscher Ingenieure in the society's Zeitschrift, 1897, Nos. 6 and 7, states that the most reliable data concerning the coefficient of transmission, k, between steam and water are as follows :—

In the case of water which is *not boiling*, according to experiments by Ser on a horizontal tube of 10 mm. bore and 314 mm. long, the transference of heat increases approximately with the cube root of the velocity of the liquid, v_f, in m. per second.

Molier calculated k_e from the experiments of Ser:

$$k_e = 3300 \sqrt[3]{v_f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

From numerous researches by Joule on vertical tubes of narrow bore,

$$k_e = 1750 \sqrt[3]{v_f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

According to the experiments of G. A. Hagemann (Nogle Transmissions-Forsög) on an externally heated vertical tube, 49 mm. in external, 45 mm. in internal diameter and about 900 mm. long, through which water was passed at various velocities, in the case of non-boiling liquids the quantity of heat transmitted increases not only with the velocity of the liquid but also with the height of the temperature at which the transference of heat is effected. The higher the temperature of the hot steam, t_d, and the temperatures of the liquid, t_{fa}, and t_{fe}, the more heat is transferred in one hour per sq. m. per 1° C. difference in temperature. Molier deduces from Hagemann's experiments the following expression for k_e :—

$$k_e = 50 + \left\{ 1000 + 10\left(t_d + \frac{t_{fa} + t_{fe}}{2}\right)\right\} \sqrt{v_f} \quad . \quad . \quad (43)$$

The figures, obtained by Nichol from experiments on a brass tube of 20 mm. bore, show a considerably greater transference of heat in the horizontal than in the vertical position. In the horizontal position about 1·5 times as many calories were transmitted as in the vertical, yet the values found by Nichol are lower than those of Ser.

It would appear that at higher temperatures the liquid is somewhat more mobile, and hence that greater differences of temperature may occur between its parts, which would then cause a greater movement over the heating surface. That the horizontal position of the hot pipe is favourable may well be explained by the immediate removal of heated particles of liquid from the hot surface, thus at once making place for fresh particles. In or about a vertical pipe many particles of liquid must remain in contact with the surface in rising.

In regard to the *transference of heat to boiling water from saturated steam*, experiments by C. Long, J. B. Morison and the brothers Sulzer, are quoted in the same paper; the results of these experiments, which were certainly carefully executed, cannot, however, well be considered from the same point of view.

From a consideration of the above-mentioned experiments, those of Jelinek (Z. d. V. für Rübenzucker-Industrie, December, 1894), and some number of the author's own, the author comes to the conclusion that the empirical equation

$$k_s = \frac{1900}{\sqrt{dl}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

most accurately expresses the transmission of heat between steam and boiling water, in so far as cylindrical copper pipes, with steam inside, are concerned.

With all due regard to such careful workers as Joule and Ser, the author is of the opinion that, from such small apparatus as that with which they worked, safe conclusions cannot be drawn as to the relations between steam and liquid on the much greater proportions of the industrial scale.

It is quite certain that the temperature and pressure of the steam at the end of a long pipe surrounded by water in violent ebullition are considerably lower than at the beginning. It is also proved that those heating surfaces, or portions of heating surfaces, transmit the

of molecules of steam. Similarly, steam at rest gives up the least heat.

Steam which is blown into a large heating space, spreads out on all sides immediately after its entry; it does not pass over the hot surface in a regular manner, and thus gives out its heat very slowly.

In the author's opinion, observation teaches that the transmission of heat increases with decreasing diameter and with decreasing length of the tube, and apparently in such a manner that the transmission is inversely proportional to the square root of the product of these quantities. The smaller the diameter of the heating tube the more molecules of those which are passing through will come into contact with the walls. Since the largest quantity of heat is given up at the beginning, every tube becomes much less active towards the end.

The equation

$$k_v = \frac{1900}{\sqrt{dl}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

is not in any way to be regarded as final; we know, indeed, that it is inaccurate. It appears that the increasing length of the heating pipe diminishes the transmission of heat in a somewhat less ratio than that of the square root. The equation is inaccurate for very short and very long tubes, but the want of results of sufficiently accurate experiments does not permit it to be corrected, and thus it must serve for the present.

For comparison with this formula certain published experimental results may be quoted:—

Jelinek, with a copper tube, 16 mm. bore, 12,000 mm. long, observed $k_v = 4494$.

$$\text{Calculated, } k_v = \frac{1900}{\sqrt{0{\cdot}016 \times 12}} = 4309.$$

Jelinek, with a copper tube, 10 mm. bore, 8200 mm. long, observed $k_v = 5890$.

$$\text{Calculated, } k_v = \frac{1900}{\sqrt{0{\cdot}01 \times 8{\cdot}2}} = 6643.$$

In this case the temperature difference was taken by Jelinek as the arithmetic mean of the initial and final temperatures of the steam, whilst it should have been calculated according to the principles laid

down in Chapter I., in which case it is less, and k_v then becomes *6750, instead of 5890.*

Jelinek, with a copper tube, 16 mm. bore, 3000 mm. long, observed $k_v = 8680$.

$$\text{Calculated, } k_v = \frac{1900}{\sqrt{0.016 \times 3}} = 8675.$$

Sulzer, with a copper tube, 100 mm. bore, 3000 mm. long, observed $k_v = 3400$.

$$\text{Calculated, } k_v = \frac{1900}{\sqrt{0.1 \times 3}} = 3480.$$

C. Long, with a copper tube, 31·4 mm. bore, 2500 mm. long, observed $k_v = 6500$.

$$\text{Calculated, } k_v = \frac{1900}{\sqrt{0.0314 \times 2.5}} = 6840.$$

In Table 12 are contained the coefficients of transmission, calculated by means of equation 44, for copper tubes of 10-150 mm. bore and 1-30 m. long. These values for k_v only apply to the evaporation of water. The thicker the liquid to be evaporated becomes, the less becomes the influence of the form and species of the heating surface upon the efficiency.

For wrought-iron pipes the coefficient, k_v, should be taken at about 0·75, for cast-iron pipes about 0·5, and for lead pipes about 0·45 of the coefficients for copper, in which values allowance has been made for the greater thickness in wall of these metals.

For application in practice only $\frac{2}{3}$ of the value of k_v as so found should be used.

When not pure water, but dilute solutions of 10-25 per cent. strength are to be evaporated, the coefficient of transmission generally decreases by 20-30 per cent.

For thick, pasty, viscous or sticky liquids, or liquids largely mixed with crystals, the value of k_v may become much less. The dimensions of the heating tubes are then found to be of little influence; for such cases the following values should be taken for k_v in practice:—

Long heating coils, about 650-750.
Short ,, ,, ,, 800-900.
Thin heating tubes (steam pipes), about 1000.
Vertical systems of pipes (steam outside), about 600-700.

TABLE 12.

The coefficient of transmission of heat, k_v, for one hour, 1° C. and 1 sq. m., between steam and *boiling* water, for copper heating coils of 10-150 mm. bore and 1-30 m. length.

Bore of the tube in mm.	Length, l, of the tube in m.								
	1	2	4	6	8	10	15	20	30
	Coefficient of transmission of heat, k_v, for copper steam pipes, heated inside.								
10	19000	13470	9500	7714	6730	6012	4912	4290	3570
15	15580	11000	7713	6333	5495	4910	3950	3408	2833
20	13470	9500	6730	5490	4750	4220	3408	3007	2455
25	12000	8520	6012	4910	4250	3800	3100	2687	2190
30	11000	7714	5490	4510	3875	3408	2835	2455	2004
35	10190	7272	4900	3900	3500	3200	2640	2270	1850
40	9500	6730	4750	3875	3363	3007	2455	2110	1743
45	8950	6333	4510	3600	3165	2835	2300	2004	1610
50	8520	6012	4253	3408	3007	2687	2190	1900	1558
60	7714	5490	3875	3170	2740	2455	2004	1743	1415
70	7200	5080	3600	2930	2540	2270	1890	1610	1310
80	6730	4750	3363	2740	2375	2125	1711	1490	1225
90	6333	4510	3170	2580	2245	2004	1610	1410	1157
100	6012	4290	3007	2455	2135	1900	1558	1364	1100
125	5714	3800	2687	2191	1820	1700	1390	1202	982
150	4910	3408	2455	2004	1743	1555	1266	1100	905

The thickness of metal of the copper tubes is taken at about 2 mm.

For wrought-iron pipes, about 3·5-4 mm. thick, the coefficient, k_v = 0·75 of that for copper.

" cast " " " 10 mm. thick, the coefficient, k_v = 0·50 of that for copper.

" lead " " " 10 mm. thick, the coefficient, k_v = 0·45 of that for copper.

In determining the dimensions of the heating surfaces of apparatus for the evaporation of water, the coefficient k_v should only be taken at about $\frac{2}{3}$ of the above values, *i.e.*,

For copper tubes	- -	0·66 of the figures in the table.
„ wrought-iron tubes	-	0·50 „ „ „
„ cast-iron tubes	- -	0·33 „ „ „
„ lead tubes	- - -	0·30 „ „ „

For liquids which contain 10-25 per cent. of solid matter in solution, the coefficients, k_v, are only about ¾ as large as those just given, *i.e.*,

For copper tubes	- -	0·5 of the figures in the table.
„ wrought-iron tubes	-	0·4 „ „ „
„ cast-iron tubes	- -	0·25 „ „ „
„ lead tubes	- - -	0·225 „ „ „

The equation (44) may now be somewhat transformed. Multiplying numerator and denominator by $\sqrt{\pi}$, the expression under the square root sign becomes equal to the heating surface, H_v, thus

$$k_v = \frac{1900\sqrt{\pi}}{\sqrt{dl}\sqrt{\pi}} = \frac{1900\sqrt{\pi}}{\sqrt{d\pi l}} = \frac{1900 \times 1{\cdot}772}{\sqrt{H_v}} = \frac{3367}{\sqrt{H_v}} \quad (45)$$

If we now insert this value for k_v in the equation for the total transmission of heat by the surface H_v—

$$C = H_v \,.\, \theta_m \,.\, k_v,$$

we obtain

$$C = 3367\sqrt{H_v}\,\theta_m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

which may be expressed in words: the heat transmitted in unit time by the surface, H_v, is proportional to the square root of the surface.

As has been said above, this equation is not quite correct, but the efficiency of larger surfaces is somewhat greater, and of smaller surfaces somewhat smaller, than would correspond to the equation. But the results obtained by its means, of all known to the writer, agree most nearly with the reality.

Having regard to the diminution in efficiency caused by incrustations, incomplete removal of air, etc., we may take for the calculation of the actual heating surfaces the equations

$$C = 2200\,\theta_m\sqrt{H_v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

or

$$H_v = \left(\frac{C}{2200\,\theta_m}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

which may be applied with some confidence to copper heating tubes for the evaporation of water.

Table 13 has been calculated by means of these equations, it gives the number of kilos. of water evaporated in one hour by copper tubes of 10-150 mm. diameter and 2-40 mm. length, with 1° difference in temperature between the steam and boiling water.* This table will serve for the rapid calculation of the proper dimensions of the heating tubes in any case under consideration.

With sufficiently short tubes the real temperature difference, θ_m, to be expected, is only about 10 per cent. less than the calculated.

If not water, but a thin solution of 10-25 per cent. strength is to be evaporated, copper coils give about 0·75, wrought-iron about 0·6, cast-iron about 0·4, and lead about 0·33 of the results quoted in the table.

From viscid, thick and crystallising liquids, containing very little water, the hourly evaporation of water by means of heating coils is much smaller, *viz.*, for copper about 0·5, wrought-iron about 0·40, cast-iron about 0·25, and lead about 0·225 of the weights given in Table 13.

Steam at a pressure of 3-4 atmospheres, in narrow and not too long copper coils, is found in practice to *evaporate* to the atmosphere about 100 litres of water in one hour per 1 sq. m.; with very small heating surfaces more (up to 130 litres), and with larger, less.

With 1 sq. m. of heating surface, heated by steam at 3-4 atmospheres, 800-1200 litres of water may be *heated* in 1 hour from 10° to 100° C. when the water is not specially moved, yet the efficiency of the heating surface varies greatly and depends on the velocity of the steam (see Chapter XXI.).

B. The Dimensions of Steam Tubes (Coils).

The ratio of the diameter to the length of a tubular heating surface is far from being without influence on the proper action of the surface. In very long pipes, in which the steam moves with great velocity, the pressure falls considerably towards the end and thus the available temperature difference sinks appreciably.

When the steam enters at high velocities the coefficient of transmission of heat is greater than when the velocity is lower, but the pressure and temperature, which sink rapidly in the first case,

TABLE 13.

Heating surface, H_v, in sq. m., and hourly evaporation of water, W, of copper heating tubes of 10-150 mm. diameter and 2-40 m. length, with 1° C. difference in temperature.

Length of tube in m.		Internal diameter of the heating tube in mm.											
		10	20	30	40	50	60	70	80	90	100	125	150
2	H_v	0·08	0·14	0·21	0·27	0·34	0·40	0·46	0·53	0·59	0·65	0·82	0·98
	W	1·12	1·48	1·83	2·07	2·32	2·52	2·71	2·91	3·07	3·20	3·60	3·96
3	H_v	0·12	0·21	0·31	0·41	0·50	0·60	0·69	0·80	0·89	0·99	1·22	1·47
	W	1·36	1·83	2·22	2·56	2·83	3·09	3·32	3·56	3·77	3·97	4·40	4·84
4	H_v	0·16	0·28	0·42	0·54	0·68	0·80	0·92	1·06	1·18	1·30	1·64	1·96
	W	1·60	2·11	2·58	2·93	3·29	3·57	3·84	4·09	4·32	4·56	4·96	5·60
5	H_v		0·36	0·51	0·68	0·85	1·00	1·16	1·34	1·49	1·65	2·04	2·46
	W	—	2·40	2·85	3·29	3·68	4·00	4·03	4·60	4·88	5·12	5·71	6·26
6	H_v		0·43	0·62	0·81	1·01	1·21	1·39	1·60	1·78	1·97	2·45	2·94
	W	—	2·62	3·12	3·60	4·00	4·40	4·71	5·04	5·32	5·60	6·26	6·85
7	H_v		0·49	0·73	0·95	1·18	1·40	1·61	1·86	2·07	2·29	2·86	3·43
	W	—	2·80	3·41	3·89	4·32	4·72	5·08	5·45	5·75	6·09	6·76	7·40
8	H_v		0·56	0·84	1·08	1·36	1·60	1·84	2·12	2·36	2·60	3·28	3·92
	W	—	2·98	3·66	4·16	4·64	5·04	5·41	5·84	6·13	6·46	7·24	7·90
9	H_v			0·93	1·22	1·53	1·81	2·09	2·41	2·69	2·97	3·68	4·41
	W	—	—	3·75	4·41	4·92	5·38	5·78	6·20	6·56	6·89	7·65	8·43
10	H_v			1·03	1·35	1·69	2·01	2·32	2·67	2·98	3·29	4·08	4·90
	W	—	—	4·04	4·64	5·20	6·02	6·08	6·52	6·90	7·24	8·08	8·85
11	H_v			1·13	1·48	1·86	2·21	2·55	2·94	3·27	3·61	4·48	5·39
	W	—	—	4·24	4·84	5·45	6·04	6·38	6·84	7·25	7·60	8·46	9·28
12	H_v			1·24	1·62	2·03	2·41	2·78	3·20	3·57	3·94	4·90	5·88
	W	—	—	4·44	5·08	5·68	6·20	6·66	7·06	7·55	7·93	8·85	9·69
13	H_v			1·35	1·76	2·19	2·61	3·00	3·46	3·85	4·26	5·31	6·37
	W	—	—	4·64	5·28	5·92	6·46	6·92	7·44	7·84	8·15	9·20	10·09
14	H_v			1·46	1·90	2·36	2·80	3·22	3·72	4·14	4·58	5·72	6·86
	W	—	—	4·80	5·39	6·12	6·69	7·07	7·71	8·13	8·49	9·56	10·48
15	H_v			1·53	2·03	2·55	3·00	3·48	4·02	4·47	4·95	6·12	7·38
	W	—	—	4·93	5·68	6·38	6·92	7·45	8·00	8·45	8·86	9·89	10·86
16	H_v				2·16	2·72	3·20	3·68	4·24	4·72	5·20	6·56	7·84
	W	—	—	—	5·88	6·58	7·30	7·67	8·23	8·68	9·14	10·24	11·20
17	H_v					2·89	3·41	3·93	4·53	5·05	5·57	6·96	8·35
	W	—	—	—	—	6·80	7·38	7·93	8·48	8·98	9·44	10·55	11·55
18	H_v					3·06	3·62	4·18	4·82	5·38	5·94	7·36	8·82
	W	—	—	—	—	6·99	7·60	8·17	8·78	9·28	9·74	10·05	11·88
19	H_v					3·22	3·82	4·41	5·08	5·67	6·26	7·76	9·31
	W	—	—	—	—	7·17	7·80	8·40	9·01	9·52	10·00	11·14	12·20
20	H_v					3·38	4·02	4·64	5·34	5·96	6·58	8·16	9·80
	W	—	—	—	—	7·35	8·01	8·60	9·24	9·76	10·32	11·40	12·52

TABLE 13—(*continued*).

Length of tube in m.		Internal diameter of the heating tube in mm.											
		10	20	30	40	50	60	70	80	90	100	125	150
21	H_r						4·32	4·87	5·61	6·25	7·00	8·56	10·29
	W	—	—	—	—	—	8·31	8·80	9·47	10·00	10·58	11·70	12·84
22	H_r						4·42	5·10	5·88	6·51	7·28	8·96	10·78
	W	—	—	—	—	—	8·40	9·04	9·69	10·22	10·74	12·00	13·12
23	H_r						4·62	5·33	6·14	6·84	7·55	9·38	11·27
	W	—	—	—	—	—	8·59	9·20	9·90	10·46	10·98	12·24	13·44
24	H_r						4·82	5·56	6·40	7·14	7·88	9·80	11·76
	W	—	—	—	—	—	8·78	9·48	10·10	10·69	11·20	12·52	13·72
25	H_r							5·78	6·66	7·42	8·20	10·21	12·25
	W	—	—	—	—	—	—	9·60	10·32	10·89	11·45	12·80	14·00
26	H_r							6·00	6·92	7·70	8·52	10·62	12·74
	W	—	—	—	—	—	—	9·79	10·52	11·09	11·65	13·04	14·28
27	H_r							6·22	7·18	7·99	8·84	11·03	13·23
	W	—	—	—	—	—	—	9·97	10·71	11·29	11·89	13·28	14·56
28	H_r							6·44	7·44	8·28	9·16	11·44	13·72
	W	—	—	—	—	—	—	10·14	10·90	11·48	12·10	13·52	14·84
29	H_r							6·70	7·74	8·61	9·53	11·84	14·24
	W	—	—	—	—	—	—	10·35	11·09	11·73	12·34	13·76	15·08
30	H_r								8·04	8·94	9·90	12·24	14·76
	W	—	—	—	—	—	—	—	11·34	12·00	12·56	14·00	15·36
31	H_r								8·26	9·10	10·15	12·68	15·22
	W	—	—	—	—	—	—	—	11·49	12·06	12·72	14·24	15·60
32	H_r								8·48	9·44	10·40	13·12	15·6[illegible]
	W	—	—	—	—	—	—	—	11·88	12·28	12·92	14·48	15·84
33	H_r									9·77	10·77	13·52	16·19
	W	—	—	—	—	—	—	—	—	12·50	13·12	14·62	16·08
34	H_r									10·10	11·14	13·92	16·70
	W	—	—	—	—	—	—	—	—	12·72	13·36	14·92	16·36
35	H_r									10·43	11·51	14·32	17·17
	W	—	—	—	—	—	—	—	—	12·92	13·60	15·12	16·56
36	H_r									10·76	11·88	14·72	17·64
	W	—	—	—	—	—	—	—	—	13·12	13·80	15·36	16·80
37	H_r										12·20	15·12	18·13
	W	—	—	—	—	—	—	—	—	—	14·00	15·56	17·04
38	H_r										12·52	15·52	18·62
	W	—	—	—	—	—	—	—	—	—	14·16	15·76	17·28
39	H_r										12·84	15·92	19·11
	W	—	—	—	—	—	—	—	—	—	14·32	15·96	17·78
40	H_r										14·16	16·32	19·60
	W	—	—	—	—	—	—	—	—	—	15·04	16·16	18·72

diminish the temperature difference to such an extent that the heat transferred per sq. m., with an excessive initial velocity of the steam, is really smaller than when it retains its full pressure to the end of the pipe.

The connection between diameter and length of tube, velocity and pressure of steam, may be explained in the following manner:—

The heat passing through the walls of a steam tube into the surrounding boiling water is equal to the heat set free by the condensation of the steam. Thus we have the equation:

$$2200\theta_m \sqrt{d\pi l} = \frac{d^2\pi}{4} v_a 3600 c\gamma \quad . \quad . \quad . \quad . \quad . \quad (49)$$

[illegible]meter of the tube, l its length, v_a the velocity of the steam on entering the tube (all in m.), c the heat of evaporation of 1 kilo. of steam, γ the weight of 1 cub. m. of steam, θ_m the difference in temperature.

By a transformation of this equation (49) we obtain the connection between the length and diameter of the tube.

$$\sqrt{\frac{l}{d}} = \frac{v_a 3600 c\gamma d \sqrt{\pi}}{4\theta_m 2200} = 0{\cdot}725 \frac{v_a c\gamma d}{\theta_m} \quad . \quad . \quad . \quad . \quad (50)$$

The external surface of the tubes should have been taken here as the heating surface, but in equation (50) the thickness of the metal was neglected in order to obtain a compact formula, the internal diameter of the tube being taken as equal to the external. This inaccuracy makes the calculated lengths of pipe about 10 per cent. too great, which must be remembered in applying equation (50).

The velocity with which the steam enters is conditioned by the dimensions of the tube, the difference in temperature and the fall in pressure in the tube. The latter cannot, however, well be calculated, not even by means of equation (143), which does not hold good for complete condensation, thus the proper ratio, $\frac{l}{d}$, cannot be found with certainty from equation (50). It must suffice to assume the greatest advisable length of pipe from the results of experiment.

The lower the pressure of the steam, and the greater the temperature difference between steam and boiling liquid, the shorter must the tube be. For differences in temperature of 30°-40° C., the following values of the ratio $\frac{l}{d}$ are suitable:—

Absolute pressure of steam, atmos.,	5	4	3	2	1·5	1·25	0·8324	0·466
$\frac{l}{d}=$	275	250	225	200	175	150	125	100

For any other difference in temperature, θ_m, the highest value of the ratio $\frac{l_1}{d_1}$ is then

$$\frac{l_1}{d_1} = \frac{6l}{d\sqrt{\theta_m}}.$$

For the sake of convenience in calculation it may be stated that the values of $0{\cdot}725c\gamma$ for the above steam pressures are

997, 817, 631, 438, 340, 288, 203, 116.

If the steam is to be used in the heating tube at its original high pressure, and, consequently, its highest temperature, it must not be throttled on entering the tube. The valve admitting the steam must be of fair dimensions.

If the highest available steam pressure is required to be exerted in the coil, then the velocity of the steam on entering may be 30 m. If, on the other hand, a certain fall in pressure from the main steam pipe to the heating tube is permissible, the steam may enter with a velocity of 50-60 m. The latter is regularly the case, when the available steam pressure is higher than is required in the coil.

Table 14 may assist in the choice of the steam valve. In it are given the weights of steam at different pressures which pass in one hour with a velocity of 30 m. through valves of 10-350 mm. diameter. For higher or lower velocities the weight of steam admitted is naturally proportionately larger or smaller.

Example.—The dimensions of a steam coil are to be determined, by which in one hour 300 kilos. of water, or 300 kilos. of dilute alcohol (50 per cent. by weight), or 300 kilos. of ether, can be evaporated, when the available steam is at a pressure of 4 or 1·25 atmos. absolute.

The heat of evaporation of 1 kilo. of dilute alcohol vapour of 50 per cent. strength by weight is 375 calories, *i.e.*, as large as for $\frac{375}{540} = 0{\cdot}7$ kilo. of water. Thus, in regard to the consumption of heat, 300 kilos. of the vapour of water + alcohol are equivalent to 210 kilos. of steam.

The heat of evaporation of 1 kilo. of ether is 97 calories, thus 300 kilos. of ether are equivalent to

$$300\,\frac{97}{540} = 54 \text{ kilos. of steam.}$$

TABLE 14.

The weight of steam which enters with the velocity $v_a = 30$ m. and at mm. diameter, without

Steam pressure, Atmos. absolute.	Steam temperature, °C.	Diameter										
		10	15	20	25	30	35	40	45	50	55	60
		Weight of steam, in kilos. per										
1·00	100	5	12	20	32	46	63	82	103	126	154	184
1·25	106	6·3	14·3	25	40	57	78	101	132	158	191	278
1·50	112	7·5	17	30	47	68	92	120	164	188	227	270
2	121	10	23	39	63	88	120	157	200	245	298	355
2·5	128	12	28	48	76	110	149	194	245	304	367	438
3	134	14	32	56	89	128	173	225	285	353	428	510
4	144	19	43	76	130	170	231	300	280	471	570	680
5	152	27	53	93	146	210	285	372	472	583	705	841

Thus there are to be evaporated

	300 kilos. of water,	300 kilos. of alcohol + water,	300 kilos. of ether.
or	300 „ „	210 „ water,	54 „ water.
The boiling point is	100°	92·5°	37°

(*a*) For saturated steam at 3 atmos. (= 4 atmos. absolute) the temperature = 144° C.

The temp. diff. is thus	44°	51·5°	107°

We shall assume that in reality the temperature difference is about 10 per cent. less,

i.e.,	40°	46°	96°

For 1° temperature difference the heating tube must evaporate

$$\frac{300}{40} = 7{\cdot}5 \text{ kilos.,} \quad \frac{210}{46} = 4{\cdot}56 \text{ kilos.,} \quad \frac{54}{96} = 0{\cdot}506 \text{ kilo. of water.}$$

From Table 13 we now find that there is required

1 tube of	60 mm. × 18 m.	40 mm. × 10 m.	10 mm. × 0·6 m.
	= 3·62 sq. m.	= 1·35 sq. m.	= 0·025 m.
or 2 tubes of	46 mm. × 7 m.	25 mm. × 4 m.	—
	= 1·92 sq. m.	= 0·72 sq. m.	—
or 3 „	30 mm. × 4 m.	—	—
	= 1·29 sq. m.	—	—

(*b*) For saturated steam of 0·25 atmos. (= 1·25 atmos. absolute) the temperature = 106·38° C.

The temp. diff. is	6·38°	13·88°	69·38°

TABLE 14.

pressures of 1-5 atmos. absolute in one hour, through valves of 10-350 sensible loss of pressure.

of the eteam valve in mm.											
65	70	80	90	100	125	150	175	200	250	300	350
hour, wbich enters with a velooity of 30 m.											
215	250	325	413	505	802	1144	1560	2192	3206	4576	6254
267	320	403	527	632	998	1422	1932	2529	3972	5688	7745
317	367	429	657	752	1172	1679	2292	3000	4686	6714	9188
415	483	628	795	980	1533	2209	3014	3933	6148	8816	
513	595	774	980	1214	1895	2726	3717	4862	7600		
597	693	900	1144	1412	2209	3180	4406	5764			
796	926	1204	1520	1881	3004	4254	5820				
985	1143	1485	1888	2332	3704	5247					

The real temperature difference ie again assumed to bo about 10 per cent. lees.

i.e., 5·5° 12° 63°

Thus for 1 temperature difference the hot tube must evaporate

$$\frac{300}{5{\cdot}5} = 54{\cdot}6 \text{ kiloe.} \quad \frac{210}{12} = 17{\cdot}5 \text{ kilos.} \quad \frac{54}{63} \quad 0{\cdot}86 \text{ kilo.}$$

From Table 13 we now find there are required

3 tubes of 150 mm. × 40 m. = 57 sq. m.	1 tube of 150 mm. × 39 m. = 19·1 sq. m.	1 tube of 10 mm. × 1 m. = 0·04 sq. m.
or 4 „ 150 mm. × 24 m. = 47 sq. m.	2 tubes of 100 mm. × 15 m. = 9·9 sq. m.	—
or 6 „ 100 mm. × 15 m. = 29·7 sq. m.	3 „ 60 mm. × 11 m. = 6·6 sq. m.	—
or 8 „ 80 mm. × 12 m. = 25·8 sq. m.	—	—
or 15 „ 40 mm. × 6 m. = 12·2 sq. m.	—	—

A heating surface for evaporating may be constructed to consist of a single tube, diminishing in *diameter* towards the end either gradually or in steps, or of several parallel tubes, the *number* of which is diminished towards the end (*e.g.*, from 4 to 3, to 2, to 1).

The researches published up to the present show that the coefficient of transmission for such heating surfaces is not less than for short tubes of equal length of the same section throughout.

Since, however, as soon as the length becomes somewhat considerable in proportion to the diameter ($l = 600\ d$ to $800\ d$), the pressure of steam in the tube sinks to a great extent towards the end, the difference in temperature between steam and liquid also sinks inconveniently, and the evaporation per sq. m. becomes small.

Short tubes of relatively small diameter make the most efficient heating surface

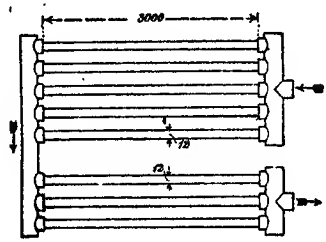

Fig. 6.

Example.—An actual case (see Fig. 6). Eight equal horizontal brass tubes (70 per cent. of copper), of 10 mm. bore, 12 mm. external diameter and 3000 mm. length, supplied with steam at 111·9° C. on entering, 103·2° C. on leaving, evaporated in one hour at 100° C. 141 litres of water, originally at 23°. The total heating surface is $H_v = \cdot 90$ sq. m.

The difference in temperature at the beginning is $\theta_a = 11{\cdot}9°$.

" " end is $\theta_e = 3{\cdot}2°$.

The mean temperature difference would be obtained from Table 1: (since $\frac{3{\cdot}2}{11{\cdot}9} = 0{\cdot}269$), $\theta_m = 0{\cdot}56 \times 11{\cdot}9 = 6{\cdot}68°$.

Since, however, the first portion of the heating surface is larger than the second, θ_m must be taken as 7·1°, hence the *observed* coefficient of transmission,

$$k_v = \frac{141(635 - 23)}{7{\cdot}1 \times \cdot 9} = 13{,}500 \text{ approx.}$$

The average heating surface for 1 tube is $\frac{\cdot 9}{8} = 0{\cdot}112$ sq. m., from which we obtain the *calculated* coefficient (by equation 45),

$$k_v = \frac{3367}{\sqrt{0{\cdot}112}} = 10{,}100.$$

C. Evaporation and Heating by Means of Double Bottoms and Wide Jackets.

Steam admitted to double bottoms or wide cylindrical jackets, the other surface of which is in contact with *boiling liquid*, does not pass over the whole heating surface as regularly, and is not forced on to the heating surface in the same manner, as in a coil. Immediately after it enters the wide space, the steam spreads and takes the shortest path to the open. This is probably the reason why the results of experiments on evaporation in jacketed pans do not show a regular relation between the transference of heat and the size of the heating surface, which was the case with heating coils. Large and small jacketed pans give almost the same transference of heat. The published values for k_v vary greatly, they range from $k_v = 1300$ to $k_v = 3300$. The chief cause of the variation is probably the incomplete removal of air. On an average it may be taken that, in evaporating water in a copper pan with a double bottom or jacket, $k_v = 1400$ to 1800; for bottoms up to 1 m. in diameter $k_v = 1800$, from 1 to 1·3 m. diameter $k_v = 1700$, from 1·5-2 m. diameter $k_v = 1600$, and for larger pans $k_v = 1400$. The transmission of heat by copper double bottoms for the evaporation of water is thus:—

$$C = H_m 1400 \text{ to } H\theta_m 1800 \quad . \quad . \quad . \quad . \quad . \quad (51).$$

In the case of small pans up to 1 m. in diameter, the mean difference in temperature during boiling may be assumed to be about 0·85 of that at the steam entrance; with pans of 1-2 m. diameter about 0·75, and with larger pans about 0·65 of the same amonnt. But all these figures are somewhat variable, and it is not yet possible to ascertain what causes produce, now a larger, and then a smaller, fall in pressure in the double bottom in each case. The distance from the boiler, the bore of the steam pipe, the loss of heat in it, the kind of pan, the form and nature of the steam entrance and its width all play a part.

With steam at 3-4 atmospheres pressure in the boiler it will be found that, in an open pan with a double bottom of about 1-2 sq. m., 80-100 litres of water are evaporated in one hour per sq. m. from quite dilute solutions. In larger pans the efficiency is somewhat smaller. In this case it is very advisable to arrange several entrances for the steam, by which the efficiency is considerably increased.

By means of equation (51) the following figures have been calculated, showing how great an evaporation of water per hour may be expected with copper double pans of 500-3000 mm. diameter, with one steam entrance and steam pressures of 2-5 atmospheres absolute.

Diameter of the bottom in mm.	500	800	1000	1250	1500	1750	2000	2250	2500	2750	3000
Depth of the bottom in mm.	200	300	400	500	550	600	600	700	800	900	1000
Heating surface of the bottom in sq. m.	0·33	0·79	1·26	2·02	2·7	3·62	4·3	5·5	6·8	8·5	10·36
Atmos. abs.	Water evaporated in litres per hour.										
Pressure. 2	18·5	44	56	95	127	163	190	193	238	297	360
3	30	62	92	159	212	271	300	315	388	488	590
4	44	104	132	209	280	358	400	420	503	627	766
5	50	117	156	248	330	421	500	525	583	726	888

If 2-4 steam inlets are provided for the larger pans, the hourly evaporation may be half as much again as here given.

Example.—It was observed that, in a double-bottomed pan of 3450 mm. diameter (11·2 sq. m. heating surface), in one hour there were evaporated by steam of 2-2·5 atmos. absolute pressure 1200 litres = 107 litres per sq. m.; by steam of 2·5-3 atmos. absolute, 1500 litres = 134 litres per sq. m. (four steam entrances).

If the water in a double pan is *not boiling*, but is only to be warmed by the steam, on account of the low temperature of the water, the difference in temperature between steam and water is considerably greater than when the water boils. The pressure of the steam then usually falls considerably even at the entrance, and when the heating commences is often zero at the side opposite the entrance. As the temperature of the water rises, the pressure of the steam in the steam space also increases. It may be assumed that the mean difference in temperature θ_m, between steam and water during the whole period of heating until boiling commences, is about half the difference between the temperature of the hot steam, t_d, and that of the liquid at first, t_f.

$$\theta_m = \frac{t_d - t_f}{2}.$$

The coefficient of transmission, having regard to incrustations, is $k_v = 1400$.

Thus, during the period of warming, the following quantities of heat are conveyed to the non-boiling liquid in one hour through a copper double bottom heated by steam:—

$$C = 1400H\theta_m = 700H(t_d - t_f) \quad . \quad . \quad . \quad . \quad . \quad (52)$$
$$\text{to } C = 1800H\theta_m = 900H(t_d - t_f),$$

from which the heating surface may be calculated for any case.

In most cases, in which steam of about 3-5 atmospheres pressure (130°-160° C.) is supplied to the pan, 1000 litres of water can be heated in one hour from 10° to 100° C. per 1 sq. m. of double bottom. If the liquid to be heated is thicker and less mobile than water, only a smaller efficiency can be expected. As the example in Chapter VII. shows, the transmission of heat increases as the temperature of the liquid rises.

Examples.—The following are actual observations:—

720 litres of water were heated from 13° to 100° C. in 28 mins. by 1·2 sq. m. (diameter of pan 1000 mm.) by means of steam at $3\frac{1}{2}$ atmos. pressure, *i.e.*, 1285 litres per sq. m. per hour.

640 litres of water were heated from 12° to 100° C. in 30 mins. by 1·2 sq. m. (diameter of pan 1000 mm.) by means of steam at $3\frac{1}{2}$ atmos. pressure, *i.e.*, 1068 litres per sq. m. per hour.

89·6 litres of water were heated from 20° to 100° C. in 16 mins. by 1·45 sq. m. (diameter of pan 540 mm.) by means of steam at 4 atmos. pressure, *i.e.*, 746 litres per sq. m. per hour.

1075 litres of water were heated from 19·25° to 100° C. in 47 mins. by 1·5 sq. m. (diameter of pan 1295 mm.) by means of steam at $3\frac{1}{2}$ atmos. pressure, *i.e.*, 921 litres per sq. m. per hour.

4200 litres of mash were heated from 52·5° to 100° C. in 45 mins. by 4·5 sq. m. (diameter of bottom of pan 2450 mm.) by means of steam at 100° to 139° C. in the double bottom, *i.e.*, 970 litres per sq. m. per hour.

5000 litres of mash were heated from 65° to 100° C. in 20 mins. by 5·8 sq. m. (diameter of bottom of pan 2450 mm.) by means of steam at 3·5 atmos. absolute, *i.e.*, 2596 litres per sq. m. per hour (two steam inlets and stirrer).

21,000 litres of wort were heated from 68·5° to 100° C. in 50 mins. by 11·2 sq. m. (diameter of bottom of pan 3400 mm.) by means of steam at 3·5 atmos. absolute, *i.e.*, 2256 litres per sq. m. per hour (four steam inlets).

CHAPTER IX.

EVAPORATION IN A VACUUM.

A VACUUM apparatus is a closed vessel, heated by steam, or more rarely by fire, and in which a lower pressure than that of the atmosphere is maintained by suitable arrangements. The diminished pressure—the vacuum—is obtained by leading the vapours, evolved from the liquid which is evaporating in the apparatus, through the shortest possible pipe into a second closed vessel—the condenser—where they are precipitated directly by a jet of water or on well cooled metallic surfaces.

In completely closed vessels a diminution of pressure, a vacuum, a partial absence of air, or even a perfect vacuum, would arise through the liquefaction and disappearance of vapour alone, if air did not always enter from the evaporating liquid, the injected water, or by leakages (always present) in the walls of the apparatus. Since this air must be removed, an air-pump is always essential with a vacuum apparatus.

A vacuum may be indeed obtained by condensing the vapours evolved from a closed vessel, but it will soon be decreased, since air enters from the liquid, from the water and through leaks. Without pumping out the air, a *lasting* vacuum cannot be obtained.

The dimensions of the pipes, condenser and air-pump will be treated in later chapters.

A vacuum apparatus may be made of any resistant form: spherical, egg-shaped, cylindrical, conical; it may be made of wrought-iron, cast-iron, copper, brass, lead or tin, also of earthenware, glass or porcelain; it may be heated by steam (coils, double bottoms, systems of tubes), by hot liquids, or it may stand on the open fire. Everything depends on the properties of the material which is being treated and the results to be obtained.

Since a portion of the liquid, which is drawn into the vacuum apparatus, is evaporated and the residue remains, the capacity in most cases need not be as great as the volume of the dilute liquid to be evaporated within a definite time, but only sufficiently large to contain the evaporated liquid. In order to preserve a constant level in the apparatus the dilute liquid may be fed in as required. There are, however, occasional cases in which it is not permissible to feed after the commencement, the contents of the apparatus must then be equal to the volume of the dilute liquor.

The proportion of the heating surface to the capacity depends on the object of the vacuum apparatus. For many liquids it is desirable to keep them in the vacuum as short a time as possible; large heating surfaces and a small capacity will then be used. In other cases, in order to obtain crystals, the charge may be gradually increased. Experience must here be the guide as to the proportion of heating surface, which depends on the duration of crystallisation no universal rule can be made, except that the capacity should be arranged to correspond with the desired output, and the heating surface with the time in which a definite amount of water (or of liquid) is to be removed from the contents.

The first advantage of evaporating in a vacuum over evaporation at atmospheric pressure is that *in vacuo* all liquids boil and evaporate at considerably lower temperatures than under atmospheric pressure thus there is a greater difference in temperature between the heating steam and the boiling liquid, and, consequently, a much greater transmission of heat per sq. m. of heating surface. In fact for heating purpose *in vacuo* steam of very low pressure, at 100° C. or lower, may be used with great success. The exhaust steam from engines and other sources may be profitably utilised, for since the boiling points of most liquids are 40° C., or more, lower *in vacuo*, there is nearly always sufficient difference in temperature.

Liquids, which boil at higher temperatures (180°-200°-210° C.), can generally not be evaporated under atmospheric pressure by means of high pressure steam, since steam would be required of such high temperatures, and, therefore, high pressures, that its application would be inconvenient, if not dangerous. The boiling points of these liquids fall, however, in the vacuum apparatus, so that steam of moderate pressure, as generally employed, may be used. In a vacuum, rapid evaporation may be expected if there is a difference

in temperature of 10° C., or even of 5° C., if the liquid is not too viscous.

The vapour pressures of liquids in a vacuum (and under pressure) may be calculated by means of a rule found by U. Dühring and published by E. Dühring in *Neue Grundzuge zur rationellen Physik und Chemie*, Leipzig, 1878. This rule, which does not appear to be quite reliable in all cases, runs:—

The difference between the boiling points (t_f and t^1_f) of a liquid at any two pressures, divided by the difference between the boiling points (t_w and t^1_w) of any other liquid at the same two pressures, is a constant q for these two liquids:

$$q = \frac{t_f - t^1_f}{t_w - t^1_w} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

Example.—The boiling point of mercury is 357° C. at 1 atmos., 261° C. at 100 mm. pressure. The boiling point of water is 100° C. at 1 atmos., 52° C. at 100 mm. pressure.

$$\text{Then } q = \frac{357 - 261}{100 - 52} = \frac{96}{48} = 2.$$

The boiling point of mercury is 214·5° C. at 30 mm. pressure, 154·4 C. at 5 mm. The boiling point of water is 29·1° C. at 30 mm. and 1·2° C. at 5 mm. pressure, hence

$$q = \frac{214{\cdot}5 - 154{\cdot}4}{29{\cdot}1 - 1{\cdot}2} = \frac{60{\cdot}1}{27{\cdot}9} = 2{\cdot}12.$$

Similar results are obtained for other pressures and liquids.

The inaccuracy of the constant q is perhaps to be referred to insufficient knowledge of the boiling points.

Thus, if the boiling point of one liquid be known at two pressures, the boiling point of another liquid at one of these pressures, and also the constant q for these two liquids, by means of this rule the boiling point of the second liquid at all other pressures may be calculated.

Now if water be taken as the standard liquid, since its boiling points at different pressures are most accurately known, and, further, if 1 atmos. absolute be taken as one of the common pressures, since the boiling points of most liquids at this pressure have been carefully determined, then by means of this rule we can calculate the boiling points of all these liquids for all pressures, for which the constant q is known, or we can calculate the constant q for all the liquids, of which the boiling point has been observed at a second pressure.

Let t_f = the boiling point of one liquid at a pressure of 1 atmos. absolute,

t^1_f = the required boiling point of the same liquid at another pressure,

t_w = the boiling point of water at 1 atmos. pressure,

t^1_w = „ „ „ at the other pressure,

then $$t_f - t^1_f = q(100 - t^1_w)$$

or $$t^1_f = t_f - q(100 - t^1_w) \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

Example.—The boiling point of alcohol at a pressure of 1 atmos. is $t_f = 78{\cdot}26°$ C., that of water at 60 mm. pressure is $t^1_w = 40°$ C., the constant for alcohol is $q = 0{\cdot}904$ (Dühring), thus the boiling point of alcohol at 60 mm. pressure is

$$t^1_f = 78{\cdot}26 - 0{\cdot}904(100 - 40) = 24{\cdot}02° \text{C}.$$

The constants q for about forty different liquids are given in Dühring's book (see above), by means of them Table 15 has been calculated, it gives for a number of liquids the boiling points under several diminished pressures, *viz.*, at vacua of 526, 611, 710 and 750 mm.

TABLE 15.

The boiling points of certain liquids at vacua of 526, 611, 710 and 750 mm., calculated by Dühring's rule.

	Constant. q	760 mm. abs.	230 mm. abs. 526 mm. vac.	139 mm. abs. 611 mm. vac.	50 mm. abs. 710 mm. vac.	10 mm. abs. 750 mm. vac.
		Boiling points, t^1_f.				
Water - - - - -	—	100	70	60	40	10
Alcohol - - - -	0·904	78·26	51·14	42·1	24·02	–3·1
Ether - - - - -	1·0	34·97	4·97	–5·03	–25·02	–55·09
Acetic acid - - -	1·164	119·7	84·58	73·17	49·84	15
Benzene - - - -	1·125	80·36	46·61	35·36	12·86	–20·9
Turpentine (oil of) -	1·329	159·15	119·28	106	79·81	29·54
Butyric acid - - -	1·228	161·70	124·86	111·6	87·02	51·2
Glycerin - - - -	1·25	290	252·5	240	215	177·5
Mercury - - - -	2	357·25	297·25	277·25	237·25	177·25
β-Naphthol - - -	2	290	230	210	170	110
Carbolic acid - - -	1·2	178	142	130	104	70
Cresol - - - - -	1·2	190	154	145	118	82

The second great advantage of evaporating in a vacuum is that the liquid does not become as hot as at atmospheric pressure, and that also the heating surfaces, since steam of a lower pressure is used, remain at a lower temperature—both great advantages, and even necessary for certain industries which deal with organic materials, such as milk, blood, gelatine, albumin. These substances require, if they are not to turn brown, or coagulate, not only that they themselves shall be evaporated at a low temperature (60°, 50°, 40° C.), but also that the heating surface shall not be too hot, in fact, shall not exceed certain limits which are different for each liquid. Now, as we have always observed, the side of the heating surface in contact with the liquid is always at a lower temperature than the side in contact with the heating medium, so that the latter may *be* somewhat warmer than the liquid may *become*, since the liquid never attains the highest temperature. This is, however, only the case when the liquid moves rapidly over the heating surface, so that its molecules have not time to attain a higher temperature and be injured thereby. Stirrers and violent ebullition afford a good protection against local overheating in liquids; however, these means are often insufficient, and then the best method consists in keeping the temperature of the steam so low that no damage may be done under the most unfavourable conditions. This result is achieved by the evaporation apparatus of C. Heckmann, Ger. Pat. No. 60,588.

The transference of heat between steam and liquid in vacuo is greater than at ordinary pressures, corresponding to the greater difference in temperature. Equation (47) may be used to calculate the heating surface, consisting of tubes containing steam, for vacuum evaporating apparatus—$H_v = \left(\frac{C}{2200\theta_m}\right)^2$.

Table 13 gives the evaporative efficiency of copper heating coils for vacuum apparatus also.

In the case of double bottoms it may be assumed that the transmission of heat takes place *in vacuo* according to equation (51).

$$C_v = H\theta_m k_v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

in which, For water, $k_v = 1600$;
,, thin liquors, $k_v = 1200$;
,, thick ,, $k_v = 900\text{-}500$.

Experience shows that in a vacuum apparatus at 650 mm. vacuum, there are evaporated in one hour per 1 sq. m. of heating surface :—

With exhaust steam at 110° C.,	from water	-	100-110 litres.
„ „ „ „ „	thin liquors	-	60- 70 „
„ „ „ „ „	thick „	-	30- 45 „
„ high pressure steam at 180° C.,	from water	-	130-175 „
„ „ „ „ „	thin liquors	-	80-100 „
„ „ „ „ „	thick „	-	40- 55 „

CHAPTER X.

THE MULTIPLE EFFECT EVAPORATOR.

THE processes which occur in a multiple evaporator, both in regard to the efficiency and the consumption of steam, are somewhat more complicated than in a simple evaporator, and not at first sight comprehensible. They will, therefore, be treated at some length. In considering these evaporators there are two questions of principal importance, which will be dealt with in the present chapter:—

A. How much water is converted into steam in each separate vessel of the multiple evaporator, and how much heating steam does each consume?

B. What is the composition (percentage of solid or dry matter) of the liquor in each vessel?

A. The Evaporative Capacity of Each Vessel

depends on the following conditions:—

1. The temperature and pressure of the heating steam.
2. The temperature and pressure of the steam produced in each separate vessel.
3. The extent to which the liquid is to be thickened, and its specific gravity.
4. The nature of the liquid, with regard to the ease with which it evolves steam.
5. The height of the boiling layer of liquid in each vessel.
6. Whether steam is withdrawn only from the first, or also from the following vessels ("extra steam," which may be used for heating other apparatus).
7. Whether the condensed water, from the steam used for heating, is separately removed from each vessel or whether it all leaves with the temperature of the last vessel.

It will be assumed at first that the liquid to be evaporated is introduced into the first vessel at the temperature therein prevailing, so that no expenditure of heat is required for raising the temperature in the first vessel.

It will be at once seen that the influence of all the above-mentioned conditions on the evaporative capacity cannot be expressed in figures, if the results of experience and experiment are not especially employed to assist. However, the conditions of each case, though expressed definitely in figures, may change so entirely and produce so many variations, that conclusions applicable in *all* cases cannot be drawn from a few cases, without great inaccuracy.

The process of evaporation is as follows:—

The steam from the liquor in the first vessel, D_1, produced by the action of the hot steam, D_0, which is supplied externally, passes into the heating chamber of the second vessel, there in its turn produces vapour from the liquid, and is condensed, escaping with the temperature, t_{u2}, prevailing in the lower part of the liquid in that second vessel. The weight of liquid, W, which has lost the weight of water, D_1, by evaporation in the first vessel, and which, consequently, now weighs $W - D_1$, passes, at the mean temperature, t_{m1}, of the first vessel, into the *second* vessel, in which the mean temperature is only t_{m2}. Thus, in cooling from t_{m1} to t_{m2} it must form steam. If c_2 be the total heat of the steam in the second vessel, then by reason of the hotter liquid entering from the first vessel

$$s_2 = \frac{(W - D_1)(t_{m1} - t_{m2})}{c_2 - t_{m2}} \quad . \quad . \quad . \quad . \quad . \quad (55)$$

kilos. of steam must be evolved.

In the *second* vessel steam is thus evolved *both* by reason of the heat of the hot liquid itself and *also* because of the steam, D_1, coming from the first vessel.

In the *third* vessel steam is produced *both* by the heat of the entering liquor ($W - D_1 - D_2$) and *also* by reason of the heat of the steam, D_2, which is the total steam produced in the second vessel.

In the *fourth* and following vessels similar actions are produced, so that, in addition to the repeated action of the hot steam, there is also the repeated action of the steam produced by the decrease

in temperature of the liquor. Since 1 kilo. of steam at 100° C. contains more heat than 1 kilo. of steam at 60° C., it follows that 1 kilo. of hot steam at 100° will produce more than 1 kilo. of steam at 60°. Neglecting the effects of higher boiling points and high columns of liquid, and considering simply the action of the steam, we find that 1 kilo. of steam, evolved in one vessel, must always produce more than 1 kilo. of steam in the next vessel, since the total heat (sensible and latent) of the hot steam is used, *minus* the quantity of heat carried away in the condensed water, the temperature of which is equal to that of the boiling liquid in the second vessel. In order to produce 1 kilo. of steam from this boiling liquid, there is thus required the heat proper to 1 kilo. of steam *minus* the quantity of heat contained in the liquid.

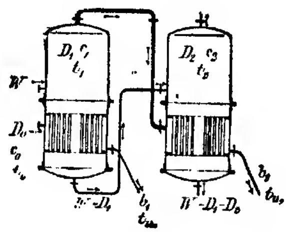

Fig. 7.

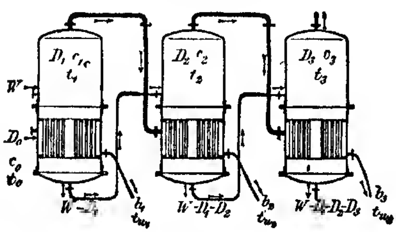

Fig. 8.

This purely schematic process suffers alterations by reason of the conditions enumerated above.

Although, as we shall see later, the somewhat complicated formulæ, based on the principles just laid down for estimating the evaporative capacity of each single vessel, have no great practical value, yet they will be given here.

Figs. 7 and 8 give diagrammatic pictures of double and triple effect evaporators, in which the subscripts represent the conditions at their respective positions:—

W = the weight of liquid introduced into the first vessel.
U = the weight of liquid drawn from the last vessel.
t_f = the temperature of the liquid to be taken into the first vessel.
D_0 = the weight of heating steam used in the first vessel.
c_0 = the total heat in 1 kilo. of this steam.
D_1, D_2, D_3 = the total weights of steam evolved in the vessels.
c_1, c_2, c_3 = the total heat in 1 kilo. of each of these quantities of steam.
t_1, t_2, t_3 = the temperatures in the steam spaces of the vessels I., II., III.
t_{m1}, t_{m2}, t_{m3} = the temperatures of the middle layers of the liquor.
t_{u1}, t_{u2}, t_{u3} = the temperatures in the lowest layers of the liquor.
b_1, b_2, b_3 = the weight of condensed water running out of the vessels.

The temperature of an evaporating liquid of any considerable depth is not the same at all parts; it is lowest at the top, highest at the bottom and has a mean value about the middle, since the specific gravity (which is almost always more than 1 and may reach 1·4), and the height of the column of liquid under which the vapour is evolved, cause a higher vapour pressure at the bottom, and thus a higher temperature of vapour and liquid.

In order to obtain the equations representing the consumption of heat in the separate vessels, the following facts are utilised:—

1. In the condition of equilibrium the quantity of heat supplied to one vessel must be equal to that which it gives out.
2. The weight of the heating steam used in each vessel is equal to the weight of the condensed water formed in that vessel.

For the *double effect* evaporator the following equations are deduced from these conditions:—

$$D_0 = b_1,\ D_1 = b_2,\ U = W - D_1 - D_2 \qquad (56)$$

$$D_2 = W - U - D_1$$

$$(W - D_1)t_{m1} + D_1c_1 = D_1t_{u2} + D_2c_2 + (W - D_1 - D_2)t_{m2}$$

$$D_1c_1 + Wt_{m1} - D_1t_{m1} = D_1t_{u2} + D_2c_2 + Ut_{m2}$$

$$D_1(c_1 - t_{m1} - t_{u2}) = Ut_{m2} - Wt_{m1} + Wc_2 - Uc_2 - D_1c_2$$

$$D_2 = W - U - D_1 \qquad (57)$$

$$D_1 = \frac{W(c_2 - t_{m1}) - U(c_2 - t_{m2})}{c_1 + c_2 - t_{m1} - t_{u2}} \quad \ldots\ldots\ldots\ldots \quad (58)$$

$$D_0 c_0 + W t_f = D_0 t_{u1} + D_1 c_1 + (W - D_1) t_{m1}$$

$$D_0 = \frac{D_1(c_1 - t_{m1}) + W(t_{m1} - t_f)}{c_0 - t_{u1}} \quad \ldots\ldots\ldots\ldots \quad (59)$$

For the *triple effect* evaporator the following equations are deduced from the same conditions:—

$$D_1 c_1 + (W - D_1) t_{m1} = D_2 c_2 + (W - D_1 - D_2) t_{m2} + D_1 t_{u2}$$

$$D_1 c_1 + W t_{m1} - D_1 t_{m1} = D_2 c_2 + W t_{m2} - D_1 t_{m2} - D_2 t_{m2} + D_1 t_{u2}$$

$$D_1(c_1 - t_{m1} + t_{m2} - t_{u2}) + W(t_{m1} - t_{m2}) = D_2(c_2 - t_{m2})$$

$$D_1 = \frac{D_2(c_2 - t_{m2}) - W(t_{m1} - t_{m2})}{c_1 - t_{m1} + t_{m2} - t_{u2}} \quad \ldots\ldots\ldots\ldots \quad (60)$$

$$D_2 c_2 + (U + D_3) t_{m2} = D_2 t_{u3} + D_3 c_3 + U t_{m3}$$

$$D_2 c_2 + U t_{m2} + D_3 t_{m2} = D_2 t_{u3} + D_3 c_3 + U t_{m3}$$

$$D_2(c_2 - t_{u3}) + U(t_{m2} - t_{m3}) = D_3(c_3 - t_{m2})$$

$$D_3 = \frac{D_2(c_2 - t_{u3}) + U(t_{m2} - t_{m3})}{c_3 - t_{m2}} \quad \ldots\ldots\ldots\ldots \quad (61)$$

$$D_1 + D_2 + D_3 = W - U$$

$$\frac{D_2(c_2 - t_{m2}) - W(t_{m1} - t_{m2})}{c_1 - t_{m1} + t_{m2} - t_{u2}} + D_2 + \frac{D_2(c_2 - t_{u3}) + U(t_{m2} - t_{m3})}{c_3 - t_{m2}} = W - U$$

$$D_2\left(1 + \frac{c_2 - t_{m2}}{c_1 + t_{m2} - t_{m1} - t_{u2}} + \frac{c_2 - t_{u3}}{c_3 - t_{m2}}\right) + \frac{U(t_{m2} - t_{m3})}{c_3 - t_{m2}} - \frac{W(t_{m1} - t_{m2})}{c_1 - t_{m1} + t_{m2} - t_{u2}} = W - U \quad \ldots \quad (62)$$

$$D_0 c_0 + W t_f = D_1 c_1 + D_0 t_{u1} + (W - D) t_{m1}$$

$$D_0 c_0 + W t_f = D_1 c_1 + D_0 t_{u1} + W t_{m1} - D_1 t_{m1}$$

$$D_0(c_0 - t_{u1}) + W(t_f - t_{m1}) = D_1(c_1 - t_{m1})$$

$$D_0 = \frac{D_1(c_1 - t_{m1}) - W(t_f - t_{m1})}{c_0 - t_{u1}} \quad \ldots\ldots\ldots\ldots \quad (63)$$

It must be admitted that the formulæ for the double effect are not very elegant, and for the triple effect are already exceedingly complicated; for the quadruple effect quite cumbrous formulæ would be obtained, which are therefore not given here, and which, moreover, would not be applicable in practice.

It would be *possible*, by means of these equations for the double and triple effect evaporators, to calculate the evaporative efficiency of

each single element, and the consumption of steam for the whole apparatus for any definite case, *if* the temperatures prevailing in each vessel were known. This is, however, *à priori* not the case, for in order to calculate the efficiency of an evaporator only the following are given :—

1. The evaporation, $W - W$, to be accomplished in unit time.
2. The temperature, t_r, at which the liquid enters.
3. The temperature of the heating steam, t_0, and its total heat, c_0.
4. The vacuum in the last vessel, hence t_3 and c_3.

The formulæ require, however, as has been said, a knowledge of a number of temperatures, which are conditioned by the form and size of the heating surfaces, the height of the boiling layer of liquid, and the specific gravity of the liquid, all of which are not known *à priori*.

It would thus be necessary, if the above equations were to be utilised, to assume arbitrary values to these temperatures, without warranty that they would really be attained in the constructed apparatus.

Thus the only possible way of recognising the influence of all these conditions, on the result, lies in calculating the evaporative capacity of the single parts of the apparatus for a large number of different conditions, chosen arbitrarily, with particular attention to limiting values. If the results so calculated be arranged in tabular form, then it will be fairly easy to see in each case how the result is altered when those conditions (temperatures, pressures, etc.), are varied which are independent of the data.

It is first necessary to consider in some detail the processes in the apparatus, before performing the calculations and arranging the tables.

It is at once evident the amount of evaporation in each vessel is not the same, but rather is different in each, since the liquor, in passing from a warmer to a colder vessel, must use its excess of heat in evaporating water. The larger is the difference in temperature between two vessels, the larger will be this evaporation, which we may call the *self-evaporation*. The difference in temperature between the single vessels of an evaporator may be very different.

It is of considerable importance to know how much hot steam must be supplied to the first vessel in order to accomplish a certain desired evaporation in the whole apparatus. Other conditions being the same, this necessary consumption of heating steam will be the

smaller, the more self-evaporation takes place in the separate vessels. On this account, also because a more accurate idea of the procedure of the evaporation will be obtained, and finally because it is the simplest course (especially if certain approximations be permitted), in the next place we shall find how much water is changed into steam by *self-evaporation* in each vessel of a multiple evaporator in different cases arbitrarily chosen, and then how much *heating steam* is used in each vessel, and especially in the first.

An inspection of Fig. 9 will facilitate the formation of the equations given below.

The specific heat, σ_f, of the liquid will in what follows always be taken as unity. Its boiling point will be taken as equal to that of water; if it is higher, the self-evaporation is somewhat larger.

In the first vessel, by means of the admitted heating steam, d_h, the weight of liquor, W, is first heated from its original temperature, t_f, to the temperature, t_{m1}, prevailing in the first vessel, and then by more heating steam, d_0, the weight of water, d_1, is converted into vapour. The condensed heating steam, $d_h + d_0 = b_1 = D_0$, flows away at the temperature, t_{u1}.

The consumption of heating steam in the first vessel is thus

$$D_0 = d_h + d_0 = \frac{W(t_{m1} - t_f) + d_1(c_1 - t_{m1})}{c_0 - t_{u1}} \quad . \quad . \quad . \quad (64)$$

In the *first* vessel the steam, d_1, is produced,

$$d_1 = D_1.$$

The liquor of weight $(W - d_1)$, at the temperature t_{m1}, enters the *second* vessel, in which the temperature is t_{m2}, and hence evolves steam from itself, forming the amount of steam, s_2, from its excess of heat $(W - d_1)(t_{m1} - t_{m2})$.

Thus $$s_2 = \frac{(W - d_1)(t_{m1} - t_{m2})}{c_2 - t_{m2}} \quad . \quad . \quad . \quad . \quad . \quad (65)$$

The steam from the first vessel, $d_1 = D_1$, enters the heating chamber of the second and produces steam in the second vessel:

then $$d_1(c_1 - t_{u2}) = d_2(c_2 - t_{m2})$$

therefore $$d_2 = \frac{d_1(c_1 - t_{u2})}{c_2 - t_{m2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

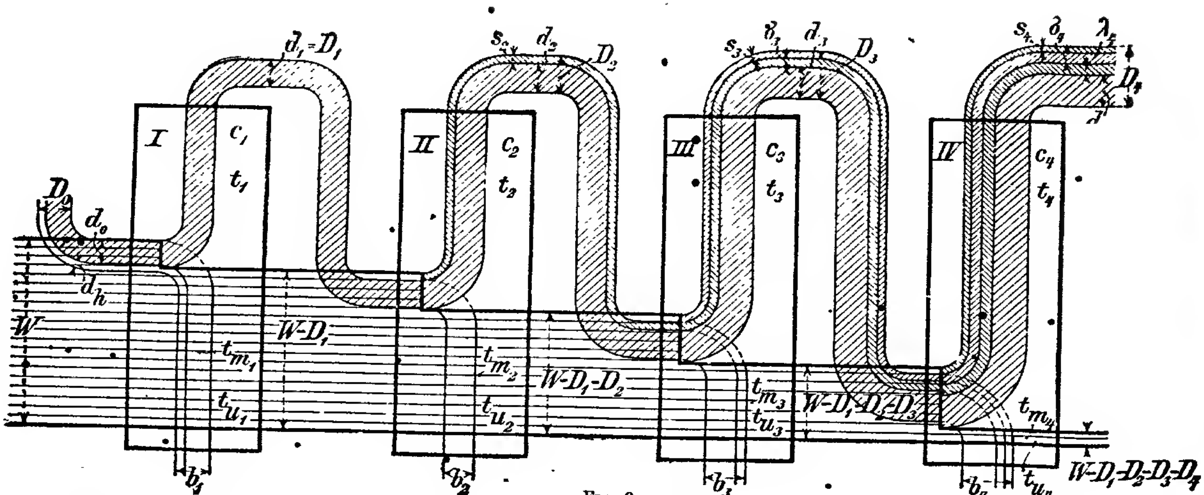

FIG. 9.

W = quantity of liquor which enters.
D_0 = total hot steam for vessel I.
d_h = steam for heating.
d_0 = steam for evaporating (produces $d_1 = D_1$).
D_1 = steam from vessel I. (produces d_2).
$D_2 = d_2 + s_2$ = total steam from vessel II.
d_2 = steam produced from d_1 (produces d_3).
s_2 = produced by self-evaporation in vessel II. (produces σ_3).

$D_3 = d_3 + s_3 + \sigma_3$ = total steam from vessel III.
d_3 = produced by d_2 (produces d_4).
σ_3 = produced by s_2 (produces λ_4).
s_3 = produced by self-evaporation in vessel III. (produces σ_4).
$D = d_4 + s_4 + \sigma_4 + \lambda_4$ = total steam from vessel IV.
s_4 = produced by self-evaporatio n vessel IV.

b_1, b_2, b_3, b_4 = condensed water from the four vessels.
t_1, t_2, t_3, t_4 = temperaturee of the steam in the four vessels.
$t_{m1}, t_{m2}, t_{m3}, t_{m4}$ = temperaturee of the liquor in the middle of each vessel.
$t_{u1}, t_{u2}, t_{u3}, t_{u4}$ = temperatures at the bottom of each vessel.
c_1, c_2, c_3, c_4 = total heat in 1 kilo. of steam.

Thus, in the *second* vessel the weight of steam, D_2; is formed:

$$D_2 = s_2 + d_2 = \frac{(W - D_1)(t_{m1} - t_{m2})}{c_2 - t_{m2}} + \frac{D_1(c_1 - t_{u2})}{c_2 - t_{m2}} \quad . \quad (67)$$

From the *second* vessel there goes *into the third* the weight $W - D_1 - D_2 = W - d_1 - s_2 - d_2$. This liquor is at the temperature t_{m2} and falls in the third vessel to the temperature t_{m3}. The difference in heat produces the weight of steam, s_3.

$$s_3 = \frac{(W - d_1 - s_2 - d_2)(t_{m2} - t_{m3})}{c_3 - t_{m3}} \quad . \quad . \quad . \quad . \quad (68)$$

The steam, s_2, produced by self-evaporation in the second vessel has the quantity of heat, c_2; in the *third* vessel it evaporates the weight of water, σ_3.

$$\sigma_3 = \frac{s_2(c_2 - t_{u3})}{c_3 - t_{m3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

Finally, there comes into the third vessel the steam, d_2, which in its turn produces the steam, d_3.

$$d_3 = \frac{d_2(c_2 - t_{u3})}{c_3 - t_{m3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

The total weight of steam, D_3, produced in the *third* vessel is thus $D_3 = s_3 + \sigma_3 + d_3$

$$= \frac{(W - d_1 - s_2 - d_2)(t_{m2} - t_{m3}) + (s_2 + d_2)(c_2 - t_{u3})}{c_3 - t_{m3}} \quad . \quad (71)$$

In the *fourth* vessel there is formed by self-evaporation the steam, s_4,

$$s_4 = \frac{(W - D_1 - D_2 - D_3)(t_{m3} - t_{m4})}{c_4 - t_{m4}} \quad . \quad . \quad . \quad . \quad (72)$$

also the weight of steam, σ_4, produced by the steam, s_3,

$$\sigma_4 = \frac{s_3(c_3 - t_{u4})}{c_4 - t_{m4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

and the weight of steam, λ_4, produced by the steam, σ_3,

$$\lambda_4 = \frac{\sigma_3(c_3 - t_{u4})}{c_4 - t_{m4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

Finally, the steam, d_3, produces in the fourth vessel the weight of steam, d_4,

$$d_4 = \frac{d_3(c_3 - t_{u4})}{c_4 - t_{m4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

In the *fourth* vessel there is thus produced the total weight of steam, D_4,

$$D_4 = s_4 + d_4 + \sigma_4 + \lambda_4$$
$$= \frac{\{W - (D_1 + D_2 + D_3)\}(t_{m3} - t_{m4}) + (d_3 + s_3 + \sigma_3)(c_3 - t_{m4})}{c_4 - t_{m4}} \quad (76)$$

It is now necessary to make a deviation, in order to simplify these still very complex equations, especially in regard to the many different temperatures.

It is known that the temperature of the boiling liquid is not the same in all parts; at its surface the boiling liquid has the temperature of the vapour evolved—t_1, t_2, t_3 or t_4—but at the bottom the steam bubbles have to penetrate the layer of liquid, they must therefore overcome a pressure corresponding to the column of liquid. Thus the steam must have a greater pressure at the bottom of the liquid than at the top, and to this pressure corresponds a higher temperature of the steam.

If s_f be the specific gravity of the boiling liquid, h_f its height in metres, B the height of the water barometer = 10·333 m., then the hydrostatic pressure at the lowest level of the liquid is, in atmospheres,

$$p = \frac{s_f \cdot h_f}{B} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (77)$$

or in millimetres of mercury,

$$b = \frac{s_f \cdot h_f \cdot 760}{B} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (78)$$

By means of this equation, the pressures of columns of liquid 0·2 to 2·0 m. in height, of specific gravities, s_f, from 1·0 to 1·4, have been calculated; the pressures are given in column 3 of Table 16. By adding to this pressure, the pressure above the liquid, the total pressure is obtained at the particular place, and thence, by means of the tables of Fliegner, Zeuner, etc. (see Table 9), the temperature of the vapour or liquid. The difference, $t_{u1} - t_1$, is the number of degrees of temperature by which the liquid at the bottom must be hotter than at the surface, in order to evolve steam.

In the diagram (Fig. 10) the abscissæ give the pressures of water vapour from 0-2 atmos. in cms., the ordinates the temperatures of the vapour at these pressures, according to Zeuner. By means of this diagram the temperatures in Table 16 were determined, by adding to the absolute pressure over the liquid the hydrostatic pressures given

in column 3, and then seeking in the diagram the temperature corresponding to the sum.

Curve showing the temperatures of steam at absolute pressures from 0 to 152 cms. of mercury.

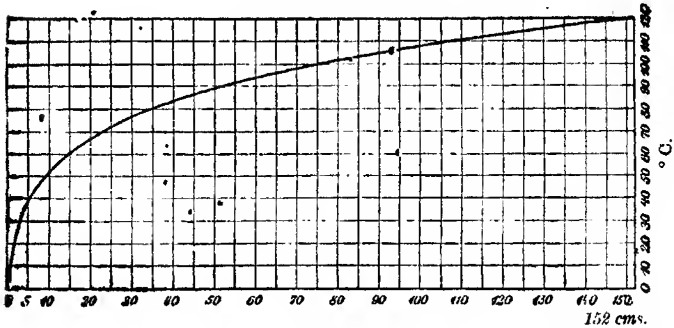

FIG. 10.

Example.—At a vacuum of 668 mm. the absolute pressure is 92 mm. of mercury, the temperature of water vapour 50° C. A column, $h = 1$ m. high, of liquid of the specific gravity, $s_f = 2$, exerts a hydrostatic pressure $b = \frac{2 \times 1 \times 760}{10 \cdot 333} = 147 \cdot 1$ mm. (equation 78). The total pressure at the bottom of the liquid is thus $92 + 147 \cdot 1 = 239 \cdot 1$ mm. At this pressure the diagram in Fig. 10 gives 70° C. The temperature of the liquid at the top is 50° C., thus the difference in temperature between top and bottom is $t_{u1} - t_1 = 70° - 50° = 20°$ C.

It will be seen from Table 16 that in the case of liquids under a pressure of 1 atmos. or more, the differences between the boiling points at the top and bottom are not very great, and are even quite moderate when the specific gravity and the height of the column of boiling liquid are great. If, however, there is a vacuum above the liquid, the difference between the upper and lower boiling points increases considerably, and, in the case of heavy liquids and high vacua, has a very disturbing effect.

There is, as we shall at once see, a circumstance which makes the retarding action on the heat transference of high columns of liquid less sensible, but in spite of that the rule remains that *it is in the interest of a great evaporative capacity to diminish as far as possible the height of the boiling layer of liquid, in order to lose as little as possible of the fall in temperature.*

The reason why the lower layers of violently boiling liquids, which are under the whole pressure of the column of liquid, are not at a temperature corresponding to their hydrostatic pressure, is the following:—

Consider a steam bubble rising through the liquid as divided by a horizontal plane at its greatest section, then a greater pressure is exerted on the lower half from below than on the upper from above. If the steam bubble had the shape of a cylinder with vertical axis and horizontal ends, the difference in pressure would be equal to the pressure of a column of liquid of the height of the cylinder. If the bubble were spherical, the difference in pressure would be equal to the height of a column of liquid of half the diameter of the sphere. (The upward force itself is equal to the weight of a quantity of liquid equal in volume to the bubble.)

In large vessels, in which many steam bubbles are rising at all parts, the hydrostatic pressure is not altered on this account, also in tubular heaters a small layer of liquor on the wall of the tube, connecting the liquid above and below the steam bubble, transmits the total hydrostatic pressure below. The larger and higher the bubble, the greater is the difference between the pressures acting on it from below and above, and this excess of pressure rapidly drives up the bubble and the liquid above it.

The kinetic energy of the liquid thus produced often raises considerable quantities above the surface, which then fall back and sink down at less heated parts of the apparatus. There is thus produced a circulation: the boiling liquid rises rapidly on and above the heating surface, gives off its steam and excessive heat and then returns cooled to the bottom.

The falling liquid is thus in fact cooler than it must be in order to form steam at the bottom, since it is only at the temperature of the surface. The difference in temperature (fall in temperature) between it and the heating steam is thus at first greater than it should be as a consequence of the hydrostatic pressure.

It should not be assumed that the differences of temperature, given in Table 16, between the upper and lower layers of boiling liquids, quite represent the actual conditions. These differences are in fact always less and only hold good for liquids at rest, which are not considered here.

Since the heights of the columns of liquid are generally made as

TABLE 16.

Increase in vapour pressure and rise in boiling point in the lowest gravities, $s_{,}$, of 1·0-1·40, and steam pressures

Temperature of evaporation at top - C. Absolute pressure at top - mm. Vacuum at top - mm.			116·4° 1330 —	111·7° 1140 —	106·3° 950 —	100° 760 —
Height of the liquid, h_f. Metres.	Specific gravity of the liquid. s_f.	Hydrostatic pressure of the liquid. mm. of mercury.	Temperature, in degrees Centigrade.			
0·20	1·0	15·49	0·0	0·5	0·5	0·5
	1·1	17·03	0·0	0·5	0·5	0·5
	1·2	18·58	0·0	0·5	0·5	0·5
	1·3	20·13	0·5	0·5	0·5	1
	1·4	21·68	0·5	0·5	0·5	1
0·50	1·0	38·73	0·5	0·5	1	1·5
	1·1	42·60	0·5	1	1	1·5
	1·2	46·76	0·5	1	1	2
	1·3	50·34	0·5	1	1·5	2
	1·4	54·22	0·5	1	1·5	2
0·75	1·0	58·10	0·5	1·5	1·5	2
	1·1	63·90	1	1·5	1·5	2·5
	1·2	69·72	1	1·5	1·5	3
	1·3	75·53	1	1·5	2	3
	1·4	81·34	1·5	2	2	3·5
1·0	1·0	77·47	1·5	2	2	3·5
	1·1	85·21	1·5	2	2·5	3·5
	1·2	92·96	1·5	2·5	2·5	3·5
	1·3	100·71	2	2·5	2·5	3·5
	1·4	108·45	2	2·5	3	4
1·5	1·0	111·20	2	2·5	3	4·5
	1·1	122·30	2·5	3	3·5	5
	1·2	133·44	2·5	3	3·5	5
	1·3	144·56	3	3·5	3·5	5
	1·4	151·68	3	3·5	3·5	5
2·0	1·0	154·91	3·5	3·5	3·5	5
	1·1	170·40	3·5	4·5	4·5	6
	1·2	185·89	3·5	4·5	5	6
	1·3	201·38	4	4·5	5	7
	1·4	216·87	4·5	5	5·5	7·5

TABLE 16.

layers of evaporating liquids at depths of $h_{,} = 0·2$-$2·0$ m., specific over the liquid of 1310 to 31·5 mm. of mercury.

95°	90°	80°	70°	60°	50°	40°	30°
633	525	354	233	148·7	92	54·9	31·5
126	234	405	526	611	668	705	728
by which the boiling point of the liquor is higher at the bottom than at the top.							
0·5	0·5	1	1	2·5	2·5	5	6·5
0·5	0·5	1·5	1·5	2·5	3	5	7
1	1	1·5	1·5	2·5	3	5	8
1	1	1·5	1·5	2·5	3·5	5·5	8·5
1	1	2	2·5	3	4	5·5	9
2	1·5	2·5	3·5	4·5	6·5	10	15
2	2·5	2·5	4	5	7	10	15·5
2·5	2·5	3	4·5	5·5	9	11	16
2·5	2·5	3	5	6	9·5	12	17
2·5	3	3·5	5	6·5	10	13	18
2·5	3	4	5	7	10·5	14	19
3	3·5	4·5	5·5	7·5	11	15	20
3	3·5	5	6	8	12	16	21
3	4	5	6·5	9·5	12·5	17	22
3·5	4·5	5	7	10	13	18	24
3·5	4·5	5	7	9·5	13	18	22
4	4·5	5	7·5	10·5	13·5	19·5	24·5
4	5	5·5	7·5	11	15	20	26
4·5	5	6	8	12	15·5	21	27·5
4·5	5	6·5	9	12·5	16·5	22	29
5	5·5	6·5	9·5	12·5	17	22·5	29·5
5	6	7	10	13·5	18	23	31
5	6·5	7·5	11	14·5	19·5	25	32
6·5	7	8·5	12	15	20·5	26	34
6	7	9	12·5	16	21	27·5	35
5·5	7·5	9	12·5	16	21	27·5	35·5
6·5	7·5	10	13	17·5	23	29·5	36·5
7	8	10	14	18·5	24·5	30	38·5
8	9	11	15	20	25·5	32	39
8·5	9·5	12	15·5	21	26·5	33·5	41

small as possible, and further, since the liquor in the first vessels of the apparatus rarely has a high specific gravity, in most cases in *calculating the quantity of steam developed in each vessel* this difference in temperature between the top and bottom may be neglected without introducing any considerable error. In fact the error due to this approximation is for the first vessel rarely more than 0·25 per cent., for the last vessel about 1 per cent., of the steam produced by self-evaporation, and may thus safely be neglected.

In determining the *efficiency* of the heating surface *per sq. m. and the temperature difference*, this difference between the temperature at top and bottom of the liquid should *not* be neglected.

To return to the equations. In agreement with the preceding remarks, by neglecting the differences in the temperatures of the liquor, and thus removing those temperatures which are *à priori* unknown, the equations previously given may now be written as below.

Consumption of heating steam in vessel I. :—

$$D_0 = \frac{W(t_1 - t_i) + d_1(c_1 - t_1)}{c_0 - t_1} \quad \ldots \ldots \quad (79)$$

Steam from vessel I. :—

$$D_1 = d_1 \quad \ldots \ldots \ldots \quad (80)$$

Steam from vessel II. :—

$$D_2 = \frac{(W - d_1)(t_1 - t_2) + d_1(c_1 - t_2)}{c_2 - t_2} \quad \ldots \quad (81)$$

$$s_2 = \frac{(W - d_1)(t_1 - t_2)}{c_2 - t_2} \qquad d_2 = \frac{d_1(c_1 - t_2)}{c_2 - t_2} \quad \ldots \quad (82)$$

Steam from vessel III. :—

$$D_3 = \frac{(W - d_1 - s_2 - d_2)(t_2 - t_3) + (s_2 - d_2)(c_2 - t_3)}{c_3 - t_3} \quad \ldots \quad (83)$$

$$s_3 = \frac{(W - d_1 - s_2 - d_2)(t_2 - t_3)}{c_3 - t_3} \qquad d_2 = \frac{d_1(c_1 - t_2)}{c_2 - t_2} \quad \ldots \quad (84)$$

$$\sigma_3 = \frac{s_2(c_2 - t_3)}{c_3 - t_3} \qquad d_3 = \frac{d_2(c_2 - t_3)}{c_3 - t_3} \quad \ldots \ldots \quad (85)$$

Steam from vessel IV. :—

$$D_4 = \frac{(W - D_1 - D_2 - D_3)(t_3 - t_4) + (d_3 + s_3 + \sigma_3)(c_3 - t_4)}{c_4 - t_4} \quad (86)$$

$$s_4 = \frac{(W - D_1 - D_2 - D_3)(t_3 - t_4)}{c_4 - t_4} \qquad d_2 = \frac{d_1(c_1 - t_2)}{c_2 - t_2} \quad . \quad (87)$$

$$\sigma_4 = \frac{s_3(t_3 - t_4)}{c_4 - t_4} \qquad d_3 = \frac{d_2(c_2 - t_3)}{c_3 - t_3} \quad . \; . \; . \; . \; . \quad (88)$$

$$\lambda_4 = \frac{\sigma_3(c_3 - t_4)}{c_4 - t_4} \qquad d_4 = \frac{d_3(c_3 - t_4)}{c_4 - t_4} \quad . \; . \; . \; . \; . \quad (89)$$

Steam from vessel V. :—

$$D_5 = \frac{(W - D_1 - D_2 - D_3 - D_4)(t_4 - t_5) + (s_4 + \sigma_4 + \lambda_4 + d_4)(c_4 - t_5)}{c_5 - t_5} \quad (90)$$

$$s_5 = \frac{(W - U)(c_4 - t_5)}{c_5 - t_5} \qquad d_2 = \frac{d_1(c_1 - t_2)}{c_2 - t_2} \quad . \; . \; . \; . \quad (91)$$

$$\sigma_5 = \frac{s_4(c_4 - t_5)}{c_5 - t_5} \qquad d_3 = \frac{d_2(c_2 - t_3)}{c_3 - t_3} \quad . \; . \; . \; . \quad (92)$$

$$\lambda_5 = \frac{\sigma_4(c_4 - t_5)}{c_5 - t_5} \qquad d_4 = \frac{d_3(c_3 - t_4)}{c_4 - t_4} \quad . \; . \; . \; . \quad (93)$$

$$\theta_5 = \frac{\lambda_4(c_4 - t_5)}{c_5 - t_5} \qquad d_5 = \frac{d_4(c_4 - t_5)}{c_5 - t_5} \quad . \; . \; . \; . \quad (94)$$

We proceed now, by the aid of these equations, to calculate the steam evolved in each vessel in any special case: for this calculation only the following are known:—

1. The quantity of liquor introduced, W, and its temperature, t_r.
2. The quantity of evaporated liquor drawn off, U, and its temperature, t_n (*i.e.*, t_2, t_3, t_4, or t_5).
3. The temperature and heat of the heating steam, t_0 and c_0.
4. The temperature and heat in the last vessel, t_n and c_n.

All the remaining values, especially the temperatures and pressures prevailing in the separate vessels, are unknown, for they depend essentially upon the ratio of the heating surfaces of the separate vessels to one another, and this ratio is different in almost every apparatus. It must thus be our next endeavour to ascertain the *most favourable* proportion of the heating surfaces, in order that the conditions for the least consumption of steam (D_0) may be found, and also that dimensions corresponding to its evaporative capacity may be given to each vessel. However, it is impossible at present to calculate these values for any special cases, because of the want of knowledge of the temperatures, consequently the only course is to *assume* the temperatures in the separate vessels for many cases, and

especially for the limiting cases, and on these assumptions to calculate the corresponding evaporative capacity of each vessel. When these many cases have been arranged in tabular form, it will be easy to select the best in each case. It will also appear from the calculations that *the amount of evaporation effected in the first vessel, and also the actual consumption of heating steam by the multiple effect evaporator, are not to any considerable extent proportional to the fall in temperature.*

In Table 17 is given the amount of evaporation obtained in double, triple and quadruple effect evaporators, in the separate vessels of which different falls in temperature are assumed. The figures are for the evaporation of 100 litres of liquor to one tenth (0·1), and one quarter (0·25); intermediate cases are not given, since it is found that the extent of the evaporation has not much influence upon the output, the reason being that the larger the portion of the original liquor which is *not* to be evaporated, the larger is the volume of liquor taken from vessel to vessel, and consequently also its self-evaporation in the next vessel. But this self-evaporation (which is the cause of the greater evaporation in the later vessels than in the earlier) is always but a small fraction of the whole evaporation. The method of calculating Table 17 will at once be illustrated by means of an example. It is always assumed that the liquor enters at the temperature of the first vessel, t_1. A lower temperature of the entering liquor, which frequently occurs in practice, must naturally be compensated in constructing the apparatus by increasing the heating surface of the first vessel; we shall afterwards return to this point.

In Table 17 are first given the temperatures t_1, t_2, t_3, t_4 (in separate columns), which are assumed as prevailing in each vessel. This is done for many cases, as far as possible for the limiting conditions. Also apparatus is considered which works at pressures above atmospheric, without an air pump, *e.g.*, in the second line for the triple effect:—

Vessel I., 130°; vessel II., 115°; vessel III., 100°.

Then, corresponding to each temperature, are given the total calories, c_0, c_1, c_2, c_3, c_4, contained in 1 kilo. of steam at these temperatures.

Example.—100 litres of liquor are to be evaporated to 10 litres in a quadruple-effect evaporator, in the elements of which the temperatures 100°, 95°, 85° and 50° C. are maintained. How much water is evaporated in each vessel?

In accordance with what has gone before, the problem can only be solved by a process of trials.

If 90 litres are to be evaporated, were there no self-evaporation, each vessel would evaporate $\frac{90}{4} = 22{\cdot}5$ litres; we know, however, that, as a matter of fact, by self-evaporation, the following (unknown) weights of steam are produced in the later vessels: s_2, $s_3 + \sigma_3$, $s_4 + \sigma_4 + \lambda_4$. Let us, therefore, *assume as a preliminary* that the evaporation is divided as follows:—

Vessel	I.	II.	III.	IV.	
Evaporation	20	22	23	25	litres.
Liquor introduced	100	80	58	35	"
The self-evaporation is then	0	$s_2=0{\cdot}75$	$s_3=1{\cdot}06$	$s_4=2{\cdot}14$	"
			$\sigma_3=0{\cdot}745$	$\sigma_4=1{\cdot}08$	"
				$\lambda_4=0{\cdot}756$	"

These weights of steam produced by self-evaporation are found from equations 79-89, *assuming* the total evaporation in each vessel, as follows.—

The self-evaporation in vessels II., III., and IV. is

$$s_2 = \frac{(W - d_1)(t_1 - t_2)}{c_2 - t_2} = \frac{80(100 - 95)}{635{\cdot}5 - 95} = 0{\cdot}74 \text{ kilo.}$$

$$s_3 = \frac{(W - D_1 - D_2)(t_2 - t_3)}{c_3 - t_3} = \frac{58(95 - 85)}{632 - 85} = 1{\cdot}06 \text{ kilo.}$$

$$s_4 = \frac{(W - D_1 - D_2 - D_3)(t_3 - t_4)}{t_4 - c_4} = \frac{35(85 - 50)}{691{\cdot}7 - 50} = 2{\cdot}14 \text{ kilos.}$$

The evaporation produced in vessel III. by means of the steam, s_2, is

$$\sigma_3 = \frac{s_2(c_2 - t_3)}{c_3 - t_3} = \frac{0{\cdot}74(635{\cdot}5 - 85)}{632 - 58} = 0{\cdot}745 \text{ kilo.}$$

In the vessel IV. s_3 evaporates

$$\sigma_4 = \frac{s_3(c_3 - t_4)}{c_4 - t_4} = \frac{1{\cdot}06(632 - 50)}{621{\cdot}7 - 50} = 1{\cdot}08 \text{ kilo.}$$

Finally, σ_3 effects in vessel IV. the evaporation, λ_4,

$$\lambda_4 = \frac{\sigma_3(c_3 - t_4)}{c_4 - t_4} = \frac{0{\cdot}745(632 - 50)}{621{\cdot}7 - 50} = 0{\cdot}756 \text{ kilo.}$$

Thus the preliminary calculation gives the following series of results:—

Vessel	I.	II.	III.	IV.	
Evaporation	20·87	21·62	22·67	24·85	litres.
Liquor introduced	100	79·13	57·51	34·85	kilos.

These results do not differ considerably from the assumptions made. If they are made the basis of a fresh calculation, in order to obtain greater accuracy, we have in a similar manner:—

$$s_2 = \frac{79\cdot13(100 - 95)}{635 - 95} = 0\cdot7325 \text{ litre.}$$

$$s_3 = \frac{57\cdot51(95 - 85)}{632 - 85} = 1\cdot051 \quad ,,$$

$$s_4 = \frac{34\cdot85(85 - 50)}{621\cdot7 - 50} = 2\cdot133 \quad ,,$$

$$\sigma_3 = \frac{0\cdot7325(635 - 85)}{632 - 85} = 0\cdot736 \quad ,,$$

$$\sigma_4 = \frac{1\cdot051(632 - 50)}{621\cdot7 - 50} = 1\cdot07 \quad ,,$$

$$\lambda_4 = \frac{0\cdot736(632 - 50)}{621\cdot7 - 50} = 0\cdot749 \quad ,,$$

From this final calculation we obtain the figures :—

Vessel	I.	II.	III.	IV.	
Self-evaporation	0	$s_2 = 0\cdot7325$	$s_3 = 1\cdot051$	$s_4 = 2\cdot133$	litres.
			$\sigma_3 = 0\cdot736$	$\sigma_4 = 1\cdot07$	,,
				$\lambda_4 = 0\cdot749$	,,
			Total, 1·787	Total, 3·952	,,

Self-evaporation and its consequences thus produce an evaporation of 0·7325 + 1·787 + 3·952 = 6·4715 litres of water; there remain still to evaporate 90 − 6·4715 = 83·5285 kilos., which weight is divided almost, but not quite, equally between the four vessels, in such a manner that the steam from one vessel always evaporates rather *more* than its own weight from the next vessel.

$$83\cdot5285 = d_1 + d_1\frac{c_1 - t_2}{c_2 - t_2} + d_1\frac{c_1 - t_2}{c_2 - t_2}\cdot\frac{c_2 - t_3}{c_3 - t_3}$$
$$+ d_1\frac{c_1 - t_2}{c_2 - t_2}\cdot\frac{c_2 - t_3}{c_3 - t_3}\cdot\frac{c_3 - t_4}{c_4 - t_4}.$$
$$= d_1\left(1 + \frac{637 - 95}{635\cdot5 - 95} + \frac{637 - 95}{635\cdot5 - 95}\cdot\frac{635\cdot5 - 85}{632 - 85} + \frac{637 - 95}{635\cdot5 - 95}\cdot\frac{635\cdot5 - 85}{632 - 85}\cdot\frac{632 - 50}{621\cdot7 - 50}\right).$$
$$= d_1(1 + 1\cdot004 + 1\cdot004 \times 1\cdot006 + 1\cdot004 \times 1\cdot006 \times 1\cdot02).$$
$$= d_1\, 4\cdot044.$$

Therefore $d_1 = \frac{83\cdot5285}{4\cdot044} = 20\cdot655$ litres of water.

$d_2 = 20\cdot655 \times 1\cdot004 = 20\cdot731$ litres of water.

$d_3 = 20\cdot731 \times 1\cdot006 = 20\cdot850$,,

$d_4 = 20\cdot850 \times 1\cdot020 = 21\cdot26$,,

Thus each vessel, including the self-evaporation, evaporates the following quantities of water :—

Vessel	I.	II.	III.	IV.	
Regular evaporation	20·655	20·731	20·850	21·26	litres.
Self-evaporation	0	0·7325	1·787	3·952	,,

Total - - 20·655 + 21·4635 + 22·637 + 25·212 = 89·9676 litres of water.

Table 17.

The Weights of Steam evolved in each separate vessel of a double, triple and quadruple effect evaporator per 100 litres of liquor: d_1, d_2, etc.; s_1, s_2, etc.; σ_2, σ_3, λ_4; by transference of heat and by self-evaporation, when the liquor is evaporated to 0·1 and 0·25 of its original weight. Regular evaporation (without extra steam) in apparatus with different falls of temperature.

Double effect.				Evaporation to 0·1 *W.*				Evaporation to 0·25 *W.*			
t_1	c_1	t_2	c_2	D_1	s_2	d_2	D_2	D_1	s_2	d_2	D_2
100	637	50	621·7	41·6	4·98	43·42	48·40	33·97	5·7	35·33	41·03
100	637	60	624·8	42·15	4·05	43·8	47·85	34·52	4·58	35·9	40·48
100	637	70	627·8	42·61	3·03	44·33	47·36	35·08	3·44	36·48	39·92
95	635·5	50	621·7	41·9	4·5	43·6	48·1	34·20	5·23	35·57	40·60
95	635·5	60	624·8	42·4	3·49	44·11	47·6	34·82	3·99	36·18	40·17
95	635·5	70	627·8	42·9	2·52	44·58	47·1	35·3	2·86	36·7	39·56
90	634	50	621·7	42·3	3·71	43·99	47·70	34·7	4·23	36	40·23
90	634	60	624·8	42·29	2·49	45·22	47·71	35·17	3·24	36·59	39·83
90	634	70	627·8	43	1·99	45·01	47·0	36·13	2·28	37·59	39·87
85	632	50	621·7	42·3	3·7	44·0	47·70	34·95	3·7	36·35	40·05
85	632	60	624·8	42·29	2·49	45·22	47·71	35·3	2·82	36·7	39·52
85	632	70	627·8	43·4	1·46	45·14	46·60	35·95	1·65	37·4	39·05
80	631	50	621·7	42·15	2·96	44·89	47·85	35·1	3·36	36·54	39·90
80	631	60	624·8	43	2·00	45	47	35·69	2·18	37·13	39·31
80	631	70	627·8	43·6	1·00	45·4	46·4	36·22	1·11	37·67	38·78
135	647·7	100	637	42·3	3·67	44·03	47·7	34·72	4·16	36·12	40·28
122·5	643·8	100	637	42·9	2·34	44·76	47·1	35·46	2·65	36·89	39·54
108	639·6	70	627·8	42·3	3·84	43·86	47·7	34·65	4·31	36·04	40·34
102·5	637·3	60	624·8	42	4·25	43·76	48	34·40	4·81	35·79	40·60
97·5	636·5	50	621·7	41·8	4·72	43·48	48·2	34·10	5·33	35·57	40·90
115	641·6	50	621·7	40·8	6·77	42·43	49·2	32·56	7·49	33·95	41·44
115	641·6	60	624·8	41·4	5·60	43·00	48·6	33·64	6·37	34·99	41·36
115	641·6	70	627·8	41·9	4·59	43·51	48·1	34·2	5·23	35·57	40·80
			Average	42·30	3·486	44·2	47·67	34·38	3·945	35·92	40·15

	Evaporation to 0·1 *W.*	Evaporation to 0·25 *W.*
	$D_1 : D_2 = 1 : 1·127$	$D_1 : D_2 = 1 : 1·17$
Minimum	$1 : 1·206$	$1 : 1·272$
Maximum	$1 : 1·07$	$1 : 1·07$
	$D_1 : d_2 = 1 : 1·045$	$D_1 : d_2 = 1 : 1·041$
Minimum	$1 : 1·07$	$1 : 1·042$
Maximum	$1 : 1·04$	$1 : 1·04$

TABLE 17—(*continued*).

Triple effect.						Evaporation to 0·1 W.				
t_1	c_1	t_2	c_2	t_3	c_3	D_1	s_2	d_2	D_2	s_3
140	649	130	646	100	637	27·8	1·39	28	29·39	2·84
130	646	115	641·6	100	637	27·7	2·04	28	30·04	1·17
130	646	115	641·6	50	621·7	26·56	2·07	26·82	28·89	4·78
130	646	115	641·6	60	624·8	26·8	2·07	27	29·07	4·10
130	646	115	641·6	70	627·8	26·8	2·07	27·1	29·17	3·39
125	644	105	638·5	60	624·8	26·56	2·60	26·82	29·42	3·4
125	644	105	638·5	70	627·8	26·56	2·60	26·82	29·42	2·8
120	643	110	640	100	637	28·37	1·32	28·65	29·97	0·78
120	643	95	635·5	50	621·7	26·17	3·38	26·43	29·81	3·3
120	643	95	635·5	60	624·8	26·4	3·38	26·6	29·98	2·6
120	643	95	635·5	70	627·8	26·64	3·38	26·96	30·34	1·86
115	641·6	95	635·5	70	627·8	27·16	2·6	27·43	30·03	1·86
115	641·6	90	634	60	624·8	26·8	3·1	27·06	30·16	1·94
115	641·6	85	632	50	621·7	25·96	4·03	26·22	30·26	2·60
105	638·5	95	635·5	50	621·7	27·54	1·33	27·81	29·13	3·3
105	638·5	95	635·5	60	624·8	27·72	1·33	28·04	29·37	2·6
105	638·5	95	635·5	70	627·8	28	1·33	28·2	29·53	1·86
100	637	90	634	50	621·7	27·78	1·31	28·05	29·36	2·6
100	637	90	634	60	624·8	28·03	1·31	28·31	29·62	1·94
100	637	90	634	70	627·8	28·30	1·31	28·48	29·79	1·30
100	637	80	631	50	621·7	27·03	2·62	27·30	29·92	2·20
100	637	80	631	60	624·8	27·28	2·62	27·55	30·17	1·45
100	637	80	631	70	627·8	27·54	2·62	27·81	30·43	0·75
97	636	84	632	70	627·8	27·94	1·70	28·17	29·87	1·00
95	635·5	80	631	50	621·7	27·43	1·9	27·70	29·60	2·2
95	635·5	80	631	60	624·8	27·74	1·9	27·94	29·84	1·45
93	635	76	630	60	624·8	27·61	2·25	27·88	30·13	1·18
90	634	80	631	50	621·7	27·91	1·30	28·18	29·48	2·2
90	634	70	627·8	50	621·7	27·31	2·58	27·58	30·16	1·45
95	635·5	85	632	50	621·7	27·78	1·31	28·05	29·36	2·60
95	635·5	85	632	60	624·8	28·02	1·31	28·30	29·61	1·85
					Average	27·33	2·147	27·59	29·72	2·22

TABLE 17—(*continued*).

$D_1 : D_2 : D_3 =$ 1 : 1·088 : 1·2048; $D_1 : d_2 : d_3 =$ 1 : 1·01 : 1·041			Evaporation to 0·25 W.				$D_1 : D_2 : D_3 =$ 1 : 1·106 : 1·26; $D_1 : d_2 : d_3 =$ 1 : 1·01 : 1·039			
σ_3	d_3	D_3	D_1	s_2	d_2	D_2	s_3	σ_3	d_3	D_3
1·44	29	32·78	22·62	1·49	22·84	24·33	3	1·51	23·54	28·05
2·12	29·1	32·39	22·62	2·20	22·84	25·04	1·5	2·24	23·54	27·28
2·15	27·62	34·55	21·10	2·23	21·31	23·54	6·15	2·27	21·95	30·35
2·15	27·95	34·20	21·395	2·23	21·6	22·83	5·25	2·27	22·26	29·78
2·15	28·49	34·03	21·74	2·23	21·95	24·18	4·18	2·27	22·63	29·08
2·7	27·62	33·72	21·31	2·9	21·52	24·42	4·18	2·96	22·18	29·34
2·7	27·62	33·12	21·57	2·9	21·78	24·68	3·35	2·96	22·44	28·75
1·37	29·77	31·92	23·34	1·4	23·57	24·97	1·0	1·42	24·27	26·69
3·51	27·22	34·03	20·83	3·6	21·03	24·63	4·2	3·67	21·67	29·54
3·51	27·5	33·61	21·10	3·6	21·31	24·91	3·36	3·67	21·96	28·99
3·51	27·71	33·08	21·41	3·6	21·62	25·22	2·42	3·67	22·28	28·37
2·7	28·25	32·81	21·91	2·85	22·12	24·97	2·42	2·9	22·80	28·12
3·2	27·64	32·78	21·31	3·53	21·52	25·05	2·9	3·6	22·18	27·68
4·19	27	33·79	20·63	4·31	20·83	25·14	3·37	4·39	21·47	29·23
1·38	28·65	33·33	22·27	1·42	22·49	23·91	4·2	1·44	23·17	28·81
1·38	28·88	32·86	22·53	1·42	22·75	24·17	3·36	1·44	23·56	28·30
1·38	29·2	32·44	22·86	1·42	22·08	24·50	2·42	1·44	23·78	27·64
1·36	28·90	32·86	22·41	1·41	22·63	24·04	3·78	1·44	23·34	28·55
1·36	29·25	32·45	22·70	1·41	22·92	24·33	2·9	1·44	23·64	27·97
1·36	29·35	30·01	23·04	1·41	23·27	24·68	1·89	1·44	23·96	27·98
2·72	28·12	33·04	21·77	2·83	21·28	24·81	2·89	2·88	22·65	28·42
2·72	28·38	32·55	22·09	2·83	22·31	25·14	1·89	2·88	23·00	27·77
2·72	28·65	32·12	22·40	2·83	22·62	25·45	0·97	2·88	23·30	27·15
2·1	29	32·13	22·94	1·81	23·16	24·97	1·35	1·84	23·90	27·09
2·25	28·52	32·97	22·31	2·0	22·53	24·53	2·89	2·04	23·23	28·16
2·25	28·79	32·49	22·64	2·0	22·86	24·86	1·89	2·04	23·57	27·5
2·34	28·79	32·26	22·52	2·36	22·74	25·10	1·53	2·4	23·45	27·38
1·35	29·06	32·61	22·73	1·37	22·95	24·32	2·89	1·39	23·67	27·95
2·68	28·41	32·54	22·13	2·77	22·35	25·12	1·90	2·82	23·03	27·75
1·36	28·90	32·86	22·58	1·39	22·81	24·20	3·31	1·41	23·49	28·21
1·36	29·16	32·37	22·89	1·39	23·11	24·50	2·40	1·41	23·80	27·61
2·244	28·46	32·925	22·12	2·295	22·335	24·47	2·89	2·335	22·99	27·89

TABLE 17—(*continued*).

Quadruple effect.								Evaporation to 1·0 *W*.								
t_1	c_1	t_2	c_2	t_3	c_3	t_4	c_4	D_1	s_2	d_2	D_2	s_3	σ_3	d_3	D_3	s_4
140	649·7	135	647·6	125	644·6	100	637	20·9	0·732	21·0	21·73	1·051	0·735	21·15	22·9	1·63
134	647·3	123	644	112	640·5	100	637	20·15	1·66	20·25	21·91	1·17	1·67	20·35	23·19	0·78
130	646·6	115	641·6	100	637	50	621·7	19	2·20	19	21·2	1·597	2·92	19·1	22·91	3·06
130	646·6	115	641·6	100	637	60	624·8	19·25	2·20	19·44	21·64	1·597	2·92	19·6	23·41	2·49
130	646·6	115	641·6	100	637	70	627·8	19·46	2·20	19·51	21·71	1·597	2·92	19·7	23·51	1·89
135	647·6	125	644·6	115	641·6	50	621·7	19·6	1·47	19·6	20·07	1·051	1·478	19·7	22·22	4·0
135	647·6	125	644·6	115	641·6	60	624·8	19·8	1·47	19·8	21·27	1·051	1·478	19·9	22·42	3·41
135	647·6	125	644·6	115	641·6	70	627·8	20	1·47	20	21·47	1·051	1·478	20·1	22·62	2·84
126·5	645·0	108	639·7	89·5	633·8	70	627·8	19·02	2·78	19·2	21·98	1 95	2·79	19·3	24·04	1·22
124	644	103	638	82	631·2	60	624·8	18·45	3·14	18·63	21·77	2·19	3·17	18·8	21·16	1·36
121·5	644·6	98	636·7	74·5	629·5	50	621·7	18·09	3·50	18·2	21·7	2·40	3·53	18·38	24·31	1·51
115	641·6	100	637	80	631	50	621·7	19·07	2·206	19·15	21·35	2·105	2·23	19·34	23·67	1·83
115	641·6	100	637	80	631	70	627·8	19·42	2·206	19·5	21·7	2·105	2·23	19·6	23 93	0·62
105	638·5	100	637	90	634	50	621·7	20·64	0·732	20·74	21·47	1·051	0·735	20·94	22·72	2·49
105	638·5	100	637	90	634	60	624·8	20·8	0·732	20·8	21·53	1·051	0·735	21	23·72	1·83
105	638·5	100	637	90	634	70	627·8	20·95	0·732	20·95	21·68	1·051	0·735	21·15	22·93	1·24
105	638·5	90	634	80	631	50	621·7	19·67	2·206	19·67	21·87	1·051	2·22	19 77	23·04	1·83
105	638·5	90	634	80	631	60	624·8	19·85	2·206	19·94	22 14	1·051	2·22	20·05	23·32	1·24
105	638·5	90	634	80	631	70	627·8	20	2·206	20·1	22·3	1·051	2·22	20·2	23·47	1·62
105	638·5	95	635·5	85	632	70	627·8	20·48	1·47	20·58	22·05	1·051	1·47	20·68	22·15	0·94
100	637	95	635·5	85	632	50	621·7	20·65	0·732	20·73	21·46	1·051	0·736	20·85	22·64	2·13
100	637	95	635·5	90	634	60	624·8	21·06	0·732	21·06	21·79	0·525	0·736	21·17	22·43	1·83
100	637	95	635·5	85	632	70	627·8	21·06	0·732	21·03	21·76	1·051	0·736	21·13	22·91	0·94
100	637	90	634	80	631	50	621·7	20·2	1·47	20·30	21·77	1·051	1·47	20·40	22·92	1·83
100	637	90	634	80	631	70	627·8	20·55	1·47	20·65	22·12	1·051	1 47	20·75	23·27	0·62
100	637	95	635·5	80	631	60	624·8	20·68	0·732	20·78	21·51	1·597	0·736	20·88	23·21	1·24
97·5	636·3	85	632	72·5	629·5	60	624·8	20·12	1·83	20·22	22·05	1·300	1·84	20·36	23·46	0·77
95	635·5	85	632	75	630	50	621·7	20·25	1·47	20·35	21·82	1·051	1·47	20·45	22·97	1·58
95	635·5	85	632	75	630	60	624·8	20·48	1·47	20·58	22·05	1·051	1·47	20·68	23·20	0·49
95	635·5	90	634	85	631	50	621·7	20·93	0·732	20·93	21·66	0·525	0·735	20·03	22·29	0·14
95	635·5	80	635·5	65	626·3	50	621·7	19·35	2·206	19·44	21·64	1·595	2·22	19·52	23·33	0·94
							Average	20·0	1·326	20·07	21·74	1·29	1·67	20·19	23·14	1·60

TABLE 17—(*continued*).

$D_1 : D_2 : D_3 : D_4 =$ $1 : 1·087 : 1·157 : 1·258$ $D_1 : d_2 : d_3 : d_4 =$ $1 : 1·0033 : 1·0095 : 1·024$				Evaporation to 0·25 W.								$D_1 : D_2 : D_3 : D_4 =$ $1 : 1·16 : 1·215 : 1·375$ $D_1 : d_2 : d_3 : d_4 =$ $1 : 1·008 : 1·016 : 1·017$				
σ_4	λ_4	d_4	D_4	D_1	s_2	d_2	D_2	s_3	σ_3	d_3	D_3	s_4	σ_4	λ_4	d_4	D_4
1·05	0·738	21·2	24·61	16·77	0·89	16·85	17·74	1·28	0·89	16·93	19·1	2·22	1·28	0·89	17·01	21·4
1·17	1·68	20·45	24·08	16·31	1·76	16·39	18·15	1·35	1·77	16·47	19·59	1·09	1·35	1·78	16·15	20·37
1·64	2·28	19·5	27·48	14·77	2·40	14·91	17·31	1·79	2·42	14·99	19·2	4·36	1·84	2·49	15·53	24·22
1·62	2·26	19·9	26·27	15·02	2·40	15·17	17·57	1·79	2·42	15·25	19·46	3·43	1·82	2·46	15·69	23·40
1·61	2·24	19·9	25·64	15·28	2·40	15·43	17·83	1·79	2 42	15·65	19·86	2·57	1·80	2·44	15·80	22·61
1·08	1·52	20·2	26·82	15·50	1·62	15·57	17·19	1·23	1·63	15 65	18·51	5·58	1·26	1·68	16·10	24·62
1·07	1·52	20·5	26·50	15·50	1·62	15·57	17·19	1·23	1·63	15·65	18·51	4·78	1·26	1·68	16·10	23·82
1·07	1·50	20·5	25·91	15·77	1·62	15·85	17·47	1·23	1·63	15·92	18·78	3·90	1·25	1·66	16·22	23·03
1·96	2·81	19·5	25·49	14·87	2·95	15·01	17·96	2·19	2·98	15·08	20·25	1·72	2·21	3·01	15·23	22·17
2·21	3·20	18·98	25·74	14·44	3·35	14·58	17·96	2·37	3·38	14·72	20·47	1·88	2·39	3·41	14·86	22·54
2·24	3·56	18·55	25·86	13·95	3·75	14·08	17·83	2·66	3·78	14·22	20·66	2·10	2·68	3·81	14·36	22·95
2·13	2·26	19·55	25·77	15	2·40	15·02	17·42	2·35	2·42	15·17	19·94	2·57	2·38	2·45	15·35	22·75
2·12	2·25	19·8	24·79	15·4	2·40	15·49	17·89	2·35	2·42	15·64	20·54	0·88	2 37	2·43	15·82	21·50
1·07	0·746	21·15	25·45	16 54	0·757	16·62	17·37	1·23	0·76	16 78	18·77	3·40	1·25	0·77	17·1	22·52
1·065	0·742	21·3	24·93	16·52	0·757	16·52	17·27	1·23	0·76	16·68	18·67	2·60	1·24	0·76	16·87	21·47
1·06	0·74	21·36	24·40	17	0·757	17	17·75	1·23	0·76	17·08	19·07	1·76	1·24	0·76	17·15	20·91
1·06	2·25	20·07	25·21	15·41	2·30	15·42	17·72	1·20	2·31	17·50	19 01	2·57	1·21	2·34	15·76	21·85
1·06	2·24	20·25	24·79	15·86	2·30	15·94	18·24	1·20	2·31	16·02	19·53	1·73	1·21	2·33	16·18	21·45
1·06	2·24	20·40	24·33	16·07	2·30	16·15	18·45	1·20	2·31	16·23	19·74	0·88	1·21	2·33	16·39	20·81
1·06	1·48	20·88	24·38	16·46	1·56	16·43	17·99	1·17	1·56	16·51	19·24	1·68	1·18	1·58	16·77	21·21
1·07	0·749	21·27	25·21	16·66	0·78	16·66	17·44	1·17	0·78	16·74	18·69	3·00	1·19	0·79	17·06	22·04
0·532	0·750	21·47	24·58	17·04	0·78	17·04	17·82	0·60	0·78	17·12	18·50	2·60	0·61	0·79	17·36	21·36
1·06	0·746	21·34	24·08	17·1	0·78	17·1	17·88	1·20	0 78	17·19	19·17	1·32	1·21	0·78	17·26	20·57
1·065	1·49	20·70	25·08	16·25	1·53	16·33	17·86	1·20	1·54	16·41	19·15	2·57	1·21	1·56	16·57	21·91
1·06	1·48	20·95	24·11	16·64	1·53	16·60	18·31	1·20	1·54	16·68	19·42	0·90	1·21	1·55	16·84	20·51
1·610	0·742	21·08	24·67	16·70	0·75	16·70	17·45	1·80	0·75	16·78	19·33	1·83	1·81	0·75	16·94	21·38
1·31	1·85	20·52	24·45	15·91	2·33	15·99	18·32	1·38	2·34	16·07	19·79	1·08	1·39	2·36	16·23	21·06
1·068	1·49	20·75	24·88	16·29	1·55	16·37	17·92	1·17	1·56	16·45	19·18	2·14	1·18	1·58	16·69	21·54
1·06	1·48	20·88	24·38	16·54	1·55	16·62	18·17	1·17	1·56	16·70	19·43	1·30	1·18	1·57	16·86	20·91
0·535	0·749	21·45	24·87	16·90	0·75	16·9	17·65	0·60	0·76	16·98	18·33	3·00	0·61	0·76	17·20	21·57
1·61	2·24	20·71	25·50	15·71	2·29	15·78	18·07	1·74	2·30	15·85	19·89	1·28	1·75	2·32	16·00	21·85
1·303	1·64	20·48	25·07	15·94	1·77	16·06	17·79	1·46	1·75	16·19	19·34	2·35	1·48	1·79	16·204	21·909

Results of Table 17.

	W = 100 litres of liquor are to be evaporated down to 0·1 W. There must thus be evaporated from it 90 litres of water,				to 0·25 W. 75 litres of water.			
In vessel - -	I.	II.	III.	IV.	I.	II.	III.	IV.
If divided equally. Double effect -	45	45	—	—	37·5	37·5	—	—
If divided equally. Triple " -	30	30	30	—	25	25	25	—
If divided equally. Quadruple " -	22·5	22·5	22·5	22·5	18·75	18·75	18·75	18·75
According to Table 17 each vessel actually evolves								
Double effect. Total - -	43·33	47·67	—	—	34·38	40·15	—	—
Double effect. Thus in the ratio	1 :	1·127	—	—	1 :	1·167	—	—
Double effect. Through heating alone - -	42·33	44·2	—	—	34·38	36·22	—	—
Double effect. Thus in the ratio	1 :	1·045	—	—	1 :	1·046	—	—
Triple effect. Total - -	27·33	29·72	32·925	—	22·12	24·47	27·89	—
Triple effect. Thus in the ratio	1 :	1·088 :	1·2048	—	1 :	1·106 :	1·26	—
Triple effect. Through heating alone - -	27·33	27·59	30·90	—	22·12	22·335	25·335	—
Triple effect. Thus in the ratio	1 :	1·006 :	1·1306	—	1 :	1·009 :	1·145	—
Quadruple effect. Total - -	20	21·74	23·14	25·17	15·94	17·79	19·34	21·929
Quadruple effect. Thus in the ratio	1 :	1·087 :	1·157 :	1·258	1 :	1·16 :	1·215 :	1·375
Quadruple effect. Through heating alone - -	20	20·07	21·86	23·42	15·94	16·06	17·94	19·47
Quadruple effect. Thus in the ratio	1 :	1·0083 :	1·093 :	1·171	1 :	1·008 :	1·125 :	1·223

In the mean the total evolution of steam is in the

Double effect - - $D_1 : D_2 = 1 : 1·147$
$= 0·4658 : 0·5442.$

Triple effect - $D_1 : D_2 : D_3 = 1 : 1·097 : 1·233$
$= 0·3003 : 0·3294 : 0·3703.$

Quadruple effect - $D_1 : D_2 : D_3 : D_4 = 1 : 1·123 : 1·187 : 1·316$
$= 0·2161 : 0·2427 : 0·2565 : 0·2844.$

In the mean the evaporative capacity (without self-evaporation) is in the

Double effect - $D_1 : d_2 = 1 : 1·045.$

Triple effect - $D_1 : d_2 : (d_3 + \sigma_3) = 1 : 1·0075 : 1·138.$

Quadruple effect $D_1 : d_2 : (d_3 + \sigma_3) : (d_4 + \sigma_4 + \lambda_4) = 1 : 1·0055 : 1·109 : 1·196.$

Table 17 has been calculated in the manner indicated in this example (p. 80). It is now possible to make a satisfactory inspection of the evaporative action of double, triple and quadruple effect evaporators, and to see without trouble how much water each vessel really vaporises, how much heating steam is used by each vessel, and in particular how much heating steam must be supplied to the first element, in order to bring 100 litres of liquor from the initial to any desired concentration. It is assumed that the liquid enters at the temperature t_{m1}.

If an *average* be taken of the figures in Table 17 for the whole quantity of water, *D*, evaporated in each vessel, and the quantity of steam, *d*, evolved by *heating* in each vessel (these averages are given at the bottom of the table), an extraordinary regularity in the evaporative capacity is seen, the extreme cases hardly varying by 5 per cent. from the average. The figures (also given in the Table) for the mean ratios of the total quantities, *D*, evaporated in the separate vessels, to the portions, *d*, evaporated by heating alone in the same vessels also vary very little from one another in the extreme cases, so that these figures may well be taken as a basis for the general case in practice.

These proportions of the amounts of steam in each vessel, d_1, d_2, d_3, d_4, will form the basis for the estimation of the necessary heating surfaces of the evaporator, to be given later.

Five important conclusions may be drawn from Table 17 to assist in the division of the heating surfaces in the most efficient manner:—

1. *The smallest amount of heating steam required to produce a certain amount of evaporation is used in all multiple evaporators, when the fall in temperature is the same in each vessel.*

2. *However the fall in temperature in the separate vessels be arranged, the weight of heating steam to be supplied to the first vessel always varies within very narrow limits. Thus the manner in which the available fall in temperature is distributed amongst the separate vessels has no great influence on the economy of steam. No considerable saving in steam can be obtained by any definite division of this fall in temperature.*

3. *The quantity of water to be evaporated in the first vessel is, on an average, of the total evaporation of the multiple evaporator:—*

In the double effect . $\frac{1}{2 \cdot 147} = 0 \cdot 466 \quad D_1 = (W - U)\, 0 \cdot 466.$

In the triple effect · $\frac{1}{3 \cdot 333} = 0 \cdot 300 \quad D_1 = (W' - U)\, 0 \cdot 300.$

In the quadruple effect $\frac{1}{4 \cdot 626} = 0 \cdot 216 \quad D_1 = (W'' - U)\, 0 \cdot 216.$

The extreme cases are :—

For the double effect · $D_1 = (W - U)\, 0 \cdot 434$ to $0 \cdot 484$.
For the triple effect · $D_1 = (W - U)\, 0 \cdot 2777$ to $0 \cdot 3152$.
For the quadruple effect · $D_1 = (W - U)\, 0 \cdot 1926$ to $0 \cdot 2335$.

4. *The evaporation effected by heating is in all cases the least in the first vessel, but the increase in the following vessels is not very great—at most 4 per cent. In the mean it may be assumed that this evaporation in the separate vessels is in the*

	Double effect.		*Triple effect.*			*Quadruple effect.*			
	I.	II.	I.	II.	III.	I.	II.	III.	IV.
	d_1 :	d_2	d_1 :	d_2 :	d_3	d_1 :	d_2 :	d_3 :	d_4
as	1 :	1·045	1 :	1·01 :	1·04	1 :	1·005 :	1·012 :	1·02

5. *The total quantity evaporated in the last vessel is :—*

In the double effect · · 0·534
In the triple effect · · 0·3703
In the quadruple effect · 0·284

of the total evaporation of the apparatus $(W - U)$.

B. The Percentage of Solids in the Liquid in Each Vessel of the Multiple Evaporator.

In the preceding section of the chapter it has been found that in performing a certain amount of evaporation, each separate vessel must evaporate its proper fraction, almost independently of the fall in temperature. In the next place, it is desirable to find the evaporative efficiency of each vessel and the percentage of solid matter in each, for liquors varying in strength both before and after evaporation; the results can only be approximate—never quite exact. The total evaporative capacity and the concentration in percentages are given in Table 18, which thus contains an answer to the questions :—

If a liquor of known strength (4-17 per cent.) is to be concentrated to another known strength (40-70 per cent.), how much water must with this intent be evaporated in each vessel and what is the concentration of the liquor in each vessel?

The following example illustrates the method of calculation of Table 18:—

Example.—100 kilos. of a liquor, containing 10 per cent. of solid matter, are to be evaporated to a strength of 50 per cent. in a triple effect evaporator. How much water is evaporated in each vessel and what is the concentration in each vessel?

In order to evaporate 100 kilos. of liquor from 10 per cent. to 50 per cent. strength, 100 − (10 + 10) = 80 kilos. of water must be evaporated.

Of this, according to Table 17,

Vessel I. evaporates	80 × 0·3003	= 24·02	kilos.
„ II. „	24·02 × 1·097	= 26·35	„
„ III. „	24·02 × 1·233	= 29·62	„
		79·99	„

Thus the first vessel contains

10 kilos. of solids in 100 − 24·02 = 75·98 kilos. of solution,

i.e., in the solution there is $\frac{10 \times 100}{75 \cdot 98} = 13 \cdot 16$ per cent. of solids.

The second vessel contains

10 kilos. of solids in 75·98 − 26·35 = 49·63 kilos. of solution,

i.e., in the solution there is $\frac{10 \times 100}{49 \cdot 63} = 20 \cdot 15$ per cent. of solids.

The third vessel contains

10 kilos. of solids in 49·63 − 29·62 = 20·01 kilos. of solution,

i.e., in the solution there is $\frac{10 \times 100}{20} = 50$ per cent. of solids.

Table 18.

The amount of evaporation, and the percentage of solids in the liquor, in each vessel of the double, triple and quadruple effect apparatus with regular evaporation (*i.e.*, no *extra steam* is withdrawn) for the concentration of 100 kilos. of liquor to 0·08 – 0·34 of its weight.

The *upper lines* of each pair in *ordinary* type, give the weights of water to be evaporated in each vessel.

The *lower figures*, in *heavy* type, give the corresponding percentages of dry material in the liquor in each vessel.

Initial strength of the liquor. Per cent.	Double effect.		Triple effect.			Quadruple effect.			
	D_1 I.	D_2 II.	D_1 I.	D_2 II.	D_3 III.	D_1 I.	D_2 II.	D_3 III.	D_4 IV.
	42·2	47·8	27·34	29·74	32·92	20	21·7	23·1	25 2
4	**6·92**	**40**	**5·5**	**9·32**	**40**	**5**	**6·86**	**11·4**	**40**
	40·95	46·55	26·69	29·11	32·25	19·4	21·07	22·5	24·63
5	**8·46**	**40**	**6·82**	**11·35**	**40**	**6·2**	**8·4**	**13·5**	**40**
	39·6	45·4	25·63	28·04	31·33	18·78	20·35	21·85	24 05
6	**9·93**	**40**	**8·07**	**13 03**	**40**	**7·38**	**9·86**	**15·3**	**40**
	38·35	44·15	24·83	27·25	30·52	18·24	19·71	21·11	23·44
7	**11·35**	**40**	**9·31**	**14·31**	**40**	**8·56**	**11·28**	**16·12**	**40**
	37	43	23·90	26·38	29·72	17·55	19	20·5	23
8	**12·7**	**40**	**10·51**	**16·09**	**40**	**9·7**	**12·6**	**18·6**	**40**
	35·87	41·88	23·15	25·60	29	17	18·43	19·92	22·41
9	**14·3**	**40**	**11·71**	**17·55**	**40**	**10 84**	**13·94**	**20·15**	**40**
	34·38	38·62	22·15	24·7	28·15	16·33	17·65	19·22	21·8
10	**15 4**	**40**	**12·84**	**18·76**	**40**	**11·95**	**15·1**	**21·4**	**40**
	32·82	39·43	21·23	23·77	27·25	15·67	16·86	18·56	21·16
11	**16·2**	**40**	**13·96**	**20**	**40**	**13·04**	**16·3**	**22·49**	**40**
	42·86	48·26	27·72	30·10	33·8	20·28	22	23·88	25·45
4	**7·0**	**45**	**5·53**	**9·48**	**45**	**5·02**	**6·9**	**11·68**	**45**
	41·64	47·25	26·96	29·37	32·57	19·72	21·42	22·84	24·91
5	**8·88**	**45**	**6·85**	**11·45**	**45**	**6·23**	**8·45**	**13·9**	**45**
	40·52	46·14	26·21	28·61	31·85	19·17	20·84	22·27	24·42
6	**10·09**	**45**	**8·13**	**13·28**	**45**	**7·42**	**10**	**15·85**	**45**
	39·32	45·18	25·45	27·87	31·13	18·61	20·21	21·71	23·89
7	**11·5**	**45**	**9·35**	**15·0**	**45**	**8·6**	**11·28**	**17·7**	**45**
	38·21	44·02	25·02	27·46	30·75	18·15	19·66	21·06	23·88
8	**12·94**	**45**	**10·67**	**16·90**	**45**	**9·77**	**12·85**	**19·45**	**45**
	37	48	23·90	26·38	29·72	17·5	19·1	20·50	22·9
9	**14·29**	**45**	**11·83**	**18·1**	**45**	**10·91**	**14·14**	**20·9**	**45**

TABLE 18—(*continued*).

Initial strength of the liquor. Per cent.	Double effect.		Triple effect.			Quadruple effect.			
	D_1 I.	D_2 II.	D_1 I.	D_2 II.	D_3 III.	D_1 I.	D_2 II.	D_3 III.	D_4 IV.
	86	42	23·2	25·69	29·06	17·1	18·7	20·3	22·7
10	15·62	45	13·02	19·58	45	12·06	15·57	22·8	45
	35	41	22·41	24·86	28·67	16·5	17 8	19·4	21·8
11	16·85	45	14·3	20·86	45	13·17	16·74	23·76	45
	43·3	48·7	28·04	30·76	33·62	20·5	22·2	23·6	25·7
4	7·06	50	5·55	9·7	50	5·03	6·95	11·85	50
	42·2	47·8	27·34	29·74	32·92	20	21·7	23·1	25·1
5	8·65	50	6·88	11·66	50	6·25	8·57	14·2	50
	41·2	4[illegible]·8	26 61	29·04	32·23	19·51	21·2	22·6	24·8
6	10·2[illegible]	50	8 17	13·5	50	7·45	10 1	16·3	50
	40·2	45·8	26	28·44	31·66	19·01	2[illegible]·6	22·1	24·3
7	11·7	50	9·46	15·37	50	8·64	11·58	18·3	50
	39·1	44·9	25·28	27·74	31	18·54	20	21·5	23·9
8	13·13	50	10·70	17·00	50	9·81	13·01	20	50
	38·1	43·9	24·56	27	30 32	18·04	19 5	21	23·4
9	14 54	50	11·93	18 58	50	10·9	14·4	21·7	50
	37	43	24	26·35	29·63	17·55	19	20 5	23
10	15·87	50	13·16	20·15	50	12·13	15·76	23·5	50
	36	[illegible]2	23·22	25·7	29·08	17·06	18·5	20	22·5
11	17·19	50	14·32	21·53	50	13·26	17·07	24·7	50
	35	41	22 5	25	28·41	16·58	17·9	19·5	22
12	18·5	50	15·49	22·85	50	14·37	18·31	26·29	50
	33·9	40·1	21·85	24·4	27·85	16·08	17·4	18·97	21·55
13	19·66	50	16·63	24·19	50	15·49	19·53	27·33	50
	32·8	39·2	21·45	23·4	27·26	15·5	16·9	18·5	21·1
14	20·83	50	17·82	25·4	50	16·57	20·7	28·5	50
	31 8	38·2	20·4	23	26·45	15	16·3	18	20 6
15	22	50	18·9	26·5	50	17·65	21·83	29·5	50
	30·8	37·2	19·76	22·36	25·81	14·5	15·8	17 5	20·1
16	23·12	50	19·9	27·69	50	18 71	23	30·6	50
	29·8	36·2	19·1	21·7	25·15	14·0	15·3	17	19·6
17	24·2	50	21·01	28·7	50	19·78	24·05	31·6	50
	43·76	49·07	28·3	30·66	33·81	20·68	22·42	23·78	25·83
4	7·11	55	5·57	9·74	55	5·04	7·03	12·07	55
	43·21	48·61	27·96	30·34	33·52	20·45	22·2	23·08	25·62
5	8·80	55	6 9	11·76	55	6·28	8·72	14·8	55
	41·74	47·35	27·03	29·43	32·63	19·75	21·47	22·87	24·97
6	12·9	55	8·22	13·18	55	7·47	10·2	16·9	55
	40·83	46·44	26·41	28·84	32·03	19·32	20·99	22·42	24·57
7	11·83	55	9·5	15·65	55	8·67	11·7	18·8	55

TABLE 18—(*continued*).

Initial strength of the liquor. Per cent.	Double effect.		Triple effect.			Quadruple effect			
	D_1 I.	D_2 II.	D_1 I.	D_2 II.	D_3 III.	D_1 I.	D_2 II.	D_3 III.	D_4 IV.
	39·93	45·53	25·78	28·21	31·47	18·86	20·50	21·96	24·14
8	13·31	55	10·78	17·4	55	9·86	13·2	20·6	55
	38·92	44·72	25·16	27·6	30·89	18·45	20·01	21·41	23·71
9	14·73	55	12·02	19·04	55	11·03	14·62	22·4	55
	38·01	43·71	24·38	27·02	30·36	18·01	19·55	20·95	23·27
10	16·13	55	13·22	20·57	55	12·2	16	24·1	55
	37	43	23·94	26·4	29·75	17·55	19	20·5	23
11	17·46	55	14·46	22·14	55	13·3	17·3	25·6	55
	36·09	42·09	23·30	25·77	29·2	17·13	18·55	20·05	22·45
12	18·77	55	15·64	23·56	55	14·48	18·68	27·1	55
	35·18	41·19	22·76	25·15	28·52	16·67	18·1	19·6	22
13	20·56	55	16·83	24·95	55	15·6	19·02	28·5	55
	34·07	40·48	22	24·55	28	16·22	17·54	19·14	21·65
14	21·23	55	18	26·36	55	16·71	21·14	29·7	55
	33	39·55	21·32	23·85	27·38	15·73	17·03	18·63	21·12
15	22·36	55	19·06	27·4	55	17·8	22·15	30·8	55
	32·35	40·48	20·73	23·33	26·78	15 22	16·52	18·22	20·82
16	23·7	55	20·16	28·6	55	18·87	23·41	32·16	55
	31·9	39·9	20·40	23·0	26·45	15·0	16·3	18·0	20·6
17	24·95	55	21·35	30·04	55	20	24·74	33·5	55
	44·62	49·21	28·48	30·85	34·0	20·83	22·59	23·96	25·97
4	7·15	60	5·59	9·85	60	5 05	7·06	11·9	60
	44·13	48·54	27·93	30·30	33·38	20·42	22·16	23·52	25·59
5	8·79	60	6·93	11·99	60	6·28	8·74	14·7	60
	42·2	48·59	27·34	29·74	32·92	20	21·7	23·1	25·2
6	10·39	60	8·26	13·68	60	7·5	10·29	17·05	60
	41·41	47·02	26·8	29·22	32·42	19·61	21·31	22·71	24·84
7	11·94	60	9·56	15·8	60	8·7	11·85	19·2	60
	40·53	46·14	26·21	28·61	31·85	19·07	20·84	22·27	24·42
8	13·45	60	10·84	17·7	60	9·88	13·33	21·2	60
	39·6	45·4	25·6	28·04	31·2	18·78	20·35	21·85	24·05
9	14·9	60	12·1	19·41	60	11·08	14·7	23·06	60
	38·77	44·57	25·05	27·50	30·79	18·4	19·94	21·84	23·66
10	16·33	60	13·34	21·08	60	12·25	16·22	24·8	60
	37·94	43·74	24·48	26·94	30·26	17·95	19·55	20·90	23·8
11	17·72	60	14·56	22·64	60	13·4	17·6	26·4	60
	37	43	23·94	26·4	29·75	17·55	19	20·5	23
12	19·1	60	15·78	24·15	60	14·5	18·6	27·7	60
	36·17	42·17	23·35	25·82	29·17	17·13	18·57	20·07	22·57
13	20·37	60	16·96	25·56	60	15·69	20·22	29·38	60
	35·33	41·34	22·79	25·26	28·62	16·74	18·08	19·68	22·17
14	21·65	60	18·13	26·89	60	16·81	21·48	30·77	60

TABLE 18—(*continued*).

Initial strength of the liquor. Per cent.	Double effect.		Triple effect.			Quadruple effect.			
	D_1 I.	D_2 II.	D_1 I.	D_2 II.	D_3 III.	D_1 I.	D_2 II.	D_3 III.	D_4 IV.
15	34·38 22·86	40·62 60	22·15 19·27	24·70 28·22	28·15 60	16·33 17·9	17·65 22·7	19·22 32	21·8 60
16	33·42 24·03	39·92 60	21·60 20·40	24·14 29·48	27·61 60	15·93 19·03	17·14 23·9	18·84 33·28	21·44 60
17	32·7 25·25	38·1 60	21·35 21·6	23·36 30·73	27·16 60	15·5 20·11	16·9 25·1	18·5 34·6	21·07 60
4	44·35 7·18	49·52 65	28·66 5·6	31·03 9·92	34·17 65	20·96 5·06	22·72 7·1	24·06 12·4	26·1 65
5	43·55 8·85	48·76 65	28·15 6·91	30·52 12·1	33·66 65	20·58 6·28	22·32 8·75	23·68 15	25·75 65
6	42·58 10·40	48·19 65	27·61 8·29	30 14·16	33·17 65	20·19 7·51	21·91 10·36	23·29 17·3	25·37 65
7	41·8 12·08	47·43 65	27·1 9·6	29·5 16·12	32·70 65	19·81 8·73	21·51 11·93	22·91 19·6	25·08 65
8	41 13·57	46·1 65	26·54 10·89	28·97 17·99	32·2 65	19·42 9·93	21·09 13·45	22·52 21·6	24·66 65
9	40·28 15·07	45·88 65	26·03 12·16	28·45 19·79	31·68 65	19·05 11·12	20·72 14·93	22·15 23·6	24·22 65
10	39·4 16·5	45·2 65	25·5 13·43	27·9 21·46	31·2 65	18·7 12·4	20·25 16·38	21·65 25·4	23·95 65
11	38·5 17·8	44·5 65	24·93 14·66	27·42 23·11	30·7 65	18·3 13·46	19·90 17·8	21·3 27·1	23·6 65
12	37·86 19·31	43·67 65	24·93 15·75	26·9 24·8	30·2 65	17·92 14·62	19·46 19·1	20·88 28·78	23·28 65
13	37 20·63	43 65	23·94 17·09	26·4 26·2	29·75 65	17·55 15·77	19 20·49	20·5 30·28	23 65
14	36·25 21·94	42·25 65	23·41 18·28	25·88 27·6	29·21 65	17·18 16·90	18·61 21·80	20·12 31·70	22·6 65
15	35·36 23·20	41·56 65	22·91 19·33	25·3 28·9	28·70 65	16·9 18·05	18·13 23·09	19·73 33·2	22·13 65
16	34·68 24·5	40·68 65	22·32 20·6	24·82 30·27	28·22 65	16·44 19·15	17·74 24·31	19·34 34·41	21·84 65
17	33·72 25·65	40·13 65	21·77 21·73	24·31 31·5	27·78 65	16·07 20·26	17·26 25·50	18·96 35·63	21·56 65
4	44·54 7·21	49·75 70	28·83 5·62	31·14 10	34·35 70	21·07 5·07	22·83 7·13	24·17 12·5	26·54 70
5	43·83 8·89	49·03 70	28·33 7·0	30·70 12·20	33·84 70	20·71 6·31	22·45 8·79	23·81 15·15	25·86 70
6	43·01 10·53	48·43 70	27·83 8·31	30·20 14·3	33·4 70	20·36 7·53	22·1 10·43	23·46 17·5	25·53 70

TABLE 18—(*continued*).

Initial strength of the liquor. Per cent.	Double effect.		Triple effect.			Quadruple effect.			
	D_1 I.	D_2 II.	D_1 I.	D_2 II.	D_3 III.	D_1 I.	D_2 II.	D_3 III.	D_4 IV.
	42·2	47·8	27·34	29·75	32·96	20	21·7	23·1	25·2
7	**12·11**	**70**	**9·63**	**16·31**	**70**	**8·75**	**12·01**	**20**	**70**
	41·48	47·09	26·85	29·26	32·47	19·64	21·34	22 74	24·87
8	**13·67**	**70**	**10·94**	**18·23**	**70**	**9·95**	**13·5**	**22·04**	**70**
	40·77	46·37	26·39	28·85	32·01	19·29	20·96	22·39	24·54
9	**15·2**	**70**	**12·22**	**20·11**	**70**	**11·15**	**15·06**	**24·1**	**70**
	40·05	45·66	25·86	28·3	31·56	18·93	20·57	22·03	24·21
10	**16·52**	**70**	**13·49**	**21·81**	**70**	**12·33**	**16·53**	**26**	**70**
	39·24	45·05	25·39	27·82	31·09	18·57	20·17	21·67	23·85
11	**18·1**	**70**	**14·74**	**23·5**	**70**	**13·5**	**17·9**	**27·78**	**70**
	38·52	44·31	24·88	27·33	30·62	18·3	19·81	21·21	23·51
12	**19·5**	**70**	**15·98**	**25·07**	**70**	**14·69**	**19·38**	**29·48**	**70**
	37·81	43·62	24·4	26·86	30·18	17·9	19·46	20·86	23·21
13	**20·9**	**70**	**17·19**	**26·6**	**70**	**15·83**	**20·75**	**31·11**	**70**
	37	43	23·9	26·38	29·72	17·5	19·1	20·5	22·9
14	**22·2**	**70**	**18·39**	**28·2**	**70**	**16·97**	**22 08**	**32·63**	**70**
	36·28	42·27	23 42	25·9	29·24	17·2	18·65	20·15	22·56
15	**23·54**	**70**	**19·59**	**29·6**	**70**	**18·12**	**23·38**	**34·09**	**70**
	35·57	41·57	22·95	25·43	28·79	16·74	18·29	19·79	22·31
16	**24·83**	**70**	**20·76**	**30·98**	**70**	**19·21**	**24·59**	**35·33**	**70**
	34·85	40·85	22·44	24 94	28·3	16·60	17·8	19·40	21·9
17	**26·09**	**70**	**21·92**	**32·3**	**70**	**20·38**	**25·91**	**36·9**	**70**

CHAPTER XI.

MULTIPLE EFFECT EVAPORATORS, IN WHICH STEAM ("EXTRA STEAM") IS TAKEN FROM THE FIRST AND FOLLOWING VESSELS FOR OTHER PURPOSES THAN TO HEAT THE NEXT VESSEL.

In the foregoing, those multiple evaporators have been considered, in which the steam produced in the first vessel is only used to heat the next vessel, *i.e.*, in which the operation of repeatedly using the steam is carried out without interference. It is, however, often the case that from the first, and frequently from later vessels, considerable quantities of steam are taken to be used for other manufacturing purposes. This method has the advantage of economising steam, for when steam is taken direct from the boiler for other purposes than for the evaporator, a certain consumption of fuel is necessitated. Naturally when this specially required steam is drawn from the first vessel of the evaporator, additional high pressure steam has to be supplied, since as much more heating steam must be supplied to the first vessel as is necessary to produce the steam taken from it. But then this *extra* steam is produced from the liquor, which is thus freed from the weight of water turned into steam, which weight of water has not now to be removed by a separate consumption of high pressure steam.

It is noteworthy that, when this *extra steam* is taken from the second or one of the following vessels, the economy in high pressure steam is still greater, for steam is now used for manufacturing purposes which has already removed several times its own weight of water in the evaporator. It would naturally be most advantageous to take the steam required for other purposes from the last vessel of the evaporator, which is indeed done, when practicable, but it must be remembered that the temperature of the steam falls considerably from the first to the last vessel, and the *extra steam* must thus

be drawn from that particular earlier vessel which affords a sufficiently high temperature.

The saving for every 100 kilos. of *extra steam*, taken from the vessels indicated, is as follows :—

	Double effect.	Triple effect.	Quadruple effect.	
From vessel I.	47·5	31	22·5	kilos. of heating steam.
„ „ II.	—	62	45·0	„ „ „
„ „ III.	—	—	67·5	„ „ „

Just as in the preceding section there are here two questions to answer :—

A. How much water must be evaporated in each vessel of a multiple evaporator, when *extra steam* is taken from the separate vessels?

B. What is then the strength of the solution in each vessel?

A. How much Water must be Evaporated in Each Vessel of a Multiple Effect Evaporator when Extra Steam is taken from the Separate Vessels?

The diagrammatic representation of the evolution of steam in the separate vessels given in Fig. 11 provides a clear idea of the process. We may suppose the production of *extra steam* in all the vessels completely separated from the regular evaporation of the liquor, for it may be assumed that there are separately introduced into the first vessel :—

1. The water, which is to be converted into steam in the various vessels by the extra evaporation, then to emerge partly as steam, partly as condensed water.

2. The liquor, which was originally mixed with this water but is now separate from it, and which now contains the same quantity of solid matter as originally, but less water by the amount which is to be used in the formation of *extra steam*. The liquor is thus to be supposed more concentrated from the beginning. We can find the quantity of water to be evaporated in each vessel and in all together for the purpose of producing *extra steam*. By subtracting this weight of water from the total weight of liqnor, we obtain the weight of liquor to be evaporated, on our supposition, *in the ordinary manner*.

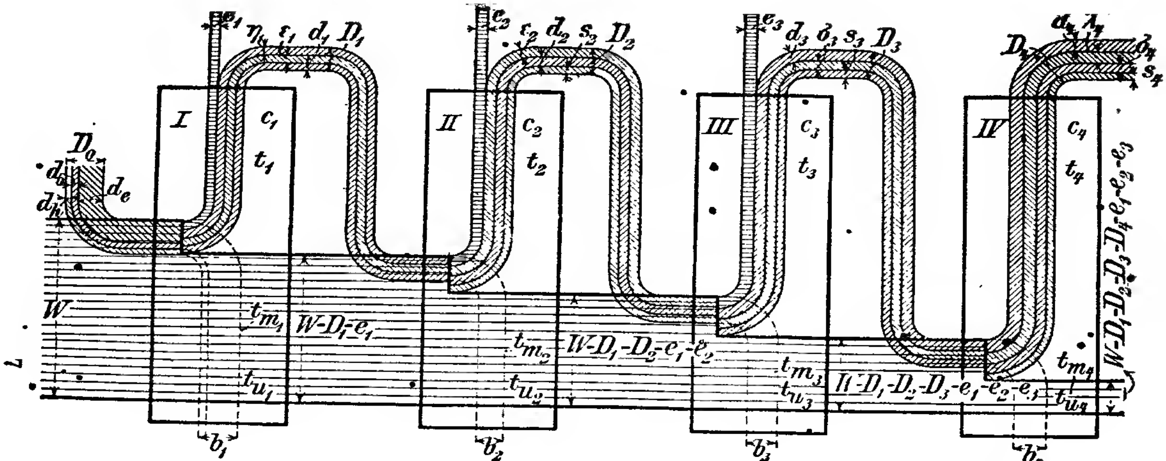

FIG. 11.

W = quantity of liquor which enters.
D_0 = total heating steam for vessel I.
d_h = steam for heating the liquor.
d_0 = steam for evaporating (produces $d_1 = D_1$).
d_e = heating steam for the product on of *extra steam* (produces e_1, ϵ_1, η_1).
e_1 = *extra steam* taken from vessel I.
η_1 = produced from d_e (produces e_2).
ϵ_1 = produced from d_e (produces ϵ_2).
d_1 = steam from vessel I. (produces d_2).
$D_1 = d_1 + \epsilon_1 + \eta_1$ = total steam from vessel I. to vessel II.

e_2 = *extra steam* taken from vessel II.
d_2 = produced from d_1 (produces d_3).
ϵ_2 = produced from ϵ_1 (produces e_3).
s_2 = produced by self-evaporation in vessel II. (produces σ_3).
$D_2 = d_2 + s_2 + \epsilon_2$ = total steam from vessel II. to vessel III.
e_3 = *extra steam* taken from vessel III. (produced from ϵ_2 which is from ϵ_1).
σ_3 = produced from s_2 (produces λ_4).
s_3 = produced by self-evaporation in vessel III. (produces σ_4).

δ_3 = produced by d_2 (produces d_4).
$D_3 = d_3 + s_3 + \sigma_3$ = total steam from vessel III. to vessel IV.
s_4 = produced by self evaporation in vessel IV.
σ_4 = produced from s_3.
λ_4 = produced from σ_3.
$D_4 = d_4 + s_4 + \sigma_4 + \lambda_4$ = total steam from vessel IV.
d_4 = produced from d_3.
b, t and c as in Fig. 9 (p. 69).

Let W = the original weight of liquid,

r_f = its original percentage strength in solid matter,

r_e = its percentage strength after the suppositious removal of the *extra steam*,

e_1 = the weight of the *extra steam* to be taken from vessel I.,

e_2 = " " " " " " II.,

e_3 = " " " " " " III.

If from the second vessel e_2 kilos. of *extra steam* are to be withdrawn, then for this purpose η_1 kilos. of steam must be produced in the first vessel. And, if e_3 kilos. of *extra steam* are to be removed from the third vessel, for that purpose ϵ_2 kilos. must be produced in the second and ϵ_1 kilos. in the first.

Thus, in order to draw-off the weights of *extra steam*, e_1, e_2, and e_3, it is necessary to develop

In vessel I. $e_1 + \eta_1 + \epsilon_1$ kilos. of steam.
" II. $e_2 + \epsilon_2$ "
" III. e_3 "

Thus the development of *extra steam* withdraws from the liquor, W, the weight of water or steam, D_e.

$$D_e = e_1 + e_2 + e_3 + \epsilon_1 + \epsilon_2 + \eta_1 \quad . \quad . \quad . \quad . \quad . \quad (95)$$

Thus there remains to be evaporated in the ordinary manner the weight of liquor,

$$W - D_e = W - (e_1 + e_2 + e_3 + {}_1 + \epsilon_2 + \eta_1) \quad . \quad . \quad (96)$$

The percentage of solids in the liquor rises thereby from r_f to r_e, and

$$r_e = \frac{100 r_f}{100 - (e_1 + e_2 + e_3 + \epsilon_1 + \epsilon_2 + \eta_1)} = \frac{100 r_f}{100 - D_e} \quad . \quad (97)$$

The weights of *extra steam*, $e_1 + e_2 + e_3$, are given; the weights, ϵ_1, ϵ_2, η_1, are now to be determined.

In order to obtain usable results we shall here, as in the preceding chapter, neglect those differences in evaporative capacity produced by differences in the fall of temperature from one vessel to another. We shall also adopt the average values previously obtained for the self-evaporation and the increased evaporation due to the diminution of the total heat of the steam in the later vessels. The errors so produced are small and negligible in practice.

The conclusions of the preceding chapter lead to the following expressions:—

Double effect.	Triple effect.	Quadruple effect.
$\epsilon_1 = \frac{1}{1\cdot045}e_2$	$\eta_1 = \frac{1}{1\cdot0075}e_2$	$\eta_1 = \frac{1}{1\cdot0055}e_2$
	$\epsilon_1 = \frac{1}{1\cdot0075}\epsilon_2$	$\epsilon_1 = \frac{1}{1\cdot0055}\epsilon_2$
		$\epsilon_2 = \frac{1}{1\cdot103}e_3$

or

$\epsilon_1 = 0\cdot957\,e_2$	$\eta_1 = 0\cdot992\,e_2$	$\eta_1 = 0\cdot995\,e_2$
	$\epsilon_1 = 0\cdot992\,\epsilon_2$	$\epsilon_1 = 0\cdot995\,\epsilon_2$
		$\epsilon_2 = 0\cdot9067\,e_3$
		$\eta_1 = 0\cdot9022\,e_3$

Thus, as a result of the removal of the *extra steam*, e_1, e_2, and e_3, from the quadruple effect, the total quantity of water withdrawn from the liquor is

$$D_e = e_1 + e_2 + e_3 + 0\cdot995\,e_2 + 0\cdot9067\,e_3 + 0\cdot9022\,e_3$$
$$= e_1 + 1\cdot995\,e_2 + 2\cdot8089\,e_3.$$

D_e gives the quantity of water (or total weight of steam) removed from the liquor, when in the first vessel e_1, in the second e_2, and in the third e_3 kilos. of *extra steam* are drawn off.

In Table 19 are given for many cases the weights of water which must be evaporated in the separate vessels of a multiple evaporator *in addition to* the ordinary evaporation of the liquor, if the weights of *extra steam* e_1, e_2, e_3, are withdrawn.

If this water, evaporated for the production of *extra steam*, be subtracted from the weight of the liquor, and the remaining water still to be evaporated divided among the single vessels as shown in Chapter X., and finally the weight of *extra steam* taken from each vessel be added, the total evaporation in each vessel is obtained.

Example.—$W = 100$ kilos. of liquor are evaporated in a quadruple effect evaporator from the concentration $r_f = 10$ per cent. to $r_u = 65$ per cent. From the first vessel $e_1 = 12$, from the second $e_2 = 6$ and from the third $e_3 = 4$ kilos. of *extra steam* are to be withdrawn per 100 kilos. of liquor.

100 kilos. of liquor of 10 per cent. strength will give

$$\frac{10 \times 100}{65} = 15\cdot38 \text{ kilos. of 65 per cent. strength.}$$

TABLE 19.

The weights of steam which must be evolved in each vessel of a multiple evaporator, and the total quantity of water lost in consequence by the liquor, if e_1, e_2 and e_3 kilos. of *extra steam* are taken from the vessels.

If e_1 kilos. of extra steam are withdrawn from vessel I. per 100 kilos. of liquor. e_1	This weight has to be evaporated in the first vessel and the liquid loses the same weight.	If e_2 kilos. of extra steam are withdrawn from vessel II. per 100 kilos. of liquor, e_2	then in vessel I. η_1 kilos. must be evaporated, $\eta_1 = 0{\cdot}993e_2$, η_1	thus the liquor loses in all $e_2 + \eta_1$ kilos. $e_2 + \eta_1$	If e_3 kilos. of extra steam are withdrawn from vessel III. per 100 kilos. of liquor, e_3	then in vessel II. ϵ_2 kilos. must be evaporated, $\epsilon_2 = 0{\cdot}9067e_3$, ϵ_2	and in vessel I. η_1 kilos. must be evaporated. $\epsilon_1 = 0{\cdot}995\epsilon_2$. ϵ_1	Thus the liquor loses in all $e_3 + \epsilon_2 + \epsilon_1$ kilos. $e_3+\epsilon_2+\epsilon_1$
2		2	1·986	3·986	2	1·813	1·804	5·617
4		4	3·972	7·972	4	3·626	3·608	11·234
6		6	5·958	11·958	6	5·439	5·412	16·851
8		8	7·944	15·944	8	7·252	7·216	22·468
10		10	9·93	19·930	10	9·067	9·022	28·089
12		12	11·916	23·916	12	10·880	10·826	33·706
14		14	13·903	27·903	14	12·693	12·630	39·323
16		16	15·888	31·888	16	14·504	14·431	44·935
18		18	17·874	35·874	18	16·321	16·240	50·561
20		20	19·86	39·860	20	18·130	18·040	56·170
22		22	21·846	43·846	22	19·960	19·861	61·824
24		24	23·832	47·832				
26		26	25·818	51·818				
28		28	27·804	55·804				
30		30	29·790	59·790				
32		32	31·773	63·773				

Thus there must be evaporated 100 − 15·38 = 84·62 kilos. of water.

Next, to determine the weight of steam which must be evolved in each vessel in order to produce the extra steam.

From Table 19 we find:—

In vessel	I.	II.	III.	
For $e_1 = 12$	$e_1 = 12$	—	—	
For $e_2 = 6$	$\eta_1 = 5{\cdot}958$	$e_2 = 6$	—	
For $e_3 = 4$	$\epsilon_1 = 3{\cdot}608$	$\epsilon_2 = 3{\cdot}626$	$e_3 = 4$	
	21·566	9·626	4	Total, 35·192 kilos.

Thus in the first vessel 21·566, in the second 9·626, in the third 4·0 kilos. of steam, in all 35·192 kilos., are withdrawn from the liquor for the formation of *extra steam*. For evaporation in the regular manner there remain

$$84{\cdot}62 - 35{\cdot}192 = 49{\cdot}428 \text{ kilos.}$$

The quadruple effect evaporates this weight (Chapter X., p. 86):—

In vessel - -	I.	II.	III.	IV.	
In the ratio -	0·2161 :	0·2427 :	0·2535 :	0·2844	Total,
	$D_1 = 10{\cdot}685$	$D_2 = 12{\cdot}000$	$D_3 = 12{\cdot}682$	$D_4 = 14{\cdot}061$	49·428 kilos.
Add for extra steam - -	21·566	9·626	4·0	0·0	
Thus the total evaporation of each vessel is	35·251	21·626	16·682	14·061	Total, 84·620 kilos.

The evaporation effected by the transference of heat, *i.e.*, without self-evaporation, in each vessel, is, on the average, according to Chapter X. (pp. 84, 85),

$$0{\cdot}931 \times 49{\cdot}428 = 46{\cdot}017 \text{ kilos.,}$$

of which are evaporated

In vessel - -	I.	II.	III.	IV.	
In the ratio - -	1 :	1·0055 :	1·109 :	1·196	Total,
	$d = 10{\cdot}685$	$d = 10{\cdot}725$	$d = 11{\cdot}837$	$d = 12{\cdot}770$	46·017 kilos.
Add for extra steam	21·566	9·626	4·0	0·0	
	32·251	20·351	15·837	12·770	Total, 81·209 kilos.

B. What is now the Concentration of the Liquor in Each Vessel?

After finding how much water the liquor loses in each vessel, its strength or the percentage of solid matter is readily ascertained.

If the original liquor contained $r_{\prime}$ per cent. of solids (in the last example, 10 per cent.), and from 100 kilos. there were evaporated in the first vessel $D_1 + e_1 + \eta_1 + \epsilon_1$ (here 32·251 kilos.), then the percentage of dry material in the first vessel would be

$$r_1 = \frac{100\, r_{\prime}}{100 - (D_1 + e_1 + \epsilon_1 + \eta_1)} = \frac{100 \times 10}{100 - 32{\cdot}251} = 14{\cdot}8 \text{ per cent.,}$$

in the second

$$r_2 = \frac{100 \times 10}{100 - (32{\cdot}251 + 21{\cdot}626)} = 21{\cdot}7 \text{ per cent.,}$$

in the third

$$r_3 = \frac{100 \times 10}{100 - (32 \cdot 251 + 21 \cdot 626 + 16 \cdot 682)} = 34 \cdot 2 \text{ per cent.},$$

and in the fourth

$$r_4 = \frac{100 \times 10}{100 - (32 \cdot 251 + 21 \cdot 626 + 16 \cdot 682 + 14 \cdot 06)} = 65 \text{ per cent.}$$

Since the cases which occur in practice are so extraordinarily different, that they cannot be brought within the limits of a table, the attempt must be abandoned; when necessary the calculation must be performed.

The commonest case in practice is that in which *extra steam* is taken only from the first vessel; the variations are not then so numerous that they cannot be tabulated. Accordingly Table 20 has been calculated for this case; the percentage strength is given of the liquid in the different vessels of the double, triple and quadruple effect evaporator for liquids which are thickened from $r_{,,} = 6$-15 per cent. to $r_u = 50$-70 per cent., when *extra steam* to the extent of 5, 10, 15, 20 or 25 per cent. is taken from the first vessel.

Finally, in order to facilitate numerous calculations, Table 21 is added. It gives the percentage strengths of solutions, which originally contained 1-30 per cent. of solids, after 1-38 per cent. of water has been withdrawn.

TABLE 20.

Percentage of solids in the contents of the separate vessels of the double, triple and quadruple effect evaporators, for liquids of originally $r_{,}$ – 6-13 per cent. strength, when in the first vessel 5, 10, 15, 20 or 25 per cent. of *extra steam* is drawn off, and in the last vessel a liquor of 50, 60 or 70 per cent. strength is to be produced.

Original strength, per cent.	Percentage of *extra steam* taken from vessel I.	The liquor is thereby brought to the percentage strength.	Double effect.		Triple effect.			Quadruple effect.			
			I.	II.	I.	II.	III.	I.	II.	III.	IV.
$r_{,}$	e_1	r_e	r_1	r_2	r_1	r_2	r_3	r_1	r_2	r_3	r_4
6	5	6·315	10·7	50	8·6	14·1	50	7·75	10·6	17	50
	10	6·66	11·2	50	8·9	14·7	50	8·25	11·1	17·4	50
	15	7·05	11·7	50	9·46	15·37	50	8·64	11·58	18·3	50
	20	7·5	12·4	50	10·1	16·2	50	9·24	12·33	19·75	50
	25	8	13·13	50	10·7	17·03	50	9·81	13·01	20	50
6	5	6·315	11·1	60	8·66	14·0	60	7·9	10·79	17·5	60
	10	6·66	11·4	60	9·06	14·3	60	8·3	11·3	18·5	60
	15	7·05	11·94	60	9·54	15·8	60	8·7	11·85	19·2	60
	20	7·5	12·69	60	10·16	16·75	60	9·3	12·6	20·2	60
	25	8	13·45	60	10·84	17·7	60	9·88	13·33	21·2	60
6	5	6·315	11·04	70	8·71	14·9	70	7·93	10·93	18·3	70
	10	6·66	11·53	70	9·15	15·4	70	8·33	11·5	19·1	70
	15	7·05	12·11	70	9·63	16·31	70	8·75	12·01	20	70
	20	7·5	12·86	70	10·28	17·25	70	9·3	12·76	21	70
	25	8	13·67	70	10·94	18·23	70	9·95	13·5	22·04	70
7	5	7·36	12·12	50	9·9	15·97	50	9·05	12·08	18·9	50
	10	7·77	12·7	50	10·35	16·8	50	9·54	12·7	19·6	50
	15	8·235	13·48	50	11·3	17·4	50	10·1	13·36	20·45	50
	20	8·75	14·1	50	11·0	18	50	10·7	14	21·32	50
	25	9·33	15	50	12·3	19·1	50	11·2	14·8	22·3	50
7	5	7·36	12·44	60	10	16·5	60	9·1	12·35	19·9	60
	10	7·77	13·05	60	10·5	17·1	60	9·6	12·75	20·7	60
	15	8·235	13·85	60	11·15	18	60	10·18	13·9	21·7	60
	20	8·75	14·55	60	11·7	18·6	60	10·78	14·2	22·67	60
	25	9·33	15·4	60	12·5	19·95	60	11·48	15·2	23·66	60
7	5	7·36	12·61	70	10·03	16·95	70	9·15	12·51	20·7	70
	10	7·77	13·1	70	10·5	17·75	70	9·65	13·20	21·5	70
	15	8·235	14	70	11·24	18·7	70	10·25	13·9	22·6	70
	20	8·75	14·87	70	11·85	19·18	70	10·85	14·65	23·55	70
	25	9·33	15·6	70	12·62	20·71	70	11·55	15·56	24·8	70
8	5	8·42	13·8	50	11·1	17·7	50	10·3	13·6	20·8	50
	10	8·88	14·4	50	11·4	18·3	50	10·7	14·15	21·3	50
	15	9·4	15·2	50	12·5	19·3	50	11·5	15·1	22·6	50
	20	10	15·87	50	13·16	20·15	50	12·13	15·76	23·5	50
	25	10·66	16·42	50	13·75	20·83	50	12·62	16·75	24·0	50

TABLE 20—(*continued*).

Original strength, per cent.	Percentage of *extra steam* taken from vessel I.	The liquor is thereby brought to the percentage strength.	Double effect.		Triple effect.			Quadruple effect.			
			I.	II.	I.	II.	III	I.	II.	III.	IV.
r_f	θ_1	r_e	r_1	r_2	r_1	r_2	r_3	r_1	r_2	r_3	r_4
8	5	8·42	14	60	11·3	18·3	60	10·3	13·9	21·9	60
	10	8·88	14·8	60	11·9	19·2	60	11	14·6	22·8	60
	15	9·4	15·6	60	12·7	20·2	60	11·7	15·6	23·9	60
	20	10	16·33	60	13·34	21·08	60	12·25	16·22	24·8	60
	25	10·66	17·03	60	13·79	21·87	60	12·9	16·92	25·6	60
8	5	8·42	14·3	70	11·5	18·8	70	10·4	14·1	22·8	70
	10	8 88	15	70	12	19·9	70	11	14·9	23·8	70
	15	9·4	15·7	70	12·8	21	70	11·85	15·8	25	70
	20	10	16·52	70	13·49	21·81	70	12·33	16·5	26	70
	25	10·66	17·12	70	14·1	22·6	70	12·93	17·25	26·9	70
9	5	9 48	15 2	50	12·5	19·3	50	11·5	15·1	22·6	50
	10	10	15·87	50	13·15	20·13	50	12·13	15·76	23·5	50
	15	10·56	16 18	50	13·75	20·83	50	12·62	16·76	24·1	50
	20	11·25	17·5	50	14·6	21·93	50	13·56	18	25·1	50
	25	12	18·5	50	15·49	22·85	50	14·37	18·31	26·29	50
9	5	9·48	15 6	60	12·7	20·2	60	11·7	15·5	23·9	60
	10	10·1	16·33	60	13·34	21·08	60	12·25	16·22	24·8	60
	15	10·56	17·03	60	13·79	21·87	60	12·9	16·92	25·6	60
	20	11·25	18·1	60	14·86	23·04	60	13·7	17·85	26·7	60
	25	12	19·1	60	15·78	24·15	60	14·5	18·6	27·7	60
9	5	9 48	15·7	70	12·8	21	70	11·85	15·8	25	70
	10	10·1	16·52	70	13·49	21·81	70	12·33	16·53	26	70
	15	10·56	17·12	70	14·1	22 6	70	12·93	17·25	26·9	70
	20	11·25	18·5	70	15·05	23·9	70	13·8	18·25	28·18	70
	25	12	19·5	70	15·95	25·07	70	14·69	19·38	29·48	70
10	5	10·52	16·5	50	13·8	20·8	50	12·7	16·5	24·1	50
	10	11·11	17·3	50	14·43	21·66	50	13·37	17·71	24·85	50
	15	11·76	18·2	50	15·2	22·5	50	14	18	25·7	50
	20	12·5	19·1	50	16·09	23·5	50	14·9	18·9	26·9	50
	25	13·33	20	50	17	24·6	50	15·7	19·8	27·6	50
10	5	10·52	17	60	13·9	21·8	60	12·8	16·9	25·6	60
	10	11·11	17·85	60	14·68	22·79	60	13·51	17·7	26·5	60
	15	11·76	18·8	60	15·5	24·8	60	14·2	18·3	27·4	60
	20	12·5	19·7	60	16·38	24·85	60	15·1	19·2	28·5	60
	25	13·33	20·77	60	17·26	25·86	60	16	20·52	29·7	60
10	5	10·52	17·3	70	14	22·7	70	12·9	17·2	26·9	70
	10	12·22	18·27	70	14·86	23·65	70	13·6	18	27·95	70
	15	12·95	19·2	70	15·6	24·6	70	14·4	19	29	70
	20	13·75	20·2	70	16·58	25·87	70	15·29	20	30·3	70
	25	14·66	21·2	70	17·5	26·9	70	16·1	21	31·6	70
11	5	11·57	17·9	50	14·9	22·2	50	13·8	17·6	25·5	50
	10	12·22	18·8	50	15·8	23·1	50	14·6	18·6	26·5	50

TABLE 20—(*continued*).

Original strength, per cent.	Percentage of *extra steam* taken from vessel I.	The liquor is thereby brought to the percentage strength.	Double effect.		Triple effect.			Quadruple effect.			
			I.	II.	I.	II.	III.	I.	II.	III.	IV.
r_f	e_1	r_e	r_1	r_2	r_1	r_2	r_3	r_1	r_2	r_3	r_4
11	15	12·95	19·6	50	16·5	24·1	50	15·4	19·5	27·3	50
	20	13·75	20·5	50	17·5	25·1	50	16·25	20·4	28·2	50
	25	14·66	21·5	50	18·5	26	50	17·2	21·4	29·1	50
11	5	11·57	18·30	60	15·1	23·3	60	13·8	18·1	27·1	60
	10	12·22	19·4	60	16	24·5	60	14·3	18·9	28	60
	15	12·95	20·3	60	16·9	25·5	60	15·6	20·2	29·3	60
	20	13·75	21·35	60	17·8	26·5	60	16·5	21·1	30·4	60
	25	14·66	21·4	60	18·8	27·5	60	17·5	22·2	31·4	60
11	5	11·57	18·8	70	15·4	23·8	70	14·1	18·6	28·6	70
	10	12·22	19·8	70	16·3	25·5	70	15	19·7	29·8	70
	15	12·95	20·8	70	17·1	26·5	70	15·8	20·7	31	70
	20	13·75	21·9	70	18·1	27·9	70	16·6	21·7	32·3	70
	25	14·66	22·9	70	19·1	29	70	17·6	22·7	33·4	70
12	5	12·63	19	50	16·1	23·5	50	14·9	18·9	26·8	50
	10	13·33	20	50	17	24·6	50	15·49	19·8	27·6	50
	15	14·11	20·95	50	17·93	25·5	50	16·68	20·8	28·6	50
	20	15	22	50	18·9	26·5	50	17·65	21·8	29·5	50
	25	16	23·12	50	19·9	27·69	50	18·71	23	30·6	50
12	5	12·63	19·7	60	16·4	24·8	60	15·1	19·5	28·6	60
	10	1[illegible]·33	20·77	60	17·36	25·87	60	15·99	20·63	29·7	60
	15	14·11	21·77	60	18·24	27·03	60	16·92	21·63	30·9	60
	20	15	22·86	60	19·27	28·22	60	17·9	22·7	32	60
	25	16	24·03	60	20·40	29·48	60	19·03	23·9	33·28	60
12	5	12·63	20·3	70	16·6	25·8	70	15·3	20	30·3	70
	10	13·33	21·3	70	17·59	27·1	70	16·23	20·35	30·61	70
	15	14·11	22·4	70	18·53	28·3	70	17·1	22·21	32·77	70
	20	15	23·54	70	19·59	29·6	70	18·12	23·28	34·09	70
	25	16	24·83	70	20·76	30·98	70	19·21	24·59	35·33	70
13	5	13·68	20·3	50	17·2	24·9	50	16	20·1	27·9	50
	10	14·44	21·3	50	18·3	25·9	50	17	21·2	29	50
	15	15·28	22·8	50	19·7	27·3	50	18·4	22·7	30·3	50
	20	16·25	23·4	50	20·2	27·9	50	19	23·3	30·9	50
	25	17·33	24·5	50	21·4	29	50	20	24·4	32	50
13	5	13·68	21	60	17·6	26·3	60	16·3	20·9	30·1	60
	10	14·44	22·1	60	18·6	27·4	60	17·3	22	31·2	60
	15	15·28	23·1	60	19·6	28·5	60	18·2	23	32·3	60
	20	16·25	24·3	60	20·7	29·8	60	19·3	24·2	33·6	60
	25	17·33	25·6	60	22	31·1	60	20·5	25·5	35	60
13	5	13·68	21·6	70	17·8	27·4	70	16·4	21·4	31·9	70
	10	14·44	22·6	70	18·8	28·7	70	17·5	22·6	33·2	70
	15	15·28	23·9	70	19·9	29·9	70	18·4	23·7	34·4	70
	20	16·25	25·1	70	21	31·3	70	19·5	24·9	35·7	70
	25	17·33	26·4	70	22·3	32·2	70	20·7	26·3	37·5	70

TABLE 21.

Percentage of solid matter, $r_{\prime\prime}$, in liquors,
solids, after 1·38 per

Original strength, per cent. $r_{\prime}$	If there be taken from 100 kilos. of											
	1	2	3	4	5	6	7	8	9	10	11	12
	the residue contains $r_{\prime\prime}$ per											
1	1·01	1·02	1·03	1·04	1·05	1·06	1·08	1·09	1·10	1 11	1·12	1·14
2	2·02	2·04	2·06	2·08	2·11	2·13	2·15	2·17	2·20	2·22	2·25	2·27
3	3·03	3·06	3·09	3 13	3·16	3·19	3·23	3·26	3·30	3·33	3·37	3·41
4	4·04	4·08	4·12	4·17	4·21	4·26	4·30	4·35	4·40	4·44	4·49	4·55
5	5·05	5·10	5·15	5·21	5·26	5·32	5·38	5·43	5·49	5·55	5·62	5·68
6	6·06	6·12	6 19	6·25	6·32	6·38	6·45	6·52	6·59	6·66	6·74	6·82
7	7·07	7·13	7·21	7·29	7·36	7·45	7 53	7·6	7·69	7·77	7·8	7·95
8	8·08	8·16	8·25	8·34	8·42	8·52	8·60	8·7	8·79	8·88	8·98	9·09
9	9·09	9·18	9·27	9·37	9·48	5·57	9·67	9·78	9·89	8·99	10·11	10·23
10	10·10	10·20	10·31	10·41	10·52	10·64	10·75	10·87	10·99	11·11	11·23	11·36
11	11·11	11·22	11 34	11·46	11·57	11·70	11·82	11·95	12·08	12·22	12·36	12·5
12	12·12	12·24	12·37	12·5	12·63	12·77	12·90	13·04	13·19	13·33	13·49	13·64
13	13·13	13·26	13·40	13·54	13·68	13·82	13·98	14·13	14·28	14·44	14·60	14·77
14	14·14	14·26	14·43	14·58	14·73	14·89	15·05	15·20	15·38	15·55	15·55	15·91
15	15·15	15·30	15·46	15·61	15·78	15·96	16·12	16·31	16·48	16·66	16·84	17·04
16	16·16	16·32	16·49	16·68	16·84	17·04	17·2	17·4	17·58	17·77	17·94	18·18
17	17·17	17·35	17·52	17·70	17·89	18·08	18·28	18·48	18·68	18·88	19 20	19·32
18	18·18	18·36	18·54	18·74	18·96	19·14	19·34	19·56	19·78	20·00	20·20	20·46
19	19·19	19·39	19·59	19·78	20	20·21	20·43	20·65	20·88	21·11	21·35	21·59
20	20·20	20·40	20·62	20·82	21·04	21·28	21·5	21·74	21·98	22·22	22·46	22·73
21	21·21	21·44	21·55	21·88	22·1	22·34	22·58	22·82	23·07	23·33	23·58	23·86
22	22·22	22·45	22·68	22·92	23·15	23·40	23·65	23·91	24·17	24·44	24·75	25
23	23·23	23·47	23·71	23·96	24·21	24·46	24·73	25	25·27	25·55	25·84	26·13
24	24·24	24·44	24·74	25	25·26	45·54	25·81	26·08	26·37	26·66	26·96	27·27
25	25·25	25·50	25·77	26·04	26·31	26·59	27·09	27·17	27·47	27·77	28·09	28·41
26	26·26	26·53	26·80	27·08	27·37	27·66	27·96	28·26	28·57	28·88	29·2	29·55
27	27·27	27·55	27·85	28·12	28·42	28·72	29·03	29·34	29·67	30	30·34	30·68
28	28·28	28·53	28·87	29·17	29·46	29·78	30·1	30·4	30·76	31·11	31·46	31·82
29	29·29	29·59	29·90	30·20	30·53	30·85	31·18	31·52	31·87	32·22	32·58	32·95
30	30·30	30·60	30·93	31·23	31·56	31·92	32·25	32·61	32·97	33·33	33·69	34·08

TABLE 21.

which originally contained r_r = 1-30 per cent. of
cent. of water has been abstracted.

liquor the following weights of water, in kilos.

cent. of solids.

13	14	15	16	17	18	19	20	21	22	23	24	25	Original strength, per cent. r_r
1·15	1·16	1·18	1·19	1·20	1·22	1·23	1·25	1·27	1·29	1·30	1·31	1·33	1
2·3	2·32	2·33	2·36	2·44	2·44	2·47	2·5	2·53	2·56	2·59	2·63	2·67	2
3·46	3·49	3·52	3·57	3·62	3·66	3·7	3·75	3·79	3·85	3·90	3·95	4	3
4·5	4·65	4·7	4·76	4·82	4·87	4·94	5	5·06	5·13	5·19	5·26	5·33	4
5·74	5·81	5·88	5·95	6·02	6·09	6·17	6·25	6·33	6·43	6·49	6·58	6·66	5
6·89	6·98	7·05	7·14	7·23	7·31	7·40	7·5	7·59	7·69	7·79	6·81	8	6
8·05	8·14	8·24	8·33	8·43	8·54	8·64	8·75	8·86	8·94	9·09	9·21	9·33	7
9·2	9·3	9·4	9·52	9·64	9·74	9·88	10	10·12	10·26	10·38	10·52	40·66	8
10·35	10·47	10·56	10·71	10·84	10·98	11·1	11·25	11·37	11·55	11·68	11·85	12	9
11·49	11·63	11·76	11·9	12·04	12·19	12·35	12·5	12·65	12·86	12·97	13·13	13·33	10
12·64	12·79	12·92	13·20	13·25	13·41	13·58	13·75	13·83	14·10	14·28	14·47	14·66	11
13·79	13·95	14·11	14·29	14·46	14·63	14·81	15	15·19	15·39	15·58	15·79	16	12
14·94	15·11	15·27	15·47	15·66	15·85	16·04	16·25	16·45	16·66	16·88	17·11	17·33	13
16·09	16·28	16·47	16·66	16·86	17·08	17·28	17·5	17·72	17·95	18·18	18·12	18·66	14
17·23	17·44	17·64	17·85	18·06	18·28	18·51	18·75	18·97	19·29	19·46	19·74	19·99	15
18·4	18·6	18·8	19·04	19·28	19·48	19·76	20	20·24	20·52	20·76	21·04	21·32	16
19·54	19·77	19·99	20·21	20·46	20·73	20·99	21·25	21·52	21·79	22·08	22·37	22·66	17
20·70	20·94	21·12	21·41	21·68	21·96	22·2	22·5	22·75	23·10	23·36	23·70	24	18
21·84	22·09	22·35	22·62	22·88	23·19	23·45	23·75	24·05	24·36	24·69	25	25·33	19
22·98	23·25	23·53	23·8	24	24·38	24·69	25	25·30	25·72	25·95	26·32	26·66	20
24·14	24·42	24·75	25·08	25·3	25·61	25·92	26·25	26·58	26·91	27·50	27·63	28	21
25·29	25·58	25·85	26·19	26·5	26·83	27·16	27·5	27·87	28·20	28·57	28·95	29·33	22
26·44	26·74	27·06	27·38	27·71	28·05	28·39	28·88	29·11	29·49	29·87	30·26	30·66	23
27·5	27·9	28·22	28·57	28·92	29·26	29·62	30	30·36	30·77	31·16	31·5	32	24
28·74	29·07	29·41	29·77	30·12	30·49	30·86	31·25	31·64	32·05	32·47	32·89	33·33	25
29·89	30·33	30·57	30·95	31·32	31·70	32·09	32·5	32·91	33·33	33·77	34·21	34·66	26
31·03	31·4	31·76	32·14	32·52	32·92	33·33	33·75	34·18	34·61	35·07	35·50	36	27
32·18	32·56	32·94	33·33	33·73	34·15	34·57	35	35·44	35·9	36·36	36·84	37·33	28
33·33	33·72	34·12	34·52	34·94	35·36	35·86	36·25	36·72	37·18	37·66	38·16	38·66	29
34·47	34·88	35·28	35·70	36·12	36·57	37·03	37·5	37·95	38·58	38·92	39·48	39·99	30

TABLE 21—(*continued*).

Original strength per cent. r_i	If there be taken from 100 kilos. of liquor the following weights of water, in kilos.												
	26	27	28	29	30	31	32	33	34	35	36	37	38
	the residue contains r_{ii} per cent. of solids.												
1	1·35	1·37	1·39	1·41	1·43	1·45	1·47	1·49	1·52	1·54	1·57	1·59	1·61
2	2·7	2·74	2·77	2·82	2·86	2·90	2·94	2·99	3·03	3·08	3·13	3·18	3·23
3	4·05	4·11	4·16	4·22	4·29	4·35	4·41	4·47	4·54	4·61	4·7	4·77	4·84
4	5·4	5·48	5·55	5·63	5·71	5·80	5·88	5·97	6·06	6·15	6·26	6·36	6·45
5	6·75	6·85	6·93	7·04	7·14	7·25	7·35	7·46	7·58	7·69	7·83	7·95	8·07
6	8·10	8·22	8·33	8·45	8·57	8·69	8·85	8·95	9·08	9·23	9·39	9·54	9·68
7	9·46	9·6	9·72	9·85	10	10·14	10·29	10·45	10·6	10·77	10·96	11·13	11·29
8	10·8	10·96	11·11	11·26	11·42	11·60	11·76	11·94	12·12	12·31	12·62	12·72	12·91
9	12·15	12·33	12·48	12·66	12·87	13·05	13·23	13·41	13·63	13·83	14·09	14·31	14·52
10	13·51	13·7	13·87	14·08	14·29	14·49	14·71	14·93	15·15	15·38	15·66	15·90	16·14
11	14·79	15·07	15·15	15·21	15·55	15·94	16·18	16·41	16·66	16·92	17·22	17·49	17·75
12	16·21	16·44	16·66	16·9	17·14	17·39	17·64	17·91	18·17	18·46	18·79	19·08	19·36
13	17·56	17·81	18·55	18·31	18·57	48·84	19·13	19·33	19·69	20	20·36	20·67	20·98
14	18·92	19·17	19·44	19·71	20	20·29	20·59	20·90	21·21	21·54	21·92	22·26	22·59
15	20·16	20·55	20·84	21·12	21·13	21·74	22·06	22·40	22·72	23·07	23·5	23·85	24·21
16	21·6	21·92	22·22	22·52	22·84	23·20	23·52	23·88	24·24	24·62	25·95	25·44	24·83
17	22·97	23·29	23·61	23·94	24·29	24·64	25	25·37	25·76	26·15	26·62	27·03	27·43
18	24·30	24·66	24·99	24·35	25·71	26·08	26·46	26 86	27·25	27·69	28·28	28·62	29·05
19	25·67	26·02	26·39	26·76	27·14	27·52	27·94	28·36	28·79	29·20	29·75	30·21	30·68
20	27·02	17·4	27·74	28·16	28·58	28·98	29·42	29·86	20·30	30·76	31·32	31·80	32·28
21	28·38	28·77	29·16	29·46	30	30·42	30·87	31·35	31·80	32·31	32·88	33·40	33·89
22	29·59	30·14	30·30	30·42	31·10	31·88	32·36	32·82	33·33	33·84	34·45	34·98	35·50
23	31·08	31·51	31·94	32·39	32·86	33·33	33·82	34·33	34·85	35·38	36·0	36·57	37·12
24	32·42	32·88	33·33	33·80	34·29	35·78	35·29	35·82	36·35	36·92	37·58	38·16	38·73
25	33·78	34·25	34·70	35·20	35·42	36·23	36·77	37·33	37·87	38·45	39·2	39·75	40·35
26	35·13	35·61	36·11	36·62	37·14	37·68	38·26	38·65	39·39	40	40·62	41·34	41·96
27	36·48	37	37·44	37·98	38·61	39·15	39·69	40·23	40·86	41·49	42·28	42·93	43·57
28	37·84	38·35	38·88	39·43	40	40·58	41·18	41·80	42·42	43·08	43·94	44·52	45·79
29	39·19	39·72	40·27	40·84	41·41	42·03	42·79	43·29	43·94	44·61	45·41	46·11	46·90
30	40·53	41·1	41·66	42·25	43·48	43·48	44·12	44·8	45·45	46·15	47·0	47·7	48·42

CHAPTER XII.

THE WEIGHT OF WATER WHICH MUST BE EVAPORATED FROM 100 KILOS. OF LIQUOR IN ORDER TO BRING ITS ORIGINAL PERCENTAGE OF SOLIDS, r_f, UP TO THE DESIRED HIGHER PERCENTAGE :

THE purpose of an evaporator is, as a rule, to increase the original strength of a liquid in solids (dry matter) from r_f per cent. to a greater strength, r_u per cent., by evaporation of water. How much water must be evaporated in each case?

If there are r_f kilos. of solids in 100 kilos. of liquid, and if this r_f kilos. is to become r_u per cent. in the concentrated liquor, then *the weight, U, of the concentrated liquid* is given by

$$r_f : U = r_u : 100 \text{ or } U = \frac{r_f 100}{r_u} \quad . \quad . \quad . \quad . \quad . \quad (98)$$

Thus *the weight of water to be evaporated* from 100 kilos. of liquid is

$$100 - U = 100 - \frac{r_f 100}{r_u} = 100\left(1 - \frac{r_f}{r_u}\right) \quad . \quad . \quad . \quad (99)$$

and the weight of water to be evaporated from W kilos. of a liquid, which contains r_f per cent. of solids, in order to concentrate it to the strength of r_u per cent., is

$$W - U = W\left(1 - \frac{r_f}{r_u}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

Example.—1000 kilos. of liquid, originally containing $r_f = 10$ per cent. of solids, are to be evaporated to such an extent that the residue will contain $r_u = 60$ per cent. Then

$$W - U = 1000\left(1 - \frac{10}{60}\right) = 833 \text{ kilos.}$$

In Table 22 are given the weights of water which must be evaporated from 100 kilos. of liquid containing $r_{,} = 1$-25 per cent. of solids, in order to produce a concentrated liquid containing 20-70 per cent. of solids.

TABLE 22.

The weight of water which must be evaporated from 100 kilos. of liquid in order to bring the original percentage of solids, $r_{,}$ per cent., up to the desired higher r_{u} per cent.

Original percentage of solids. $r_{,}$ per cent.	Percentage of solids, r_{u}, to be contained in the liquid after evaporation. The weight of water in kilos. to be evaporated from 100 kilos. of liquid.											
	20	22·5	25	27·5	30	32·5	35	40	45	50	60	70
1	95	95·6	96	96·4	96·7	96·9	97·2	97·5	97·8	98	98·4	98·6
2	90	91·2	92	92·8	93·8	93·8	94·3	95	95·6	96	96·7	99·1
3	85	86·7	88	89·1	90	90·8	91·43	92·5	93·3	94	95	95·7
4	80	82·3	84	85·8	86·7	87·7	88·6	90	91·1	92	93·4	94·3
5	75	77·8	80	81·8	83·3	84·6	85·8	87·5	88·9	90	91·8	92·9
6	70	73·4	76	78·2	80	81·6	83·3	85	86·7	88	90	91·4
7	65	68·4	72	74·5	76·7	78·4	80	82·5	84·5	86	89	90
8	60	64·5	68	70	73·3	75·4	77·4	80	82·3	84	87·3	88·6
9	55	60	64	67·2	70	72·3	75	77·5	80	82	85	87·1
10	50	55·6	60	63·7	66·7	69·3	71·5	75	77·8	80	83·3	85·7
11	45	51·2	56	60	63·3	66·2	68·6	72·5	75·6	78	82	84·1
12	40	46·7	52	56·4	60	63·1	66·6	70	73·4	76	80	82·8
13	35	42·3	48	52·7	56·7	60	62·9	67·5	71	74	79	81·4
14	30	37·8	44	49	53·3	56·8	60	65	68·9	72	77	80
15	25	33·4	40	45·4	50	53·8	57·3	62·5	66·7	70	75	78 6
16	20	29	36	41·8	46·7	50·8	54·4	60	64·5	68	73·4	77·1
17	15	24·5	32	38·2	43·3	48·3	51·4	57·5	62·3	66	71·7	75·7
18	10	20	28	34·6	40	44·6	50	55	60	64	70	74·3
19	5	15·6	24	31	36·7	41·6	45·7	52·5	57·8	62	68	72·9
20	—	11·2	20	27·3	33·3	38·5	43	50	55·8	60	67	71·4
21	—	6·7	16	23·7	30	35·4	40	47·5	53·4	58	65	70
22	—	2·3	12	20	26·7	32·3	37·2	45	51·1	56	63·4	68·6
23	—	—	8	16·3	23·3	29·3	34·3	42·5	48·9	54	61·7	67·2
24	—	—	4	12·8	20	26·2	31·5	40	46·6	52	60	65·8
25	—	—	—	1·8	16·7	23·1	28·5	37·5	44·5	50	58·3	64 4

CHAPTER XIII.

THE RELATIVE PROPORTIONS OF THE HEATING SURFACES IN THE ELEMENTS OF THE MULTIPLE EVAPORATOR AND THEIR REAL DIMENSIONS.

In Chapter X. we have found the ratios of the evaporative capacities (not the real quantities of steam evolved, which are somewhat larger in consequence of self-evaporation) of the separate vessels of the multiple evaporator. These ratios were found to vary with the fall in temperature in each vessel, and with the extent to which the liquid is to be concentrated, but not to deviate *far* from a certain average value even in the most extreme cases. These mean evaporative capacities were (p. 86):—

In the double effect - $D_1 : d_2 = 1 : 1{\cdot}045.$

In the triple effect - - $D_1 : d_2 : (d_3 + \sigma_3) = 1 : 1{\cdot}0075 : 1{\cdot}138.$

In the quadruple effect - $D_1 : d_2 : (d_3 + \sigma_3) : (d_4 + \sigma_4 + \lambda_4)$
$= 1 : 1{\cdot}0055 : 1{\cdot}109 : 1{\cdot}196.$

Let H_1, H_2, H_3 and H_4 be the heating surfaces in sq. m.; θ_{m1}, θ_{m2}, θ_{m3} and θ_{m4} the mean differences in temperature between steam and liquid; k_1, k_2, k_3 and k_4 the coefficients of transmission (which depend upon the viscosity, the pressure of the steam, the shape and nature of the heating surface and all the other conditions); and c the heat of evaporation of 1 kilo. of steam. Then if the first vessel evolves D_1 kilos. of steam,

$$D_1 = \frac{H_1 \theta_{m1} k_1}{c_1},$$

and the heating surface required by the first vessel is

$$H_1 = \frac{D_1 c_1}{\theta_{m1} k_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (101)$$

Thus, for the quadruple effect, according to the above,

$$1:1{\cdot}0055:1{\cdot}109:1{\cdot}196$$

$$=\frac{H_1\theta_{m1}k_1}{c_1}:\frac{H_2\theta_{m2}k_2}{c_2}:\frac{H_3\theta_{m3}k_3}{c_3}:\frac{H_4\theta_{m4}k_4}{c_4} \quad . \quad (102)$$

and consequently

$$H_1:H_2:H_3:H_4=\frac{c_1}{\theta_{m1}k_1}:\frac{1{\cdot}0055\,c_2}{\theta_{m2}k_2}:\frac{1{\cdot}109\,c_3}{\theta_{m3}k_3}:\frac{1{\cdot}196\,c_4}{\theta_{m4}k_4} \quad . \quad (103)$$

If now we assume the different values for c_1, c_2, c_3 and c_4 to be equal, although they may vary from 637 to 618, thus producing only a slight inaccuracy, and, further, if we put $H_1 = 1$ and $k_1 = 1$, expressing the values of H and k for the other vessels as fractions, since we are now only determining the ratio of the heating surfaces to one another, then

$$k_1 = 1,\ k_2 = a_2k_1,\ k_3 = a_3k_1,\ k_4 = a_4k_1,$$

and the ratio of the heating surfaces to one another is

$$\frac{H_1}{H_1}:\frac{H_2}{H_1}:\frac{H_3}{H_1}:\frac{H_4}{H_1}=1:\frac{\theta_{m1}1{\cdot}0055}{\theta_{m2}a_2}:\frac{\theta_{m1}1{\cdot}109}{\theta_{m3}a_3}:\frac{\theta_{m1}1{\cdot}196}{\theta_{m4}a_4} \quad . \quad (104)$$

If the ratio to one another of the coefficients of transmission, k, were known, the proportions of the heating surfaces could be calculated from equation 104, assuming the desired temperature differences in each vessel.

The coefficients of transmission, k, are, however, not known, they depend upon the thickness of the liquid, the construction and details of the apparatus, the completeness with which the air is extracted, the diameter of the heating tubes, whether the steam is in or outside the tubes, on the absolute size of the heating surface, its cleanliness, and finally upon the effective pressure of the heating steam in each vessel. For, whilst steam at a pressure of 1 atmos. or more strives rapidly to counteract the diminution in pressure produced by condensation on the heating surfaces, and passes over the surfaces, steam at a low pressure is little inclined to do so, and rests more sluggishly in the steam space. It is often drawn off by the air-pipe in order to conduct it more rapidly over the heating surfaces.

All these different conditions make the coefficient of transmission different for each apparatus and each vessel. At the present time sufficiently accurate estimations of the coefficient for actual apparatus are wanting. Occasional observations made on apparatus in use are

rarely quite satisfactory, since the instruments (thermometers, vacuum gauges and more rarely hydrometers) are frequently not quite correct (Zeits. angew. Chem., 5th December, 1899), and because the influence of the incrustations actually present is unknown. If we give here the coefficients of transmission calculated from a number of such observations, it is from necessity with all reserve, and merely with the object of obtaining a rough representation.

From experiments made by Dr. H. Claassen on a triple-effect evaporator of a sugar works (Zeits. des Ver. für Rübenzucker-Industrie, March, 1893), and from other observations made in similar factories, the following ratios of the transmission-coefficient for sugar juices have been calculated :—

Vessel	I.	II.	III.	IV.
Double effect	1 :	0·66	—	—
Triple effect	1 :	0·70 :	0·33	—
Quadruple effect	1 :	0·91 :	0·75 :	0·55

If these figures were to some extent reliable for average conditions, and if the same temperature difference were desired in all the vessels, then the heating surfaces would be in the ratios (Equation 104) :—

In the double effect

$$1 : \frac{1{\cdot}045}{0{\cdot}66} = 1 : 1{\cdot}58.$$

In the triple effect

$$1 : \frac{1{\cdot}0075}{0{\cdot}70} : \frac{1{\cdot}138}{0{\cdot}33} = 1 : 1{\cdot}44 : 3{\cdot}414.$$

In the quadruple effect

$$1 : \frac{1{\cdot}0055}{0{\cdot}91} : \frac{1{\cdot}109}{0{\cdot}75} : \frac{1{\cdot}196}{0{\cdot}55} = 1 : 1{\cdot}105 : 1{\cdot}48 : 2{\cdot}175.$$

Similarly, if it were desired to make the heating surfaces of all the vessels of equal dimensions, then the differences in temperature (fall in temperature) would be in the ratio just calculated for the heating surfaces.

Example.—If the total available difference in temperature is 50° C., the following differences in temperatures for each vessel would be at once deduced from the above ratio, if the heating surfaces of the apparatus were equal :—

Vessel	I.	II.	III.	IV.
Double effect	19·3°	30·7°	—	—
Triple effect	8·55°	12·31°	29·18°	—
Quadruple effect	8·68°	9·59°	11·845°	18·88°

Since thick sluggish liquids, such as are contained in the later vessels, and especially in the last, are only brought by considerable differences in temperature into violent ebullition and hence to a rapid absorption of heat, it is certainly more advisable, if the last heating surfaces are to work effectively and consequently also the first, to increase the differences in temperature (and not the heating surfaces) in these (later) vessels. It is always preferable to make the later vessels at the most as large as the first and perhaps even to make them somewhat smaller. In no case, however, should the heating surfaces of the later vessels be made larger than those of the first, if there are not special reasons to the contrary.

For convenience in manufacture and erection all the vessels may be made of the same size, but then sufficient heating surface must be added to the first vessel to raise the cold liquor entering it to the temperature of this vessel. When *extra steam* is to be taken from one vessel or more, this vessel must be given as much more heating surface as is necessary for the production of the *extra steam*, and then the corresponding increase must be given to the heating surfaces of the earlier vessels.

Example.—From 1250 litres of liquor (assumed to weigh 1250 kilos.) 1000 litres of water are to be evaporated in a quadruple effect evaporator. The initial temperature of the liquor is 30° C. below the temperature of boiling in the first vessel. From each of the first and second vessels 100 kilos. of *extra steam* are to be taken.

In order to heat 1250 kilos. of liquor, the specific heat of which is 1, through 30° C., $1250 \times 30 = 37{,}500$ calories must be communicated to it in the first vessel, *i.e.*, as much heat as would be required to evaporate $\frac{37{,}500}{540} = 70$ kilos. of water.

Further, 100 kilos. of *extra steam* are to be taken from the first vessel, which quantity also must be conveyed to it.

If the *second* vessel is also to give 100 kilos. of *extra steam*, for that purpose there must, according to Table 17 (double effect, evaporation to $\frac{1}{4}$), be developed in the *first* vessel $\frac{100}{1\cdot042} = 96\cdot96$ kilos. of steam.

Through *extra steam* and the evaporation thereby necessitated, $100 + 100 + 96\cdot96 = 296\cdot96$ kilos. of water are taken from the liquor, and there remain $1000 - 296\cdot96 = 703\cdot04$ kilos. to be evaporated *regularly* in the quadruple effect.

The single vessels evaporate this, according to Table 17 (p. 85), in the ratio,

$$1 : 1\cdot16 : 1\cdot215 : 1\cdot375 \text{ (total} = 4\cdot75).$$

Since $\frac{703\cdot04}{4\cdot75} = 148$, the single vessels must evaporate

$$148 : 171\cdot68 : 179\cdot82 : 203\cdot54. \text{ Total, } 703\cdot04 \text{ kilos. of water.}$$

Thus the actual work done by each vessel must correspond to the evaporation of the following quantities of water:—

In heating the liquor	70	—	—	—	kilos.
For *extra steam*	100	—	—	—	„
For „	96·96	100	—	—	„
Regular	148	171·68	179·82	203·54	„
Totals	414·96	271·68	179·82	203·54	„ Total, 1070·00 kilos.

The self-evaporation in the second vessel of the quadruple effect, which we must consider here in regard to the production of *extra* steam, for 100 litres of liquor (*i.e.*, for 75 litres of water), is $s_2 = 1·77$ kilos. (p. 85),

$$\text{thus in this case } \frac{196·96 \times 1·77}{75} = 4·648 \text{ kilos},$$

and in the quadruple effect (regular evaporation), for 100 litres of liquor (p. 85),

$$s_2 = 1·77,\ s_3 = 1·46,\ s_4 = 2·35,$$

thus in this case

$$s_2 = \frac{703·04 \times 1·77}{75} = 16·30,\ s_3 = \frac{703·04 \times 1·46}{75} = 13·68,$$

$$s_4 = \frac{703·04 \times 2·35}{75} = 22·02.$$

The evaporation to be effected by the heating surfaces is thus

414·96, 250·70, 166·14, 181·52 kilos.

We may now correctly assume, in order to obtain greater differences of temperature in the later vessels, as we have also done in deducing the coefficients, k, from the experiments, that 1 sq. m. of heating surface has almost the same efficiency in each vessel. Then the later vessels can undertake the greater evaporation, laid upon them by the nature of the conditions, by reason of their greater fall in temperature. The effective capacity differs in different evaporators according to construction and circumstances. If we assume for the preceding case that each sq. m. of heating surface can develop 20 kilos. of steam per hour, then the following heating surfaces are indicated:—

Vessel I. For heating, $\frac{70}{20}$ . . . . . = 3·5 sq. m.

For the development of 100 kilos. of *extra steam*, $\frac{100}{20}$ . . . . . = 5

For the 96·96 kilos. of steam required to produce *extra steam* in vessel II., $\frac{96·96}{20}$ . . . . = 4·848 „

For the regular evaporation of the quadruple effect, $\frac{148}{20}$ . . . = 7·4 „

Total . . . . . 20·748 „

Vessel II.	$\frac{100}{20} + \frac{150 \cdot 7}{20}$	= 12·54	sq. m.
Vessel III.	$\frac{166 \cdot 4}{20}$	= 8·32	„
Vessel IV.	$\frac{181 \cdot 52}{20}$	= 9·76	„
	Total	51·368	„

The weight of water, which 1 sq. m. of heating surface evaporates in one hour in the multiple-effect evaporator, cannot be stated as universally applicable, since it varies greatly on account of all the reasons previously given, which cannot be expressed in calculations. It is therefore necessary to take the figures of practical experience. Ordinary vertical evaporators, with brass heating tubes of 1000 mm. length and over, evaporate from liquids which present no obstacles to evaporation :—

In the single effect:	70-80	litres of water	per	1 hour	and 1	sq. m.
In the double effect:	30-36	„	„	„	„	„
In the triple effect:	20-25	„	„	„	„	„
In the quadruple effect:	18-21	„	„	„	„	„

The same apparatus with the liquor at a low level: about 10 per cent. more.

Apparatus with wide horizontal heating tubes: the same.

Apparatus with narrow horizontal heating tubes: about 15 per cent. more.

Iron heating tubes decrease the evaporation by 10-15 per cent., chiefly on account of the greater incrustation.

Apparatus, in which the liquor flows in a thin film over the heating surface, does not evaporate more than that in which the liquor stands at a low level.

Many liquids evaporate with difficulty; the amount of evaporation from 1 sq. m. of heating surface is then very much less.

CHAPTER XIV.

THE PRESSURE EXERTED UPON FLOATING DROPS OF WATER BY CURRENTS OF STEAM AND AIR.

LARGER or smallor quantities of evaporating liquids, and in particular drops, are always thrown above tho bubbling surface. The current of steam, rising along with the drops, exerts on them a driving or lifting force, to such an extent that they frequently rise very high in the boiling pans and may even be thrown out, thus giving rise to loss, which might bo avoided

Finely divided jets or sprays of liquid, upon which the current of gas or vapour, intentionally or naturally produced, exerts a moving action, are often intentionally produced in condensers and cooling apparatus.

The nature of this action must be known, in order that apparatus may be suitably constructed with regard to it.

The action of a current of steam upon drops is due to the pressure it exerts upon them. This pressure depends upon the velocity of the current and the density of the air or steam. We shall therefore endeavour to ascertain the aotion of gas and steam of various densities, velocities and directions, upon drops of different sizes.

It must be definitely stated, that, in consequence of the want of exact research on this subject, the following considerations are based upon certain experiments not made under quite our conditions (Grashof, Theoretische Maschinenlehre, Bd. I.), and on oertain incomplete observations of the author's, and must therefore be rogarded as only tentative.

The pressure, whioh an unbounded current of steam, moving with a velocity of not more than 10 m., exerts upon a plane surface of 0·1 to 4 sq. m. at right angles to its direction, is:—

$$D = \psi \cdot \gamma_1 \cdot Q \cdot \frac{v^2}{2g} \quad \ldots \ldots \quad (105)$$

where D = the pressure in kilos.,
Q = the plane surface in sq. m.,
γ_{ι} = the weight of 1 c. m. of air in kilos.,
v = the relative velocity between the air and plane in metres,
g = the acceleration of gravity (9·81),
ψ = a numerical coefficient.

This coefficient is, according to Grashof, dependent upon the size of the surface and is:—

For surfaces of	$Q = 0·1$	0·25	0·5	1	2	4	sq. m.
	$\psi = 1·86$	2·04	2·18	2·34	2·51	2·69	

The same values hold good for the pressure of moving water upon a plane surface.

For spheres of 100-200 mm. diameter, which move in water, according to Piobert, Hutton, Borda (Grashof), in the mean,

$$\psi = 0·54 \quad \ldots\ldots \quad (106)$$

According to experiment of Didion with spherical projectiles, of 120-150 mm. diameter, moving very rapidly through the air,

$$\psi = 0·43\ (1 + 0·0023\ v) \quad \ldots\ldots \quad (107)$$

which would give for velocities of 10-50 m. a mean value of $\psi = 0·4597$.

Now ψ decreases with decreasing surface, and hence for plane surfaces smaller than 0·1 sq. m. would be considerably less than 1·86. Also the coefficients for air and water have been found to differ little. We shall therefore take for the estimation of the pressure which air exerts upon drops of water, 0·25-10 mm. in diameter, the value $\psi = 0·6$, believing that this figure is quite on the safe side.

The pressure of air upon floating drops would accordingly be

$$D = 0·6\gamma_{\iota} \,.\, Q \,.\, \frac{v^2}{2g} \quad \ldots\ldots \quad (108)$$

whence

$$v = \sqrt{\frac{2Dg}{0·6\gamma_{\iota} \,.\, Q}} \quad \ldots\ldots \quad (109)$$

We shall assume that these equations also hold good for gases and vapours, heavier or lighter than air, when the weight of 1 cub. m. of these gases is inserted for γ_{ι}, although we believe, reasoning from known facts, that in reality the pressure of currents of air upon drops is less than that calculated from equations (108) and (109).

A drop of liquid is spherical when forces act upon it evenly; but when unequal pressures are exerted upon it, as by currents of air and steam in one direction, it is flattened upon the side on which the pressure is exerted, thus its diameter will be somewhat increased. This circumstance, which is beyond a simple calculation, must be neglected, though it increases the pressure upon the drop, *i.e.*, a smaller velocity is required to make the pressure upon the drop equal to a given fraction of its weight.

Table 23 has been calculated by means of equation (109), it gives the velocities, which currents of carbonic acid, air, and steam at 100°-10° C. must have, in order to exert upon drops of 0·1-10 mm. diameter pressures equal to, and double, their weight. In the case of drops of liquids lighter or heavier than water, these velocities will be less or greater; they may be calculated in each case by means of equation (108), putting for D the weight of a drop of the particular liquid.

Table 23 is to be used with caution, for probably the velocities really necessary in order to exert the pressures, G and $2G$, are greater than are given. However, two conclusions may be drawn:—

1. *The smaller the drop of water, the smaller is also the velocity of the current of steam which exerts a pressure upon it equal to its own weight.*

2. *The lower the pressure of the air or steam, the greater must be the velocity to exert a pressure equal to the weight of a drop.*

Or, in other words, with increasing pressure and velocity of the current of air or steam, the danger increases that floating drops will be carried away with it.

The volume of the steam and also its velocity in the same section of the apparatus increase approximately in *simple* proportion with an increase in the vacuum (*i.e.*, approximately in inverse proportion to the absolute pressure). The pressure upon the drop, and hence the danger that it will be carried away with the steam, increase, however, with the *square* of this velocity.

From these facts the conclusion follows: that the sections of the apparatus, in which floating drops of water are not to be carried away by the current of steam which meets them, must always be determined for the greatest vacuum to be expected (i.e., for the lowest possible pressure expected).

TABLE 23.

The velocities of currents of carbonic acid, air and steam of different water, 0·1-10 mm. in diameter, *equal*

Diameter of the drop in mm.		0·10	0·25	0·50
Volume of the drop in cub. mm.		0·0005233	0·00819	0·0655
Section of the drop Q in mm.		0·00785	0·049	0·196
Ratio : $\frac{\text{Weight}}{\text{Surface}} = \frac{G \text{ in kilo.}}{Q \text{ in eq. m.}}$		0·0666	0·168	0·334
$\frac{2Pg}{0{\cdot}6Q}$		2·1778	5·493	10·022
		The velocity of the current of gas or steam when		
Carbonic acid at 0° C., γ=1·873	1 atm. abs.	1·04	1·66	2·35
Air at 15° C., γ=1·225	,,	1·33	2·11	2·98
Steam at 100° C., γ=0·6059	,,	1·89	3	4·24
	Vacuum.			
,, 90° C., γ=0·42829	235 mm.	2·25	3·6	5·01
,, 80° C., γ=0·29582	406 ,,	2·71	4·3	6·07
,, 70° C., γ=0·19928	527 ,,	3·3	5·2	7·4
,, 60° C., γ=0·13114	612 ,,	4·08	6·44	9·1
,, 50° C., γ=0·08336	668 ,,	5·19	8·1	11 4
,, 45° C., γ=0·06576	689 ,,	5·74	9·1	12·8
,, 40° C., γ=0·05119	706 ,,	6·5	10·3	14·59
,, 35° C., γ=0·03975	720 ,,	7·4	11·74	16·55
,, 30° C., γ=0·03086	729 ,,	8·4	12	18·8
,, 25° C., γ=0·02320	737 ,,	9·6	15·36	21·7
,, 20° C., γ=0·01753	743 ,,	11·1	17·69	24·96
,, 15° C., γ=0·01319	747 ,,	12·8	20.4	28·70
,, 10° C., γ=0·00951	754 ,,	15·1	24	33·5
		The velocity of the current of gas or steam when its		
Steam at 100° C.	1 atm. abs.	2·67	4·2	6
	Vacuum.			
,, 90° C.	235 mm.	3·18	5·1	7·14
,, 80° C.	406 ,,	3·82	6·1	8·6
,, 70° C.	527 ,,	4·68	7·4	10·4
,, 60° C.	612 ,,	5·70	9·1	12·9
,, 50° C.	668 ,,	7·35	11·4	16·18
,, 45° C.	689 ,,	8·12	12·9	18·2
,, 40° C.	706 ,,	9·2	14·6	20·6
,, 35° C.	720 ,,	10·4	16·6	23·4
,, 30° C.	729 ,,	11·8	17·0	26·60
,, 25° C.	737 ,,	13·7	21·7	30·61
,, 20° C.	743 ,,	15·78	25	35·7
,, 15° C.	747 ,,	18·16	28·8	40·8
,, 10° C.	751 ,,	21·35	32·5	48

TABLE 23.

pressures, at which these substances exert pressures upon drops of to, and *double*, the weight of the drop.

1	2	3	4	5	6	7	8	9	10
0·525	4·2	14·15	33·6	65·4	113	179	271	382	525
0·785	3·14	7·1	12·6	19·6	28·3	38·5	50·2	63·6	78·5
0·668	1·337	2·0	2·666	3·336	4·0	4·65	5·4	6·0	6·688
21·844	43·71	65·4	87·17	109·08	130·8	152·05	176·58	196·2	218·69

its pressure is to be *equal* to the weight of the drop.

3·31	4·69	5 74	6·63	7·41	8·12	8·77	9·38	9·95	10·5
4·22	5·95	7·3	8·42	9·43	10·3	11·1	11·9	12·6	13·3
6	8·48	10·3	12	13·4	14·66	15·84	17	18	19
7·14	10·09	12·3	14·14	15·96	17·46	18·84	20·2	21·4	22·5
8·6	12·12	14·8	17·18	19·2	21	22·67	24·4	25·7	27·2
10·4	14·78	18·1	20·9	23·4	25·6	27·63	29·6	31·3	33·1
12·9	18·24	22·3	25·8	28·86	31·57	34	36·8	38·4	40·8
16.1	22·89	28	32·2	36	39	42·7	46	48·5	51·2
18·2	25·80	31·6	36·3	40·8	44	48·1	51·6	54·2	57·7
20·6	29·2	35·5	42	46·2	50·5	54·5	59·7	62	65·4
23·4	33·5	40·5	47	52·4	57·2	61·85	66·70	70·2	74·2
26·6	38	46	53·2	59·5	65	70·2	75·7	79·7	84·2
30·61	43·2	53·2	61·2	69·1	75	80·95	87·5	91·8	97·1
35·7	50	61·1	70·6	78·9	86·5	93·3	100	105·8	112
40·8	57·8	70	81·5	91	99 5	107·2	114	121·8	128
48·0	68	83	96	106·7	117	126·4	136	143·5	155

pressure is to be equal to *double* the weight of the drop.

8·48	12	14·6	16·97	18·97	20·76	22·38	24·1	25·4	26·8
10·09	14·14	17·4	20·2	22·58	24·7	26·64	28·7	30·2	32
12·12	17·18	21	24·08	27·1	29·7	32	34·2	36·4	38·4
14·78	20·9	25·6	29·59	33	36·8	39	42	43·4	47·2
18·24	25·8	31·6	36·4	40·08	44·8	48·1	52	54·3	57·7
22·9	32·2	39·2	45·6	51·1	54·6	60·4	65	68·5	72·4
25·7	36·3	44·7	51·6	57·7	63	68	73·2	77·5	81·6
29·2	42	50·5	58·5	65·3	71·8	77	83·9	87·5	92·4
33	47	57·3	66·6	74	81	87·5	94·2	99·5	104·8
37·4	53·2	65·2	75·4	84	92	99·75	107	112·6	118·7
43·3	61·2	75·3	86·7	97	106	114·4	123	130	137·0
50	70·6	86·5	100	111	122	131·9	141	149·6	158
57·5	81·5	99	114·8	128	140	151·6	163	172·3	182
67·5	96	117	135·6	151	165	178·8	193	203	220

CHAPTER XV.

THE MOTION OF FLOATING DROPS OF WATER, UPON WHICH PRESS CURRENTS OF STEAM.

A. Vertical Currents of Steam upon Falling Drops.

We shall first enquire what upward pressure a current of steam may exert upon falling drops without carrying them with it.

When a drop is loosened from a fixed point in a vacuum and falls, its velocity, v, after the time, t, and the height, h, through which it has fallen, are obtained from the well-known equations,

$$v = gt = \sqrt{2gh},\ h = \tfrac{1}{2}gt^2 = \frac{v^2}{2g},\ \ t = \frac{v}{g} = \sqrt{\frac{2h}{g}}. \quad . \quad (110)$$

in which g is the gravity acceleration = 9·81 m. per sec. per sec.

Since the attraction of the earth imparts a very small velocity to the drop in the first moment, and in the second, third, etc., moments adds a second, third, etc., equally small velocity to the first, the total velocity increases uniformly, and is, after one second, 9·81 m., after the second second $2 \times 9{\cdot}81 = 19{\cdot}62$ m. per sec., etc.

Any constant pressure exerted upon a drop in any other direction naturally gives it an accelerated motion in that direction, and this acceleration is directly proportional to the pressure, since the mass of the drop remains the same. If the constant pressure of the gas or steam is equal to the weight of the drop, then the acceleration, which it imparts to the drop in its direction of action, is also equal to the gravity acceleration, g = 9·81 m. per sec. per sec. A pressure on the drop, x times as large as its weight, communicates to it in its own direction an acceleration x times as great as gravity.

Thus if the pressure be known, which a current of air or steam exerts on a drop, the acceleration which this pressure imparts is also known. If the weight of the drop is G, and the pressure D, then the acceleration due to the pressure is

$$g_1 = \frac{D}{G} g.$$

Now that this is clear, we may follow the motion of the drop, when the known pressure is exerted upon it in its direction of motion, in the opposite direction, or at an angle.

We shall take for consideration those cases which may occur in evaporators and condensers, in order to obtain from the results a basis for calculating the dimensions of these pieces of apparatus.

If a drop is falling vertically in a uniform current of steam, which is ascending vertically, and the pressure of which upon the drop is less than the weight of the drop, the fall takes place with increasing velocity, but decreasing acceleration, until the sum of the velocities of the steam, v_d, and of the drop, v_t, causes a pressure upon the drop which is equal to its weight. The sum of the two velocities, $v_d + v_t = v$, may be calculated from equation (109), and may be obtained from Table 23 for steam of known pressure and velocity. Then the velocity of the drop alone at this moment is immediately obtained by subtraction, $v_t = v - v_d$, so that v_d and v_t are then known.

The height of fall of the drop, at the moment in which the opposing pressure is equal to its weight, is obtained from the equation $v_t = \sqrt{2 g_1 h}$, in which g_1 is variable.

If the pressure of the steam upon the drop at the top of the fall is D and at the bottom G, then g_1 alters during the fall from

$$g_1 = \frac{G - D}{G} g \text{ to } g_1 = \frac{G - G}{G} g = 0,$$

and in fact according to a function of v. Although it is not quite accurate, yet a tolerably correct representation is obtained by assuming that the mean value of g_1 is $\frac{G - D}{2G} g$. Whence we find that the height, h, through which the drop must have fallen in order to attain its greatest velocity is

$$h = \frac{v_t^2}{\frac{G - D}{2G} g} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (111)$$

If the drop has fallen so far, it will thooretically continue falling in the uniform current of steam at a uniform velocity without acceleration; as a matter of fact, friction will influence this velocity.

If the velocity of the ourrent of steam which meets the falling drop is not regular, but is large below and *zero* at the point from which the drop starts, thus diminishing from below upwards, then the height, to which the drop must fall in order to attain its greatest velocity, is found from a consideration of the law according to which the speed of the current of steam decreases, and the distance through which the decrease takes place.

In *opposite current condensers* this distance is equal to the height of the condensers from the steam entry to the water distributor. The decrease in velocity is irregular, being slower above than below; it follows approximately the law given in Chapter I. But all the factors of influence can only be introduced hypothetically into the calculation, which is therefore omitted, especially since the results are not of great practical importance. There is no great deviation from the truth if we assume that the height of fall of the drop until it attains its greatest velocity is $h = \frac{v_t^2}{g}$.

The drop falls with increasing velocity in the opposing current of steam, and reaches its greatest velocity at the point where the opposing pressure is equal to its weight; then its motion becomes slower and slower, until it reaches the point at which the opposing pressure of the steam, D, alone is equal to double the weight of the drop, *i.e.*, at which $D = 2G$. With a uniformly increasing velocity of the steam this would be at the distance, $2h$, from above. Here the velocity of the drop becomes $= 0$, bnt the pressure of the steam at once carries it up again. Its upward velocity now increases, and it finally oscillates about the point, at which the pressure of the steam is *equal* to its weight, where it may come to rest.

Although this representation of the process is not quite exact, since the velocities of the steam and the drop in the opposite current condenser are in a complicated relation to one another, and the condensation, the friction and the presence of the many other drops considerably affect the movements, yet it gives an approximate picture of the motion of the drops and allows two important conclusions to be drawn.

1. *The condensation in an opposite current condenser must always be so conducted that all the steam, at the furthest, is liquefied at the water distributor; for if steam is still present here, there will still be currents of steam, and the possibility that drops may be carried out of the condenser.*

2. *The speed at which the steam enters an opposite current condenser (without steps), ought never to be so great that it can exert a pressure equal to double the weight of a drop of water. If the condenser has several steps the velocity of the steam ought only to exert a pressure somewhat greater than the single weight of a drop.*

In the *parallel current condenser* the current of steam enters at the top, along with the falling drops of water, and follows their direction; it therefore exerts a pressure on them when it moves more rapidly than they fall, which is almost always the case. Consequently the drops fall faster—they more quickly reach the lower part of the condenser—and their time of fall is less than when they fall free.

Since the velocity of the steam diminishes to zero towards the bottom, but the speed of fall of the drop increases towards the bottom, the accelerating action of the steam is not very great. It rarely increases the velocity of the drop by more than one quarter.

The jets and sheets of water present in all condensers are very much less influenced by the steam currents, it may be because these currents meet them sideways.

B. Horizontal or Inclined Steam Currents meet Falling Drops.

When a current of air or steam moving in a horizontal direction strikes a drop of water falling vertically, the latter is deflected from its vertical path. If the side pressure upon the drop begins from the same moment as its fall and is equal to its weight, then the drop falls at an angle of 45° with the horizon, since the horizontal acceleration is equal to the vertical. With a lower pressure the angle is greater, with higher pressures smaller.

If the horizontal pressure is several times greater than the weight of the drop, the direction of fall may approach very nearly to the horizontal, but can never rise above the horizontal, since the forces act only from the side and downwards but never upwards.

Should the drop already have fallen vertically through a certain distance before the side current meets it, the deviation is considerably

less, since now in equal intervals of time the vertical velocity is greater than the horizontal. The danger that the drop will be carried with the side current is therefore less. The connection can be seen more clearly from the annexed Fig. 12, than it could be made by many words.

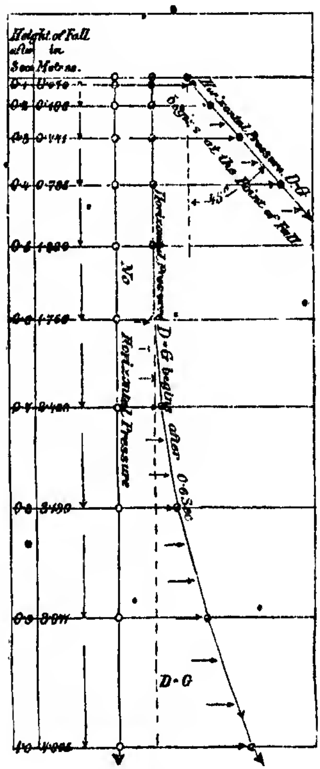

FIG. 12.

If the direction of the current of steam is inclined upwards at the angle α towards the horizon, then the drop of water will still continue to fall if the pressure of the side current, D, is less than $\frac{G}{\sin \alpha}$.

If D is *less than* G, the drop cannot be driven upwards at any angle; it always falls downwards.

If the side pressure, D, is *equal* to the weight of the drop, G, the drop falls downward when α is less than 90°. When $\alpha = 90°$ (*i.e.*, $\sin \alpha = 1$) the drop is kept exactly in its place.

If D be greater than G, the danger that the drop may be carried upwards occurs even with small values of α. When D is 1·25, 1·5 or 2·0 times as great as G, the upward angle which the current of steam may make with the horizon may not be greater than

$$\left[D \sin \alpha = G, \quad 1{\cdot}25\, G \sin \alpha = G, \quad \sin \alpha = \frac{1}{1{\cdot}25}\right]$$

$$\sin \alpha = \frac{1}{1{\cdot}25}, \ \frac{1}{1{\cdot}5} \text{ or } \frac{1}{2};$$

$$\alpha = 53°, \ 41° \text{ or } 30°.$$

TABLE 24.

The velocities of the currents of gas and steam, which, acting upwards at an angle of 30°, 45° or 60° on floating drops, drive them in a horizontal direction.

		Diameter of the drop of water in mm.												
		0·1	0·25	0·5	1	2	3	4	5	6	7	8	9	10
		Velocity of the current of gas and steam in m. per sec.												
Carbonic acid	$\alpha = 30°$	1·48	2·3	3·36	4·78	6·78	8·42	9·61	10·74	11·8	12·66	13·79	14·45	15·22
$s = 1{\cdot}529$	$\alpha = 45°$	1·24	1·98	2·82	4·01	5·69	6·98	8·09	9·00	9·9	10·64	11·48	12·10	12·77
$\gamma = 1{\cdot}873$	$\alpha = 60°$	1·12	1·80	2·56	3·64	5·27	6·34	7·33	8·18	9·0	9·67	10·44	11·0	11·61
Air	$\alpha = 30°$	1·82	2·91	4·15	5·89	8·36	10·26	11·86	13·24	14·50	15·65	16·87	17·80	18·78
$s = 1$	$\alpha = 45°$	1·52	2·43	3·45	4·92	6·99	8·57	9·91	11·06	12·16	13·00	14·10	15·00	17·44
$\gamma = 1{\cdot}293$	$\alpha = 60°$	1·39	2·22	3·16	4·44	6·39	7·83	9·06	10·11	11·12	11·95	12·90	13·62	14·82
Steam at 100° C.	$\alpha = 30°$	2·6	4·12	5·87	8·34	11·84	14·5	16·79	18·75	20·6	21·8	23·89	25·23	26·57
$s = 0{\cdot}6233$	$\alpha = 45°$	2·18	3·40	4·96	7·04	10·0	12·26	14·1	15·83	17·4	18·7	19·18	21·31	22·45
$\gamma = 0{\cdot}6059$	$\alpha = 60°$	1·85	1·96	4·21	5·99	8·51	10·43	11·98	13·04	14·8	15·9	17·17	18·1	19·04

In Table 24 are given the velocities of currents of carbonic acid, air and steam (the latter at 100° C.), at which, striking upwards at angles of 30°, 45° and 60° upon drops just beginning to fall, these

currents cause the drops to deviate into the horizontal direction. Thus if such currents are not to carry drops up with them, they should be given smaller velocities than those in the table.

A special case is that in which a drop, just falling from an edge, is met by a current moving in a circle round this edge. In this case too, D should not be greater than G, if the drop is not to be carried upwards.

Since the distance traversed by drops in apparatus is never very great, and their velocity is generally high, it follows that the time during which the drops move freely is usually very brief. Thus it often happens that before the pressure of the steam can materially deviate the course of the drop, it has arrived safely at its destination.

The cases just treated occur in dry opposite-current condensers with horizontal or inclined diaphragms. We learn that the sections between the diaphragms must be made so large, that the pressure exerted upon the drops by the velocity of the steam can never exceed their weight.

C. A Vertical Current of Steam meets a Drop thrown Obliquely.

In Heckmann's froth separator, Ger. Pat. 70,022 (Fig. 13), two other cases occur. The drops are thrown from the froth-plate either horizontally or at a downward angle and the current of steam generally meets them from below.

If the drop flies horizontally from the froth-plate, its weight draws it downwards and it falls through the space, s_f, in the time, t.

$$s_f = \frac{g}{2}t^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (112)$$

The pressure of the current of steam from below forces it upwards, and it rises in the same time, t, through the space.

$$s_p = \frac{D}{G}\frac{g}{2}t^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (113)$$

The vertical path is therefore

$$s = s_f - s_p = \frac{g}{2}t^2 - \frac{D}{G}\frac{g}{2}t^2 = \frac{gt^2}{2}\left(1 - \frac{D}{G}\right) \quad . \quad . \quad . \quad (114)$$

If $\frac{D}{G} = 1$, then $s = 0$, *i.e.*, when the upward pressure is equal to

the weight of the drop, the latter continues in the horizontal direction without deviation upwards or downwards. If the pressure D is greater than G, the drop is carried upwards by the current of steam; if the pressure is smaller, the drop falls slowly downwards.

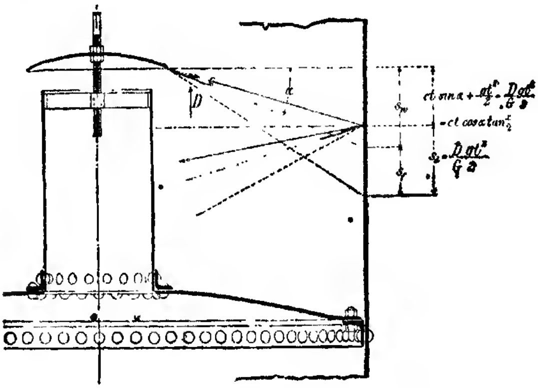

FIG. 13.

If, in consequence of the shape of the foam-plate, the drop acquires a motion inclined downwards to the horizon at the angle α, and the velocity c, whilst a current of steam acts upon it vertically from below with the pressure D, the drop describes the downward space, s_w, in the time, t, in consequence of its original velocity.

$$s_w = ct \sin \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (115)$$

The path downwards, due to the earth's attraction, is

$$s_f = \tfrac{1}{2}gt^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (116)$$

The path upwards, due to the current of steam, is

$$s_d = \frac{D}{G}\frac{g}{2}t^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (117)$$

Its total movement from the horizontal is therefore

$$s = s_w + s_f - s_d = ct \sin \alpha + \tfrac{1}{2}gt^2 - \frac{D}{G}\frac{2}{g}t^2 \quad . \quad . \quad . \quad (118)$$

or

$$s = ct \sin \alpha + \tfrac{1}{2}gt^2\left(1 - \frac{D}{G}\right) \quad . \quad . \quad \quad . \quad . \quad (119)$$

Equation (119) indicates that the curve, in which the drop moves downwards, is a parabola; we shall, however, assume now for the sake of simplicity that the path is a straight line, from which, as a matter of fact, it deviates but little in the portion considered.

From equation (119) it is also seen that, when the pressure of the steam current D from below is *less* than the weight of the drop, the latter falls *below* the direction in which it was thrown off, and that when $D = G$, it moves in that direction, *i.e.*, at the angle α with the horizon.

If D is greater than G, the drop will be carried on to the wall of the apparatus above the direction at which it was thrown off. If it is assumed that it rebounds at the same angle as that at which it hit the wall, and is now carried on the rebound by the upward current of steam to the same extent as before, this direction of rebound must not lie above the horizontal if the drop is not to be carried away upwards.

The pressure from below should thus at most have the effect of raising the drop through half the angle of inclination of the plate $\left(\text{that is, by } \frac{\alpha}{2}\right)$.

Then
$$s = ct \cos\alpha \tan\frac{\alpha}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (120)$$

Now
$$s_d = s_w + s_f - s,$$

therefore
$$s_d = \frac{D}{G}\frac{g}{2}t^2 = ct \sin\alpha + \frac{g}{2}t^2 - ct\cos\alpha\tan\frac{\alpha}{2} \quad . \quad . \quad (121)$$

Hence we obtain the relation between the pressure exerted by the steam and the weight of the drop:—

$$\frac{D}{G} - 1 = \frac{2c}{gt}\left(\sin\alpha - \cos\alpha\tan\frac{\alpha}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad (122)$$

The velocity, c, with which the drops are thrown off from the plate is rarely less than 20 m. per second, but is generally 30 m. or more. The vessels, in which this separation of drops takes place, are rarely more than 3000 mm. in diameter, the distance from the wall is thus 1200 mm. at a maximum, since the plate in this case would be more than 600 mm. in diameter. The time the drop requires in order to reach the wall under these circumstances is given by

$$20t = 1{\cdot}2$$
$$\text{or } t = 0{\cdot}06 \text{ sec.}$$

In this time of 0·06 sec. a drop may fall freely through 18 mm. If the plate has a downward inclination of 10° with the horizontal, then the drops flying off in a straight line from it would hit the wall 224 mm. below the horizontal. The pressure of the steam from below thus may raise the drop (without danger of carrying it away) through: the 18 mm. through which the attraction of the earth drags it down, and then through about half 224 mm., *i.e.*, through 18 + 112 = 130 mm., for which roughly a pressure equal to $\frac{130}{18} = 7$ times the attraction of gravity would be requisite.

If the following substitutions be made in equation (122) the results contained in Table 25 are obtained:—

$$c = 20,\ 30 \text{ and } 50 \text{ m.},$$
$$\alpha = 10^\circ,$$
$$t = 0{\cdot}06,\ 0{\cdot}03 \text{ and } 0{\cdot}01 \text{ sec.}$$

The results indicate how many times the pressure D may be greater than G before danger occurs that the drop will be carried away. It will be seen that, under ordinary circumstances, a small angle, α, is sufficient quite to exclude this danger.

TABLE 25.

t	$c = 20$ m.	$c = 30$ m.	$c = 50$ m.
	Value of $\frac{D}{G}$ when $\alpha = 10^\circ$.		
0·06	7·35	10·52	16·88
0·03	13·70	20·00	32·72
0·01	39·16	48·60	86·28

CHAPTER XVI.

THE SPLASHING OF EVAPORATING LIQUIDS.

A. The Height to which the Splashes rise when the Current of Steam acts upon them.

When liquids are in rapid evaporation, both drops and larger volumes are thrown up above the surface. These may then be carried by the ascending current of steam, thrown out of the vessel and thus readily lost.

We shall examine to what height portions of the liquid may be raised in boiling and under what circumstances losses may occur.

Three influences affect the motion of portions of the liquid:—

1. The drops, bubbles and splashes are thrown up with the constant velocity, c, by the steam bubbles produced by the boiling liquid.
2. The attraction of the earth draws them down and gives them the velocity: $v_f = gt$.
3. The current of steam rising from the liquid with the velocity, v_d, exerts an upward pressure upon the projected portions when v_d is greater than their upward velocity, c. At the level of the liquid the difference in the velocities is $v_d - c$; when the projected portions have reached the highest point of their path, at which the velocity is zero, the difference in the velocities is $v_d - 0 = v_d$.

If v_d is greater than c, the current of steam acts *from below* upon the drops, bubbles and splashes and *increases* the velocity of their ascent. If v_d is less than c, the current of steam exerts a pressure upon them *from above* and *retards* the velocity of ascent.

If we represent the pressure exerted upon the splashes by the current of steam, in consequence of this difference in velocity, by

P_u at the surface and by P_o at the highest point, then the mean pressure is approximately $\pm \frac{P_u + P_o}{2}$ and the mean acceleration they receive from this pressure is $\pm \frac{P_u + P_o}{2G} g$. Consequently the velocity imparted to them in the time, t, by the current of steam is $\pm \frac{P_u + P_o}{2G} gt$.

The *total velocity* of the splashes will therefore be

$$v_t = c - gt + \frac{P_u + P_o}{2G} gt \quad \ldots \ldots \quad (123)$$

At the highest point, at which $v_t = 0$,

$$c + \frac{P_u + P_o}{2G} gt = gt \quad \ldots \ldots \quad (124)$$

Thus the time required to reach the highest point is

$$t = \frac{c}{g\left(1 - \frac{P_u + P_o}{2G}\right)} \quad \ldots \ldots \quad (125)$$

The distance described by the drop in the time, t, *i.e.*, the height to which it has risen in the time, t, is

$$h_t = ct - \tfrac{1}{2}gt^2 + \frac{P_u + P_o}{2G} \frac{gt^2}{2} \quad \ldots \ldots \quad (126)$$

or

$$h_t = \frac{t}{2}\left(c + c - gt + \frac{P_u + P_o}{2G} gt\right). \quad \ldots \quad (127)$$

If v_t is inserted for the value in equation (123), then

$$h_t = \frac{t}{2}(c + v_t) \quad \ldots \ldots \quad (128)$$

When $v_t = 0$ (at the highest point),

$$h_t = \frac{t}{2} c \quad \ldots \ldots \quad (129)$$

or, inserting the value of t from equation (125),

$$h_t = \frac{c^2}{2g\left(1 - \frac{P_u + P_o}{2G}\right)} \quad \ldots \ldots \quad (130)$$

From this equation the height to which drops, bubbles and splashes, thrown up from boiling liquids, will rise, can be calculated in all cases for which c, P_w and P_s are known. These values must now be found.

Equation (130) *shows that the current of steam will carry drops from liquids of low specific gravity to a greater height than those from liquids of higher specific gravity.*

B. The Height to which the Splashes rise when the Current of Steam does not act on them.

We shall next consider the *velocity*, c, with which, and the *height*, h, to which, portions (*not drops*) of the evaporating liquid will be thrown above its surface, neglecting in the case of these masses the action of the rising current of steam.

1. *Steam Heaters, with Vertical Heating Tubes containing the Liquid, under Atmospheric Pressure.*

In this case, if the liquid reaches to, but does not cover, the upper end of the tube, isolated bubbles of steam are formed on heating gently; they rise in the tube, pass above the surface and burst. When the evolution of steam increases the steam bubbles form a current of steam, which continuously leaves the top of the tube.

The velocity of the emerging steam is conditioned by its volume and the section of the tube. The volume of the steam is, however, dependent upon the dimensions of the heating surface (*i.e.*, in this case the length and diameter of the tube), its evaporative capacity per sq. m., and the pressure of the steam. All these factors may vary greatly.

Now, however, steam does not escape alone from the tube; a considerable quantity of liquid accompanies it. When the steam evolved in the tube throws the liquid out, more liquid enters from below, from which, in its turn, steam is formed, which again carries with it the fresh liquid.

The velocity with which the fresh liquid enters the tube depends upon the pressure of the column of liquid outside the tube, the internal opposing pressure of the steam (which is generally small) and on the specific gravity of the liquid. The greater the height of the column of liquid and the density of the liquid, and the lower the

pressure in the tube, the greater is the velocity with which the liquid enters.

The pressure of the column of liquid is due to its height outside the tube *minus* the height of the liquid in the tube. The velocity with which the liquid enters the tube at the bottom, and consequently also the quantity of liquid carried into the tube, is greatest when the tube contains only steam throughout its entire length. This extreme case is, however, unusual. The contraction, due to sharp angles and the cylindrical form of the tube, causes the theoretical velocity of entry not to be quite attained. We shall therefore assume, by analogy with vertical jets of water, that the *greatest velocity* with which the liquid enters at the bottom is

$$v_e = 0{\cdot}8\sqrt{2gl} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (131)$$

where l is the length of the tube in metres.

The volume of liquid, V_f, in litres, which enters at the bottom of the tube in one second, is

$$\begin{aligned} V_f &= v_e\frac{d^2\pi}{4}10 \\ &= 0{\cdot}8\sqrt{2gl}\frac{d^2\pi}{4}10 \\ &= 2d^2\pi\sqrt{2gl} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (132) \end{aligned}$$

if d be the diameter of the tube in decimetres.

The *volume of steam*, in litres, formed in the tube in 1 second, and which thus must leave it at the top, is

$$\begin{aligned} V_d &= \frac{d\pi lw 1000}{10 \times 3600\gamma} \\ &= \frac{d\pi lw}{36\gamma} \text{ litres} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (133) \end{aligned}$$

in which w is the evaporative capacity in kilos. per 1 sq. m. per hour.

Thus the *total volume*, in litres, which must leave the tube in one second, is

$$V_g = V_f + V_d = 2d^2\pi\sqrt{2gl} + \frac{d\pi lw}{36\gamma} \quad . \quad . \quad . \quad . \quad (134)$$

The *velocity*, in metres, with which this volume leaves the tube, is

$$\begin{aligned} c &= \frac{2\pi d^2\sqrt{2gl} + \dfrac{d\pi lw}{36\gamma}}{\dfrac{\pi d^2}{4}10} \\ &= 0{\cdot}8\sqrt{2gl} + \frac{lw}{90\gamma d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (135) \end{aligned}$$

and the *height*, in metres, to which *the liquid would be thrown* with this initial velocity, if no other force acted on it, is theoretically

$$h_s = \frac{c^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (136)$$

This theoretical height of splashing is given in Table 26; other necessary data for its estimation will also be found in the same place, *viz.*:—

(*a*) The volumes of steam, V_d, in litres, produced in 1 second in tubes of 30, 50, 80 and 100 mm. bore and 1 m. length, when 10, 20, 30 and 50 litres of water are evaporated by 1 sq. m. of heating surface per hour, under atmospheric pressure and vacua of 234, 405, 611 and 705 mm.

(*b*) The volume of liquid, V_f, in litres, which enters at the bottom of empty tubes of 30, 50, 80 and 100 mm. bore in 1 second, when the external pressure of the liquid is 0·333, 0·5, 0·667, 1, 1·5, 2 or 3 m.

(*c*) The calculated velocities, *c*, with which steam and liquid are thrown out of the tubes, when the tubes are 1, 1·5, 2 or 3 m. long.

- (α) When the height of the liquid outside the tube is equal to the length of the tube, *i.e.*, when the hydrostatic pressure is equal to the length of the tube.
- (β) When the height of the liquid outside the tube is only $\frac{1}{3}$ of the length of the tube, *i.e.*, when the hydrostatic pressure is equal to $\frac{1}{3}$ of the length of the tube.

(*d*) Finally, in the same table are given the theoretical heights, h_s, to which the liquid would rise, without regard to the action of the current of steam, for all these cases and also for the case that liquid stands over the ends of the tubes (denoted in the table by *t.c.*—tubes covered).

In regard to the last series of figures, it is to be remarked that, when the steam and liquid emerging from the tube have to penetrate a more or less thick layer of liquid before reaching the surface, they have accordingly in proportion to overcome resistance in the layer of liquid, the steam bubbles then spread out to the sides and their velocity is retarded.

In heaters with vertical tubes, which generally stand very near together, the steam spreads out as soon as it leaves the tubes to such an extent that the isolated currents from the single tubes unite into one, the section of which is equal to the *whole* section above the tubes.

The distances apart of tubes vary in different apparatus. The distance from centre to centre may be approximately,

with tubes of	30	50	80	100 mm. bore,
about	45	65	95	115 mm.

Thus the ratio of the section of the tubes to the section of the open space above them is as

$$1 : 2{\cdot}479 : 1{\cdot}877 : 1{\cdot}573 : 1{\cdot}508 \quad . \quad . \quad . \quad (137)$$

We shall assume that the average ratio is $1 : 1{\cdot}746$; then the velocity of the current of steam above the ends of the tubes is $\frac{c}{1{\cdot}746}$ and the theoretical height of the splashes, without regard to the action of the current of steam, is

$$h_t = \frac{c^2}{(1{\cdot}746)^2 2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (138)$$

The heights of the splashes for evaporating apparatus, in which the liquid covers the ends of the tubes, have been calculated by means of this equation (Table 26D, denoted by *t.c.*).

The velocities, c, when the height of the liquid is 1, 1·5, 2 or 3 m., are divided by 1·746 in order to obtain the velocity of steam and liquid in the larger space above the tubes. The velocity so obtained is then squared and divided by $2g = 2 \times 9{\cdot}81 = 19{\cdot}32$, by which the theoretical height of the splash is obtained.

In the calculation it was assumed that the tubes were quite free from liquid; other retarding influences were also disregarded. The presence of liquid in the tubes diminishes the hydrostatic pressure and thus the velocity of entry and the quantity of liquid entering. The internal height of the liquid is naturally variable; it will be larger the more slowly the evaporation takes place.

Further, the thickness of the liquid and the height at which it stands over the plate, in which the tubes end, have been disregarded, since both conditions, in the lack of observed figures, cannot be introduced into the calculation.

The quantity of liquid above the plate, which is constantly being renewed by the steam from the sides, has also been disregarded in estimating the velocity. It somewhat increases the volume, thus the velocity, and therefore the height of the splash; it diminishes the height of the splash by absorbing kinetic energy.

It is also to be supposed that the vapours, when they become free from the somewhat compressed conditions in and over the tubes, expand and by the expansion still further throw up the liquid.

The height of the splash of the liquid is diminished by the friction to which the projected portions of the liquid are subjected, and which is disregarded here.

Thus, although the heights to which the liquid is theoretically splashed, as calculated here, cannot be regarded as absolutely exact, yet they make clear what conditions influence the height and in what manner.

Table 26 *shows that the height of the splashes from evaporating liquids increases with decreasing diameter and increasing length of the tubes, with the pressure due to the column of liquid, with the evaporative capacity of the tube per sq. m. of heating surface and with decreasing pressure above the tubes.*

2. *Evaporating Apparatus, not fitted with Vertical Tubes, but with Flat Bottoms, Double Bottoms, Steam Coils or Horizontal Tubes, or heated by Open Fire.*

In apparatus of these constructions the section available for the escape of the steam is always very much greater in proportion to the heating surface than when vertical tubes are used. Whilst with the latter the steam space is 1·5-3 sq. dcm. in section (2-2·2 sq. dcm. on the average) to 1 sq. m. of heating surface, the former constructions give a section of 5, 7, 10 or even 20 sq. dcm. per 1 sq. m. of heating surface. Table 27 gives the velocities of the currents of steam evolved from vacuum evaporators with steam coils or double bottoms.

Thus the velocity with which the steam escapes is always much lower in the latter apparatus than in evaporators with vertical tubes, but the liquid is still raised by the steam to some extent. At the point where steam enters the double bottom or heating coils and tubes, or where fire strikes directly against the wall of the vessel, a much more rapid transference of heat and evolution of steam take place; thus the liquid will be thrown up to the greatest extent near the steam entrance. Consequently there arises a current of liquid from the warmer to the colder parts and back; the velocity of this desirable motion may be very considerable. All the liquid which moves towards the place where

[*Continued on p.* 151.]

TABLES 26A, 26B, 26C, 26D.

A. *Litres of steam*, which emerge in one second from the top of vertical heated tubes, 30, 50, 80 and 100 mm. bore and 1 m. long.

B. *Litres of liquid*, which in one second enter these tubes from below.

C. *Velocities* with which boiling liquids are projected from vertical heated tubes of 30, 50, 80 and 100 mm. bore and 1, 1·5, 2 and 3 m. height, under vacua of 0, 234, 405, 611 and 705 mm., when the evaporation is 10, 20, 30 and 50 litres per sq. m. per hour, and when the height of the column of liquid is equal to the length of the tube and when it is $\frac{1}{3}$ of the same length.

D. *Heights*, h_s, to which the liquid will be splashed above the tubes under the same conditions, without regard to the assistance of the currents of steam.

TABLE 26A.

Length of tube, l.	Evaporation, w, per 1 sq. m. and 1 hour.	Vacuum.	*Litres of steam*, which leave the top of the tube in one second.			
			Bore of tube, mm.			
			30	50	80	100
			Heating surface of tube, sq. m.			
			0·094	0·157	0·251	0·314
Metres.	Litres.	mm.	Litres of steam, V_d.			
1	10	0	0·413	0·75	1·2	1·5
	20	0	0·826	1·5	2·4	3
	30	0	1·239	2·24	3·6	4·49
	50	0	2·15	3·74	6	7·48
1	10	234	0·61	1·02	1·63	2·04
	20	234	1·22	2·08	3·25	4·07
	30	234	1·83	3·05	4·88	6·1
	50	234	3·05	5·09	8·14	10·18
1	10	405	0·883	1·472	2·36	2·95
	20	405	1·766	2·944	4·72	5·9
	30	405	2·649	4·416	7·08	8·85
	50	405	4·418	7·359	11·79	14·75
1	10	611	1·992	3·333	5·32	6·656
	20	611	3·98	6·66	10·64	13·312
	30	611	5·98	9·99	15·96	19·96
	50	611	9·96	16·64	26·61	33·28
1	10	705	5·09	8·51	12·8	17·02
	20	705	10·2	17·03	25·6	34·04
	30	705	15·3	24·53	38·4	51·06
	50	705	25·47	42·54	64·02	85·09

If the heated tube is 1·5, 2 or 3 m. long, then 1·5, 2 or 3 times as many litres escape from the tube.

TABLE 26B.

Length of the tube, l.	*Litres of liquid*, which enter the tube at the bottom in one second when the velocity of entry is $v = 0{\cdot}8\sqrt{2gl}$.			
	Bore of tube, mm.			
	30	50	80	100
	Section of tube, sq. decimetres.			
	0·0706	0·196	0·502	0·785
Metres.	Litres of liquid, V_f.			
0·333	1·41	4	10	15·7
0·5	1·78	5	12·6	18·78
0·667	2·03	5·6	14·4	22·3
1	2·51	6·97	17·87	27·94
1·5	3·08	8·51	21·94	34·22
2	3·58	9·87	25·3	39·56
3	4·49	12·07	30·92	48·35

TABLE 26C.

Length of tube, l.	Evaporation, w, per 1 sq. m. and 1 hour.	Height of liquid outside tube.	Vacuum.	*Velocity*, c, with which steam and liquid leave the top of the tube. Metres per second.			
				Bore of tube, mm.			
				30	50	80	100
Metres.	Litres.	Metres.	mm.	Velocity, c.			
1	10	1	0	4	3·9	3·9	3·8
1	20	1	0	4·71	4·3	4	3·9
1	30	1	0	5·3	4·7	4·3	4·1
1	50	1	0	6·46	5·4	4·75	4·5
1·5	10	1·5	0	5·2	4·8	4·74	4·66
1·5	20	1·5	0	6·1	5·4	5·1	4·93
1·5	30	1·5	0	7	5·9	5·4	5·21
1·5	50	1·5	0	9	7·1	6·1	5·8
2	10	2	0	6·25	5·6	5·54	5·55
2	20	2	0	7·44	6·2	6	5·8
2	30	2	0	8·8	7	6·5	6·15
2	50	2	0	11·7	8·5	7·4	7·68

TABLE 26c—(*continued*).

Length of tube, *l*.	Evaporation, *w*, per 1 sq. m. and 1 hour.	Height of liquid outside tube.	Vacuum.	*Velocity, c*, with which steam and liquid leave the top of the tube. Metres per second. Bore of tube, mm. 30	50	80	100
Metres.	Litres.	Metres.	mm.	Velocity, *c*.			
3	10	3	0	8	—	—	—
3	20	3	0	10	—	—	—
3	30	3	0	11·7	—	—	—
3	50	3	0	15·7	—	—	—
1	10	1	234	4·42	3·99	3·89	3·8
1	20	1	234	5·28	4·55	4·2	4·1
1	30	1	234	6·15	5·1	4·54	4·3
1	50	1	234	7·87	6·2	5·2	4·9
1·5	10	1·5	234	5·6	5	4·8	4·8
1·5	20	1·5	234	7	5·7	5·31	5·1
1·5	30	1·5	234	8·2	6·5	5·84	5·5
1·5	50	1·5	234	10·9	8·5	6·8	6·3
2	10	2	234	6·8	5·9	5·7	5·5
2	20	2	234	8·6	6·6	6·3	6
2	30	2	234	10·3	7·3	7	6·6
2	50	2	234	13·7	9·5	8·2	7·7
3	10	3	234	9	—	—	—
3	20	3	234	11·6	—	—	—
3	30	3	234	14·3	—	—	—
3	50	3	234	19·5	—	—	—
1	10	1	405	4·78	4·3	4	3·9
1	20	1	405	6·07	5	4·5	4·33
1	30	1	405	7·03	5·8	5·3	4·7
1	50	1	405	9·82	7·3	5·9	5·44
1·5	10	1·5	405	6·2	5·4	5·1	4·92
1·5	20	1·5	405	8·1	6·5	5·8	5·48
1·5	30	1·5	405	10	7·8	6·5	6·16
1·5	50	1·5	405	13·5	10	7·9	7·46
2	10	2	405	7·62	6·5	6	5·8
2	20	2	405	10·15	7·5	6·9	6·5
2	30	2	405	12·5	8·5	7·6	7·3
2	50	2	405	17·7	11·5	9·7	9
3	10	3	405	10·2	—	—	—
3	20	3	405	14	—	—	—
3	30	3	405	17·8	—	—	—
3	50	3	405	25·3	—	—	—
1	10	1	611	6·37	5·5	4·63	4·43
1	20	1	611	9·2	6·9	5·7	5·37

TABLE 26c—(*continued*).

Length of tube, *l*. Metres.	Evaporation, *w*, per 1 sq. m. and 1 hour. Litres.	Height of liquid outside tube. Metres.	Vacuum. mm.	Velocity, *c*, with which steam and liquid leave the top of the tube. Metres per second. Bore of tube, mm. Velocity, *c*. 30	50	80	100
1	30	1	611	12·02	8·6	6·76	6·15
1	50	1	611	17·66	12	8·89	7·9
1·5	10	1·5	611	8·5	6·9	6	5·62
1·5	20	1·5	611	10·2	9·5	7·6	7·12
1·5	30	1·5	611	17	12	9·12	8·3
1·5	50	1·5	611	25·5	17	12·9	10·7
2	10	2	611	10·8	7	7·2	6·8
2	20	2	611	16·4	10·4	9·3	8·65
2	30	2	611	22	14	11·4	10·1
2	50	2	611	33·3	20	19·7	13·5
3	10	3	611	15	—	—	—
3	20	3	611	23·3	—	—	—
3	30	3	611	32·1	—	—	—
3	50	3	611	50	—	—	—
1	10	1	705	10·77	7·9	6·1	5·72
1	20	1	705	18	12	8·7	8
1	30	1	705	25	16	11·2	10·1
1	50	1	705	40	25	16·3	14·4
1·5	10	1·5	705	14·5	11	8·2	7·87
1·5	20	1·5	705	26	17·5	12	10·9
1·5	30	1·5	705	35	23	15·9	14·1
1·5	50	1·5	705	59	37	23·6	20·6
2	10	2	705	19	12	10	9·7
2	20	2	705	34	21	15·3	13·7
2	30	2	705	48	29	20·4	18·1
2	50	2	705	77	47	30·6	26·8
3	10	3	705	28	—	—	—
3	20	3	705	49·2	—	—	—
3	30	3	705	72·1	—	—	—
3	50	3	705	113·5	—	—	—
1	10	0·333	0	2·6	2·37	2·2	2·2
1	20	0·333	0	3	2·75	2·48	2·3
1	30	0·333	0	4	3·1	2·74	2·6
1	50	0·333	0	5	3·87	3·2	2·75
1·5	10	0·50	0	3·3	3	2·8	2·56
1·5	20	0·50	0	4·3	3·6	3·22	2·71

TABLE 26c—(*continued*).

Length of tube, l. Metres.	Evaporation, w, per 1 sq. m. and 1 hour. Litres.	Height of liquid outside tube. Metres.	Vacuum. mm.	Velocity, c, with which steam and liquid leave the top of the tube. Metres per second. Bore of tube, mm. 30	50	80	100
				Velocity, c.			
1·5	30	0·50	0	5	4·2	3·5	3·1
1·5	50	0·50	0	7	5·6	4·3	3·8
2	10	0·667	0	3·6	3·2	3·4	3
2	20	0·667	0	5	3·9	3·84	3·3
2	30	0·667	0	5·6	4·9	4·25	3·7
2	50	0·667	0	9	6·3	5·2	4·2
3	10	1	0	5·3	—	—	—
3	20	1	0	7·1	—	—	—
3	30	1	0	8·8	—	—	—
3	50	1	0	12·8	—	—	—
1	10	0·333	234	3	2·5	2·32	2·2
1	20	0·333	234	4	3	2·65	2·4
1	30	0·333	234	4·5	3·5	2·95	2·8
1	50	0·333	234	6·3	4·5	3·63	3·15
1·5	10	0·5	234	4	3·25	3·00	2·6
1·5	20	0·5	234	5·2	4	3·42	3·1
1·5	30	0·5	234	6·3	4·8	4	3·5
1·5	50	0·5	234	9	6·4	5	3·6
2	10	0·667	234	4·3	3·52	3·5	3·2
2	20	0·667	234	5·9	4·5	4·2	3·9
2	30	0·667	234	8	5·5	4·8	4·2
2	50	0·667	234	11·1	7·5	6	5·5
3	10	1	234	6·2	—	—	—
3	20	1	234	8·8	—	—	—
3	30	1	234	11·4	—	—	—
3	50	1	234	16·4	—	—	—
1	10	0·333	405	3·1	2·7	2·46	2·2
1	20	0·333	405	4·5	3·5	2·9	2·4
1	30	0·333	405	6	4·2	3·41	3
1	50	0·333	405	8·8	5·7	4·3	3·8
1·5	10	0·5	405	4·5	3·6	3	2·8
1·5	20	0·5	405	5·3	4·8	3·8	3·3
1·5	30	0·5	405	8	5·8	5	3·5
1·5	50	0·5	405	12	8	5·9	4
2	10	0·667	405	4·8	3·95	3·8	3·6
2	20	0·667	405	7·6	5·5	4·8	4·15
2	30	0·667	405	10	6·9	5·6	5
2	50	0·667	405	15·5	9·9	7·5	6·8

TABLE 26C—(*continued*).

Length of tube, *l*. Metres.	Evaporation, *w*, per 1 sq. m. and 1 hour. Litres.	Height of liquid outside tube. Metres.	Vacuum. mm.	Velocity, *c*, with which steam and liquid leave the top of the tube. Metres per second. Bore of tube, mm. 30	50	80	100
				Velocity, *c*.			
3	10	1·00	405	7·5	—	—	—
3	20	1·00	405	11·1	—	—	—
3	30	1	405	14·9	—	—	—
3	50	1	405	22·5	—	—	—
1	10	0·333	611	5	3·75	3	2·3
1	20	0·333	611	7·8	5·3	4·1	3·72
1	30	0·333	611	10	7	5·1	4·5
1	50	0·333	611	16	10	7·2	5
1·5	10	0·5	611	5·4	5	4	3·6
1·5	20	0·5	611	8·5	7·5	5·6	5
1·5	30	0·5	611	11	10	7·2	6
1·5	50	0·5	611	17	14·5	10·2	8·8
2	10	0·667	611	8	5·8	4·85	3·73
2	20	0·667	611	12·7	9	7·2	5·38
2	30	0·667	611	20	13	9·2	7·13
2	50	0·667	611	30·5	19	13·5	10·5
3	10	1	611	12·2	—	—	—
3	20	1	611	20·6	—	—	—
3	30	1	611	29·2	—	—	—
3	50	1	611	46·2	—	—	—
1	10	0·333	705	9	6·25	4·7	4
1	20	0·333	705	17	10·5	7·2	6·3
1	30	0·333	705	23	14·3	9·6	8
1	50	0·333	705	27·8	23	15	12·8
1·5	10	0·5	705	14	9	6·35	5
1·5	20	0·5	705	24	15·5	10	8·1
1·5	30	0·5	705	33	20·5	14·4	11·3
1·5	50	0·5	705	58	34	20	17·8
2	10	0·667	705	16	11·5	8·1	7·5
2	20	0·667	705	30	20	13	10·5
2	30	0·667	705	45	27	18	15
2	50	0·667	705	75	45	29	23·7
3	10	1	705	23	—	—	—
3	20	1	705	45	—	—	—
3	30	1	705	67	—	—	—
3	50	1	705	110	—	—	—

TABLE 26D.

Length of tube, l. Metres.	Evaporation, w, per 1 sq. m. and 1 hour. Litres.	Height of liquid outside tube. Metres.	Vacuum. mm.	Height to which the liquid is projected from the tube, h_r. Bore of tube, mm. 30. Height of splash, h_s. Metres.	50	80	100
1	10	t.c.	0	0·266	0·253	0·253	0·24
1	10	0·33	0	0·338	0·28	0·242	0·242
1	10	1·0	0	0·8	0·76	0·76	0·72
1	20	t.c.	0	0·367	0·21	0·267	0·253
1	20	0·333	0	0·450	0·373	0·3	0·265
1	20	1·00	0	1·1	0·93	0·8	0·76
1	30	t.c.	0	0·467	0·367	0·31	0·267
1	30	0·333	0	0·8	0·48	0·375	0·338
1	30	1·0	0	1·4	1·1	0·93	0·8
1	50	t.c.	0	0·667	0·483	0·37	0·333
1	50	0·333	0	1·25	0·75	0·512	0·378
1	50	1·0	0	2	1·45	1·11	1
1·5	10	t.c.	0	0·45	0·383	0·367	0·363
1·5	10	0·5	0	0·545	0·45	0·392	0·38
1·5	10	1·5	0	1·35	1·15	1·1	1·09
1·5	20	t.c.	0	0·624	0·488	0·417	0·4
1·5	20	0·5	0	0·92	0·648	0·517	0·48
1·5	20	1·5	0	1·8	1·45	1·25	1·2
1·5	30	t.c.	0	0·817	0·567	0·483	0·45
1·5	30	0·5	0	1·25	0·882	0·612	0·41
1·5	30	1·5	0	2·45	1·7	1·45	1·35
1·5	50	t.c.	0	1·35	0·817	0·617	0·56
1·5	50	0·5	0	2·45	1·57	0·924	0·722
1·5	50	1·5	0	4·05	2·45	1·85	1·68
2	10	t.c.	0	0·65	0·52	0·5	0·5
2	10	0·667	0	0·646	0·514	0·48	0·45
2	10	2·0	0	1·95	1·56	1·5	1·5
2	20	t.c.	0	0·913	0·64	0·6	0·625
2	20	0·667	0	1·25	0·761	0·7	0·55
2	20	2·0	0	2·74	1·92	1·8	1·68
2	30	t.c.	0	1·29	0·817	0·703	0·603
2	30	0·667	0	1·57	1·2	0·9	0·68
2	30	2	0	3·87	2·45	2·11	0·81
2	50	t.c.	0	2·28	1·203	0·91	0·9
2	50	0·667	0	4	1·99	1·35	0·882
2	50	2	0	6·84	3·61	2·73	2·7
3	10	t.c.	0	1·07	—	—	—
3	10	1·00	0	1·4	—	—	—

TABLE 26D—(*continued*).

Length of tube, l. Metres.	Evaporation, w, per 1 sq. m. and 1 hour. Litres.	Height of liquid outside tube. Metres.	Vacuum. mm.	Height to which the liquid is projected from the tube, h_s. Bore of tube, mm. 30	50	80	100
				Height of splash, h_r. Metres.			
3	10	3	0	3·2	—	—	—
3	20	t.c.	0	1·67	—	—	—
3	20	1	0	2·5	—	—	—
3	20	3	0	5	—	—	—
3	30	t.c.	0	2·28	—	—	—
3	30	1	0	3·87	—	—	—
3	30	3	0	6·84	—	—	—
3	50	t.c.	0	4·1	—	—	—
3	50	1	0	8·19	—	—	—
3	50	3	0	12·3	—	—	—
1	10	t.c.	234	0·32	0·267	0·25	0·233
1	10	0·333	234	0·45	0·313	0·269	0·242
1	10	1	234	0·96	0·8	0·75	0·7
1	20	t.c.	234	0·467	0·333	0·293	0·267
1	20	0·333	234	0·8	0·45	0·351	0·288
1	20	1	234	1·4	1	0·88	0·8
1	30	t.c.	234	0·633	0·433	0·333	0·31
1	30	0·333	234	1·01	0·613	0·435	0·392
1	30	1	234	1·9	1·3	1	0·93
1	50	t.c.	234	0·103	0·62	0·45	0·4
1	50	0·333	234	1·99	1·01	0·643	0·5
1	50	1	234	3·1	1·86	1·35	1·2
1·5	10	t.c.	234	0·52	0·417	0·383	0·383
1·5	10	0·5	234	0·8	0·528	0·45	0·338
1·5	10	1·5	234	1·56	1·25	1·15	1·15
1·5	20	t.c.	234	0·817	0·54	0·467	0·42
1·5	20	0·5	234	1·35	0·8	0·57	0·48
1·5	20	1	234	2·45	1·62	1·4	1·26
1·5	30	t.c.	234	1·12	0·703	0·557	0·5
1·5	30	0·5	234	1·99	1·15	0·8	0·61
1·5	30	1	234	3·36	2·11	1·67	1·5
1·5	50	t.c.	234	1·98	1·2	0·77	0·66
1·5	50	0·5	234	4	2·05	1·25	0·65
1·5	50	1	234	5·94	3·61	2·31	1·98
2	10	t.c.	234	0·767	0·58	0·54	0·5
2	10	0·667	234	0·92	0·75	0·62	0·51
2	10	2	234	2·3	1·74	1·62	1·5
2	20	t.c.	234	1·23	0·726	0·66	0·6

TABLE 26D—(*continued*).

Length of tube, l. Metres.	Evaporation, w, per 1 sq. m. and 1 hour. Litres.	Height of liquid outside tube. Metres.	Vacuum. mm.	*Height* to which the liquid is projected from the tube, h_s. Bore of tube, mm. 30	50	80	100
				Height of splash, h_s. metres.			
2	20	0·667	234	1	1·01	0·882	0·77
2	20	2	234	3·69	2·18	1·98	1·8
2	30	t.c.	234	1·77	0·887	0·817	0·727
2	30	0·667	234	3·22	1·51	1·15	0·88
2	30	2	234	5·3	2·66	2·45	2·18
2	50	t.c.	234	3·13	1·5	1·12	0·987
2	50	0·667	234	6	2·81	1·8	1·51
2	50	2	234	9·38	4·5	3·36	2·96
3	10	t.c.	234	1·35	—	—	—
3	10	1	234	1·92	—	—	—
3	10	3	234	4·05	—	—	—
3	20	t.c.	234	2·24	—	—	—
3	20	1	234	3·87	—	—	—
3	20	3	234	6·72	—	—	—
3	30	t.c.	234	3·4	—	—	—
3	30	1	234	6·5	—	—	—
3	30	3	234	10·2	—	—	—
3	50	t.c.	234	6·33	—	—	—
3	50	1	234	13·4	—	—	—
3	50	3	234	19	—	—	—
1	10	t.c.	405	0·373	0·307	0·267	0·253
1	10	0·333	405	0·47	0·365	0·302	0·242
1	10	1	405	1·1	0·92	0·8	0·76
1	20	t.c.	405	0·62	0·417	0·333	0·293
1	20	0·333	405	1·01	0·62	0·42	0·288
1	20	1	405	1·86	1·25	1	0·88
1	30	t.c.	405	0·82	0·56	0·417	0·27
1	30	0·333	405	1·8	0·882	0·578	0·45
1	30	1	405	2·46	1·68	1·23	1·1
1	50	t.c.	405	1·6	0·883	0·6	0·483
1	50	0·333	405	3·87	1·63	0·93	0·72
1	50	1	405	4·8	2·66	1·8	1·46
1·5	10	t.c.	405	0·64	0·487	0·45	0·405
1·5	10	0·5	405	1·01	0·648	0·45	0·392
1·5	10	1·5	405	1·92	1·46	1·31	1 21
1·5	20	t.c.	405	1·09	0·703	0·56	0·5
1·5	20	0·5	405	1·4	1·15	0·722	0 55
1·5	20	1·5	405	3·28	2·11	1·68	1·5

TABLE 26D—(*continued*).

Length of tube, l. Metres.	Evaporation, w, per 1 sq. m. and 1 hour. Litres.	Height of liquid outside tube. Metres.	Vacuum. mm.	*Height* to which the liquid is projected from the tube, h_s. Bore of tube, mm. 30	50	80	100
				Height of splash, h_s. Metres.			
1·5	30	t.c.	405	1·67	1·01	0·703	0·62
1·5	30	0·5	405	3·2	1·68	1·25	0·62
1·5	30	1·5	405	5	3·04	2·11	1·86
1·5	50	t.c.	405	3·07	1·67	1·04	0·93
1·5	50	0·5	405	7·2	3·2	1·74	0·8
1·5	50	1·5	405	9·2	5	3·12	2·8
2	10	t.c.	405	0·96	0·703	0·6	0·56
2	10	0·667	405	1·15	0·78	0·72	0·65
2	10	2	405	2·88	2·11	1·8	1·68
2	20	t.c.	405	1·7	0·93	0·792	0·703
2	20	0·667	405	2·89	1·51	1·15	0·86
2	20	2	405	5·1	2·81	2·38	2·113
2	30	t.c.	405	2·6	1·23	0·96	0·883
2	30	0·667	405	5	2·28	1·57	1·25
2	30	2	405	7·8	3·61	2·88	2·66
2	50	t.c.	405	5·2	2·03	1·57	1·53
2	50	0·667	405	11·3	5	2·81	2·31
2	50	2	405	15·6	6·1	4·7	4·6
3	10	t.c.	405	1·73	—	—	—
3	10	1	405	2·81	—	—	—
3	10	3	405	5·2	—	—	—
3	20	t.c.	405	5·27	—	—	—
3	20	1	405	6·16	—	—	—
3	20	3	405	9·8	—	—	—
3	30	t.c.	405	5·26	—	—	—
3	30	1	405	11·1	—	—	—
3	30	3	405	15·8	—	—	—
3	50	t.c.	405	10·7	—	—	—
3	50	1	405	25·3	—	—	—
3	50	3	405	32	—	—	—
1	10	t.c.	611	0·66	0·487	0·353	0·33
1	10	0·333	611	1·25	0·703	0·45	0·27
1	10	1	611	2	1·46	1·06	0·97
1	20	t.c.	611	1·41	0·793	0·54	0·47
1	20	0·333	611	3·04	1·4	0·81	0·68
1	20	1	611	4·23	2·38	1·63	1·4
1	30	t.c.	611	2·4	1·23	0·77	0·62
1	30	0·333	611	5	2·45	1·26	1·01

TABLE 26D—(*continued*).

Length of tube, l. Metres.	Evaporation, w, per 1 sq. m. and 1 hour. Litres.	Height of liquid outside tube. Metres.	Vacuum. mm.	*Height* to which the liquid is projected from the tube, h_t. Bore of tube, mm. 30	50	80	100
				Height of splash, h_s. Metres.			
1	30	1	611	7·2	3·7	2·3	1·86
1	50	t.c.	611	5·17	2·4	1·32	1·04
1	50	0·333	611	12·8	5	2·57	1·25
1	50	1	611	15·5	7·2	3·96	3·12
1·5	10	t.c.	611	1·203	0·793	0·6	0·523
1·5	10	0·5	611	1·46	1·25	0·8	0·65
1·5	10	1·5	611	3·61	2·38	1·8	1·57
1·5	20	t.c.	611	1·73	1·5	0·963	0·837
1·5	20	0·5	611	3·61	2·81	1·57	1·25
1·5	20	1·5	611	5·2	4·5	2·89	2·51
1·5	30	t.c.	611	0·483	2·4	1·38	1·15
1·5	30	0·5	611	7·5	5	2·59	1·8
1·5	30	1·5	611	14·5	7·2	4·14	3·45
1·5	50	t.c.	611	10·8	4·83	2·73	1·91
1·5	50	0·5	611	14·5	10·2	5·1	3·87
1·5	50	1·5	611	32·3	14·5	8·3	5·72
2	10	t.c.	611	1·94	0·817	0·8	0·77
2	10	0·667	611	3·2	1·7	1·28	0·69
2	10	2	611	5·83	2·45	2·4	2·2
2	20	t.c.	611	4·5	1·8	1·44	1·23
2	20	0·667	611	7·5	4	2·59	1·45
2	20	2	611	13·5	5·4	4·32	3·7
2	30	t.c.	611	3·07	3·27	2·17	1·7
2	30	0·667	611	15·8	8·5	4·10	2·52
2	30	2	611	24·2	7·8	6·5	5·1
2	50	t.c.	611	18·5	6·67	6·47	3·03
2	50	0·667	611	46·5	18·1	10	5·3
2	50	2	611	55·5	20	19·41	9·1
3	10	t.c.	611	3·77	—	—	—
3	10	1	611	7·4	—	—	—
3	10	3	611	11·3	—	—	—
3	20	t.c.	611	8·83	—	—	—
3	20	1	611	21·2	—	—	—
3	20	3	611	26·5	—	—	—
3	30	t.c.	611	17	—	—	—
3	30	1	611	42·6	—	—	—
3	30	3	611	51	—	—	—
3	50	t.c.	611	41	—	—	—

TABLE 26D—(*continued*).

Length of tube, l. Metres.	Evaporation, w, per 1 sq. m. and 1 hour. Litres.	Height of liquid outside tube. Metres.	Vacuum. mm.	Height to which the liquid is projected from the tube, h_s. Bore of tube, mm. 30	50	80	100
				Height of splash, h_s. Metres.			
3	50	1	611	106	—	—	—
3	50	3	611	125	—	—	—
1	10	t.c.	705	1·9	1·04	0·62	0·57
1	10	0·333	705	4	1·95	1·1	0·80
1	10	1	705	5·7	3·12	1·86	1·62
1	20	t.c.	705	5·47	2·4	1·26	1·07
1	20	0·333	705	14·5	5·2	2·60	1·28
1	20	1	705	16·4	7·2	3·78	3·2
1	30	t.c.	705	10·4	4·27	2·09	1·7
1	30	0·333	705	27	9·8	4·1	3·2
1	30	1	705	31·3	12·8	6·27	5·1
1	50	t.c.	705	26·6	10·5	4·43	3·47
1	50	0·333	705	39	26·5	9·8	7·6
1	50	1	705	80	31·5	13·3	10·4
1·5	10	t.c.	705	3·5	2·03	1·12	1·0
1·5	10	0·5	705	7·6	4·03	1·98	1·25
1·5	10	1·5	705	10·5	6·1	3·36	3
1·5	20	t.c.	705	11·3	5·1	2·4	1·98
1·5	20	0·5	705	29	12	5	3·20
1·5	20	1·5	705	33·8	15·3	7·2	5·95
1·5	30	t.c.	705	20·4	8·83	4·3	3·3
1·5	30	0·5	705	55	20	10	6·50
1·5	30	1·5	705	61	26·5	12·6	9·9
1·5	50	t.c.	705	59	22·2	9·26	7·07
1·5	50	0·5	705	156	54·5	20	15·8
1·5	50	1·5	705	178	66·5	27·8	21·2
2	10	t.c.	705	6	2·4	1·67	1·57
2	10	0·667	705	12·8	6·15	3·2	2·81
2	10	2	705	18	7·2	5	4·7
2	20	t.c.	705	19·6	7·33	3·87	3·13
2	20	0·667	705	45	20	8·5	5·7
2	20	2	705	58	22	11·6	9·4
2	30	t.c.	705	38·6	14	7	5·4
2	30	0·667	705	101	36·5	16·2	11·25
2	30	2	705	115	42	21	16·2
2	50	t.c.	705	98·5	36·3	16	11·7
2	50	0·667	705	281	100	38	28
2	50	2	705	296	110	48	35

TABLE 26D—(*continued*).

Length of tube, l. Metres.	Evaporation, w, per 1 sq. m. and 1 hour. Litres.	Height of liquid outside tubs. Metres.	Vacuum. mm.	*Height* to which the liquid is projected from the tube, h_s. Bore of tube, mm. 30	50	80	100
				Height of splash, h_s. Metres.			
3	10	t.c.	705	13	—	—	—
3	10	1	705	27	—	—	—
3	10	3	705	39	—	—	—
3	20	t.c.	705	40	—	—	—
3	20	1	705	106	—	—	—
3	20	3	705	120	—	—	—
3	30	t.c.	705	86·7	—	—	—
3	30	1	705	225	—	—	—
3	30	3	705	260	—	—	—
3	50	t.c.	705	313	—	—	—
3	50	1	705	605	—	—	—
3	50	3	705	638	—	—	—

steam is evolved must be thrown up with the steam; it therefore increases the rising volume. It is hardly possible to state how much liquid is carried up with the steam; but occasionally it may be many times the volume of the steam.

The evaporative capacity of the heating surface at the steam entrance is much greater than the mean capacity, so that in vacuum evaporators with double bottoms and heating coils the liquid is often splashed up near the steam entrance to a height as great as in an evaporator heated by vertical tubes.

C. The Influence of the Current of Steam on Projected Drops.

In determining the height to which the larger masses of liquid are projected, we neglected the action of the rising current of steam, which can only be slight. The case is different with isolated drops. The motion of small drops may be very considerably affected by currents of steam.

The velocity, c, with which the drops are splashed out of the evaporating liquid, we shall assume to be equal to that of the larger masses, although the explosion of bursting bubbles, in combination

with the action of surface tension, may cause greater initial velocities in certain cases.

The initial upward velocity of the drops thrown up from the liquid can never be less than that of the current of steam rising in the steam space; it is always somewhat, and may be considerably, greater.

Cylindrical vessels, in which the liquid is heated by direct fire, double bottoms, coils or horizontal tubes, always provide so large a section for the escaping current of steam and the rising drops that their velocities invariably decrease and become not very different from one another. The ratio of the section to the heating surface varies in this case from 1 : 1 to 1 : 20 (see Table 27).

But in the case of heaters with vertical tubes, in which the ratio of the section, available for the escaping steam, to the heating surface is much less, *viz.*, 1 : 50 to 1 : 100, the initial velocities of the liquid are very high, occasionally greater than that of the current of steam. At the maximum they are perhaps twice as great.

The highest initial velocities are rarely produced, but when they do occur they must be carefully considered. Generally the velocity, c, even with apparatus with vertical tubes, will not exceed 4-6 m. per second. The velocity of the steam is in this case approximately 4-8 m. per second. Similarly, in apparatus with coils, double bottoms, etc., the velocities of the drops and steam are fairly equal.

For this reason, and because, when the velocities c and v_d are different, the effect is to cause the drops to rise to a less extent, we shall neglect the pressure, P_u, which *opposes* the ascent of the drops (for the *highest possible rise* is alone to be determined), and assume that no such pressure is present. Equation (130) may then be written:

$$h_s = \frac{c^2}{2g\left(1 - \frac{P_o}{2G}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

This equation shows that when the velocity of the current of steam is so great that it exerts a pressure, P_o, on a drop at rest equal to twice the weight of the drop, G, ($P_o = 2G$), the drop is carried away with the steam and lost, since the denominator of the fraction then becomes = 0.

If the pressure of the steam, P_o, upon the drop = G, *i.e.*, is equal to its weight, then equation (139) becomes

$$h_s = \frac{c^2}{2g}2.$$

TABLE 27.

Velocity of the steam in the steam space of vacuum evaporators, at vacua of 0-705 mm., with evaporative capacities of 10-100 kilos. per sq. m. and ratios of section of steam space to heating surface of $\frac{1}{1}$ to $\frac{1}{20}$.

Vacuum.	Evaporation in 1 hour per sq. m.	$\frac{\text{Section in sq. m.}}{\text{Heating surface in sq. m.}}$				
		$\frac{1}{1}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{20}$
		Velocity, in metres, of the current of steam in the steam space of the vacuum apparatus.				
mm.	w.					
0	10	0·046	0·23	0·46	0·69	0·92
0	20	0·09	0·46	0·92	1·38	1·83
0	30	0·14	0·69	1·38	1·76	2·75
0	50	0·23	1·15	2·30	3·44	4·59
0	100	0·46	2·29	4·59	5·88	9·78
234	10	0·06	0·32	0·65	0·97	1·30
234	20	0·13	0·65	1·30	1·95	2·60
234	30	0·19	0·97	1·95	2·92	3·90
234	50	0·32	1·62	2·92	4·87	6·50
234	100	0·65	3·25	4·87	9·75	13·00
405	10	0·09	0·47	9·75	1·41	1·58
405	20	0·19	0·94	1·41	2·82	3·76
405	30	0·28	1·41	2·82	4·23	5·64
405	50	0·47	2·35	4·23	7·05	9·40
405	100	0·94	4·70	7·05	4·10	18·80
610	10	0·21	1·05	4·10	3·16	4·22
610	20	0·42	2·11	3·16	6·33	8·44
610	30	0·63	3·16	6·33	9·49	12·66
610	50	1·05	5·27	11·05	15·80	21·10
610	100	2·10	10·50	21·11	31·60	42·20
705	10	0·54	2·70	5·41	8·11	10·82
705	20	1·08	5·4	10·82	16·2	21·64
705	30	1·62	8·1	16·23	24·3	32·46
705	50	2·70	13·5	27·05	40·5	54·1
705	100	5·41	27·0	54·1	81·1	108·1

The drops then rise to twice the height to which they would rise *in vacuo* without the current of steam, *i.e.*, to double the height given in Table 26.

If $P_o = \frac{1}{2}G$, then the rise is $\frac{4}{3}$ of the theoretical.

$$h_e = \frac{c^2}{2g\left(1 - \frac{G}{4G}\right)} = \frac{c^2}{2g} \cdot \frac{4}{3} \quad . \quad . \quad . \quad . \quad . \qquad (140)$$

If $P_o = \frac{1}{4}G$, then the rise is $\frac{8}{7}$ of the theoretical.

These considerations and an examination of Table 26 show that the current of steam in all cases somewhat increases the height to which large drops rise, but that quite small drops must often be carried completely out of the vacuum evaporator, even with steam velocities of 5-6 m. per second. It must also be remembered that each vessel is closed at the top and has an exit pipe, of smaller section than that of the apparatus and in which, therefore, the steam will move with a greater velocity than in the steam space of the apparatus. Since the currents converge towards this exit pipe, they gradually acquire a greater velocity in the apparatus itself.

The lower the pressure of the steam, the greater must be its velocity, if equal weights are to flow in equal times through pipes of equal bore. If a certain weight of steam, at atmospheric pressure, flows through a pipe of a certain bore with 1 m. velocity, then the velocities, in order that the same weight of steam may pass through the same pipe, must be

at	234	405	611	705	mm. vacuum
	1·415	2	4·62	11·84	m. per sec.

Thus it is seen, that the current of steam in vacuum evaporators will carry with it drops the more readily, the lower the pressure, the higher the vacuum in it.

The differences in construction of apparatus, in capacities, sections and liquids do not permit us to obtain a single result for the absolute height to which liquids and drops rise. But by means of Tables 26 and 27 this height may be estimated approximately in any separate case. It is certain that, in almost all cases, the small drops are in real danger of being carried away by the steam, and since they are generally formed from valuable liquids, endeavours are made to catch them again by artificial means.

D. The Action of the Current of Steam on Projected Bubbles of Liquid (Hollow Drops) and Means for Avoiding their Loss.

We have hitherto always assumed that *whole drops* of liquid, more or less large, have been splashed up; this is, however, not the case *alone*. Under certain conditions with every liquid, and with some liquids as a rule, *hollow* drops (bubbles of steam and liquid) are thrown up in every size and in great quantity. These bubbles are projected from the liquid with the same velocity, c, as the solid drops, but the ascending current of steam has more action upon them, since with equal section they present an equal surface to the pressure, but having less weight require a lower pressure to receive the same acceleration. When projected with the same velocity as a solid drop into a current of steam flowing in the same direction but with lower velocity, the hollow drops (bubbles) are more retarded by it than the solid drops and hence rise to a lower height. But when projected into a current of steam moving in the same directionwith greater velocity, the bubbles are carried considerably further than solid drops and may readily be removed from the apparatus and lost.

These steam bubbles, together with the very small drops of liquid, constitute the real source of loss in evaporating liquids.

In order to determine the heights to which these bubbles rise, equation (130) may be used:

$$h_s = \frac{c^2}{2g\left(1 - \frac{P_o + P_u}{2G}\right)},$$

inserting, instead of the weight of the solid drop, G, that of the bubble, which may be $\frac{1}{2}$, $\frac{1}{4}$, etc., of the former.

It may be seen from this equation how rapidly the height, h_s, must rise with decreasing weight of the drop, G. Thus a tall apparatus always offers some protection against loss by drops and even bubbles, but this protection is far from sufficient for the smaller solid drops and the lighter bubbles, which must be retained by other means.

Now these steam and foam bubbles may be retained by bringing them into a position where they are converted into solid drops, against which the current of steam is powerless. Then if the solid drops

formed from the burst bubbles be given a motion in a direction different to that of the steam, directed downwards and to the side towards a protected space, they can almost all be caught and saved. The froth separating apparatus of C. Heckmann of Berlin, German Patent No. 70,022, is constructed on these principles and hence works very efficiently. See Fig. 13 (p. 129).

In order that the steam bubbles may be converted into solid drops it is necessary to let them burst. This is accomplished in this case by passing the steam, which leaves the apparatus with the pressure prevailing therein, into a space in which there is a somewhat lower pressure. The excess of pressure thus produced in the interior of the bubbles causes them to burst.

The small difference of pressure required to rupture the bubbles differs for every liquid, every degree of concentration, and for every temperature, and it cannot be exactly estimated *à priori* for any case. Thus it is necessary to arrange this foam separator in such a manner that the difference of pressure necessary in each case can be actually produced under working conditions, and can be altered when the conditions alter.

This adjustability of the foam separator is practically its indispensable property. Similar arrangements without this property are worthless.

In Table 28 are given the diameters of the central tube and of the outer vessel of this foam separator. The central tube should offer as little resistance as possible to the passage of the steam; its diameter is determined by means of the later Table 32, and with regard to the steam velocities there given, since these velocities are so low that they create very little resistance even in long tubes. The inclination of the reflecting plate is taken as 10° to the horizon; the diameter of the drops to be retained is assumed to be 0·1 mm. or more. The section of the annular space between the reflecting plate and the wall of the vessel is so determined that the velocity of the steam, obtained at the highest anticipated vacuum, may exert a pressure upon drops of 0·1 mm. not exceeding twice their weight. Thus, according to Table 25, tenfold security is obtained, so that the apparatus must retain even considerably smaller drops. By increasing the angle of inclination of the reflecting plate and the diameter of the vessel the security against loss of drops is increased.

TABLE 28.

The foam separator of Ger. Pat. No. 70,022, Fig. 13 (p. 129), diameter of the central pipe and of the outer vessel.

Evaporation of water per hour.	Vacuum.							
	0		126·2		193·7		234	
	Diameter of the central pipe, *R*, and of the outer vessel, *M*.							
Kilos.	*R*	*M*	*R*	*M*	*R*	*M*	*R*	*M*
50	50	220	50	225	70	225	70	230
100	70	230	70	230	80	235	80	240
150	80	250	80	263	90	265	90	270
200	90	275	90	290	100	300	100	310
250	100	305	100	320	100	320	100	325
300	100	330	125	350	125	355	125	359
350	120	355	125	368	125	370	125	370
400	125	370	125	385	150	400	150	407
500	125	400	150	428	150	435	150	440
600	150	440	150	458	150	470	175	480
700	150	465	150	480	175	495	175	507
800	150	488	175	519	175	525	175	530
900	175	525	175	545	175	555	200	565
1000	175	540	200	580	200	585	200	590
1500	200	640	200	675	225	690	225	705
2000	225	730	225	777	250	795	250	810
2500	250	825	250	790	275	840	275	890
3000	275	895	275	940	300	955	300	970
3500	275	955	300	1010	300	1040	325	1070
4000	300	1015	325	1100	325	1115	350	1130
4500	325	1100	325	1155	350	1175	350	1190
5000	325	1165	350	1220	350	1235	375	1250
5500	350	1215	350	1270	350	1285	375	1300
6000	350	1245	375	1330	400	1350	400	1365
6500	350	1290	375	1370	400	1390	400	1410
7000	375	1340	400	1420	425	1440	425	1460
7500	375	1380	400	1460	425	1485	425	1510
8000	400	1430	425	1520	450	1535	450	1560

TABLE 28—(*continued*).

Evaporation of water per hour.	Vacuum.							
	375·6		471		564		610	
	Diameter of the central pipe, *R*, and of the outer vessel, *M*.							
Kilos.	*R*	*M*	*R*	*M*	*R*	*M*	*R*	*M*
50	80	235	90	240	100	245	100	250
100	90	260	100	265	125	300	125	310
150	100	295	100	300	125	330	150	370
200	125	335	125	340	150	375	175	405
250	125	360	150	385	150	385	175	440
300	125	380	150	405	175	442	200	480
350	150	420	150	415	200	480	200	506
400	150	435	175	435	200	500	225	545
500	175	485	175	495	225	555	225	590
600	175	510	200	540	225	588	250	645
700	200	555	225	575	250	640	275	687
800	200	585	225	610	250	675	300	730
900	225	627	250	665	275	718	300	765
1000	225	650	250	695	300	750	325	860
1500	250	780	300	820	350	920	350	980
2000	300	890	325	969	375	966	400	1120
2500	325	1010	350	1045	400	1140	450	1245
3000	350	1090	375	1140	425	1240	500	1355
3500	350	1160	400	1160	450	1330	525	1445
4000	375	1240	425	1215	500	1420	550	1550
4500	400	1320	450	1275	525	1500	575	1620
5000	400	1380	475	1460	550	1575	600	1710
5500	425	1440	500	1510	550	1640	625	1790
6000	450	1505	500	1570	575	1705	650	1865
6500	450	1565	500	1620	600	1780	650	1930
7000	475	1600	525	1690	600	1830	675	2000
7500	500	1655	550	1740	650	1905	700	2065
8000	500	1750	550	1795	650	1960	700	2130

TABLE 28—(*continued*).

Evaporation of water per hour.	Vacuum.					
	642·5		668		705	
	Diameter of the central pipe, *R*, and of the outer vessel, *M*.					
Kilos.	*R*	*M*.	*R*	*M*	*R*	*M*
50	100	273	125	290	145	325
100	125	315	150	345	175	390
150	150	373	175	405	200	450
200	175	440	200	455	225	510
250	200	468	225	508	250	575
300	225	508	225	530	275	605
350	225	532	250	588	300	650
400	225	558	250	605	325	725
500	250	630	275	645	350	790
600	250	660	300	710	375	850
700	250	697	325	790	400	910
800	300	757	350	845	425	965
900	325	830	375	885	450	1015
1000	350	880	400	940	450	1050
1500	400	1036	450	1105	500	1250
2000	450	1160	500	1255	600	1440
2500	500	1310	550	1390	650	1590
3000	550	1430	600	1510	700	1730
3500	575	1520	625	1615	750	1855
4000	600	1620	650	1720	800	1975
4500	625	1705	700	1820	850	2095
5000	650	1800	700	1870	850	2180
5500	675	1875	750	1960	900	2290
6000	700	1960	750	2060	900	2370
6500	700	2020	800	2150	—	—
7000	725	2090	800	2220	—	—
7500	750	2155	850	2300	—	—
8000	750	2222	850	2370	—	—

E. The Change in the Size of Steam Bubbles in Boiling Liquids.

The movement of a boiling liquid is facilitated by the increase in volume, as they rise, of the steam bubbles formed in the lower layers. The volume of a small weight of steam produced at the bottom of a liquid depends upon the pressure upon it. This pressure is the sum of the pressures of the liquid and of the steam or air above it.

The pressure of the liquid upon unit section of the bubbles is proportional to the height of the layer of liquid above the bubble, h, and its specific gravity, s.

As the bubble rises, the pressure of the steam or air generally remains constant, but the height, and thence the pressure, of the layer of liquid decreases gradually. The bubble therefore increases in volume as it rises.

Table 29 shows the extent of the increase in volume of steam bubbles, when they are formed in liquids at various depths and under various pressures, and then rise upwards.

TABLE 29.

The increase in volume of a steam bubble of 1 cc. capacity, which is formed, in liquids of 1·0, 1·1 and 1·3 specific gravity, at depths of 250-2000 mm. below the surface and then rises, whilst over the liquid there is a vacuum of 0-720 mm.

Depth below the surface at which the steam bubble of 1 cc. capacity was formed.	Vacuum over the liquid.																	
	0 mm.			150 mm.			250 mm.			500 mm.			650 mm.			720 mm.		
	Specific gravity of the liquid.																	
	1	1·1	1·3	1	1·1	1·3	1	1·1	1·3	1	1·1	1·3	1	1·1	1·3	1	1·1	1·3
mm.	Volume of the bubble when it reaches the surface.																	
250	1·03	1·13	1·33	1·03	1·13	1·34	1·04	1·14	1·35	1·08	1·18	1·4	1·18	1·29	1·53	1·5	1·65	1·95
500	1·05	1·16	1·36	1·06	1·17	1·37	1·07	1·17	1·39	1·15	1·26	1·49	1·34	1·47	1·74	1·95	2 14	2·54
750	1·08	1·18	1·40	1·10	1·20	1·42	1·11	1·22	1·44	1·23	1·35	1·6	1·53	1·68	1·99	2·45	2·69	3·19
1000	1·1	1·21	1·43	1·13	1·24	1·46	1·15	1·36	1·49	1·3	1·43	1·69	1·7	1·87	2·21	2·92	3·21	3·79
1500	1·15	1·27	1·50	1·19	1·3	1·55	1·25	1·37	1·62	1·44	1·58	1·87	2·05	2·25	2·66	3·88	4·26	5·04
2000	1·2	1·32	1·56	1·25	1·37	1·56	1·3	1·43	1·69	1·61	1·77	2·09	2·2	2·42	2·86	4·85	5·33	6·31

CHAPTER XVII.

THE DIAMETER OF PIPES FOR CONVEYING STEAM, ALCOHOL VAPOUR AND AIR.

A. For Steam.

The pipes, through which gases and vapours are conducted, are made of as small diameter as is possible without ill effects, since such pipes are cheaper, lighter and more convenient. Thus it is necessary to ascertain the least diameter which the pipes may be given in any particular case.

Generally it is required to convey the gases or vapours through the pipes with a very small fall in pressure between inlet and outlet; the permissible extent of this fall limits the dimensions of the pipes.

The loss in pressure, which vapours undergo in pipes, depends on their diameter and length, on the density of the vapour and, in particular, on the velocity with which the movement takes place.

Let d = the diameter of the pipe in metres,
l = the length „ „
Q = the section „ in sq. metres,
v_d and v_l = the velocities with which steam and air respectively move in the pipe, in metres per second,
z_d and z_l = the loss of pressure, in metres of water, which the air or steam respectively suffers between inlet and outlet,
γ_d and γ_l = the weight of 1 cub. m. of steam or air respectively, in kilos.

Two formulæ are known for determining the loss in pressure:—

1. The formula of Gustav Schmidt,

$$z_l = \frac{785l}{10^{10}d}\gamma\left(5 + \frac{1}{d}\right)v_l^2 \quad . \quad . \quad . \quad . \quad . \quad (141)$$

applicable to air and tubes of 150-200 mm. bore.

2. The formula of Gutermuth and Fischer, applicable to steam in tubes of 70-300 mm. bore and velocities below 20 m. per second:—

$$z_d = \frac{15 \times 10}{10^8} \gamma_d \frac{l}{d} v_d^2 . \quad . \quad . \quad . \quad . \quad . \quad (142)$$

or

$$z_d = \frac{0{\cdot}0015}{1000} \gamma_d \frac{l}{d} v_d^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (143)$$

Unfortunately these two formulæ do not give the same result for the same conditions; if that were the case, then, when l, d, γ and v were the same, z_i would equal z_d. However, if z_i be put equal to z_d, and the equation transformed, it will be seen that both the formulæ give the same result for a pipe of diameter $d = 0{\cdot}07$ m., and different results in all other cases.

$$\frac{785}{10^{10}}\left(5 + \frac{1}{d}\right) = \frac{15 \times 10}{10^8} = \frac{15}{10^7}$$

$$\frac{785}{10^3}\left(5 + \frac{1}{d}\right) = 15$$

$$\frac{785}{d} = 15 \times 10^3 - 785 \times 5$$

$$d = \frac{785}{15 \times 10^3 - 785 \times 5} = 0{\cdot}07 \text{ m.}$$

The results obtained by Schmidt's formula (Dingl. polyt. Journal, 1880, September) are always much lower than those given by Fischer's formula (Zeits. d. V. d. Ing., 1887, pp. 718, 749). On this account the second formula must be used by preference in doubtful cases, which conclusion is strengthened by the valuable researches conducted and described by Gutermuth and others, which have shown that the values obtained by Fischer's formula correspond very closely with the reality. The equation of Fischer and Gutermuth is found to be correct for pipes of 70 – 300 mm. diameter and velocities below 20 m. per second; but, in default of any other, this formula must for the present be used for pipes of other bores and for other velocities.

Table 30 has been calculated according to the formula (143) of Fischer, in order to obtain an idea of the extent of the resistance under various conditions. For the sake of comparison and to illustrate what has been said above in regard to the two formulæ, the results (which are not used) of Schmidt's equation are also inserted. In

Table 30, a length of pipe of 20 m. is assumed, and the resistance is measured in metres of the water column. It will be seen, what the formula also expresses, how rapidly the resistance increases with the velocity, and how considerably it increases under high pressure, *i.e.*, with steam and air of high densities.

The important question for practical purpose is: What diameter of pipe must be used for any definite case? This question will at once be answered. Since, however, not only the bore of pipes for steam, but also for alcohol vapour and air, is required, these substances will be treated at the same time.

Through a tube of given section in a given time much or little steam or air may be sent; the quantity depends on the velocity with which the substance moves through the tube. But a high velocity requires also a large difference in pressure between the inlet and outlet of the pipe. In many cases the pressure applied at the inlet of the pipe is required to be transmitted with as little loss as possible to the other end, in other cases it is undesirable that the pressure at the inlet should appreciably exceed the pressure at the outlet, the difference in pressure between the inlet and outlet being generally regarded as *loss* of pressure. On the other hand too low velocities require wide and costly pipes, therefore some difference of pressure is arbitrarily chosen and the bore of the pipes determined on this assumption.

The steam pressures used in practice vary within very wide limits —20 atmos. to 0·05 atmos. Thus a constant loss of pressure cannot well be assumed for all cases. It is desirable to assume the loss of pressure as a percentage of the original pressure. If at one end of a pipe there is an absolute pressure of 50 mm. (710 mm. vacuum), then a loss of pressure of 10 mm. of mercury at the other end is quite sensible; but if there is a pressure of 4,500 mm. (5 atmos.) at one end, then 20 – 50 mm. can well be spared for the transmission of the steam through the pipe.

Since it is thus decided to devote a certain percentage of the original pressure to the transmission of the steam through the pipe, and since, if this percentage is fixed, the formula (143) at once gives the velocity and thence the weight of steam passing through the pipe in unit time, the equation (143) may more conveniently be written:

$$v_d = \sqrt{\frac{1000 z_d d}{0{\cdot}0015 l \gamma_d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (144)$$

TABLE 30.

The loss of pressure, z_d, in metres of water, experienced by steam in
and 50 m., according to Schmidt (S)

Absolute pressure, atmos.		3		1·5		0·75	
Absolute pressure, mm.		2280		1140		566·7	
Vacuum, mm.		—		—		210	
Bore of pipe d.	Velocity, V_d.	S	F	S	F	S	F
0·05	20	0·5826	0·4086	—	—	—	—
	30	1·3110	0·9194	—	—	—	—
	50	3·6411	2·5540	—	—	—	—
0·07	20	0·2947	0·2918	0·1536	0·1521	—	—
	30	0·6632	0·6566	0·3456	0·3423	—	—
	50	1·8423	1·8240	0·9600	0·9510	—	—
0·150	20	0·0831	0·1319	0·0433	0·0709	0·0224	0·0368
	30	0·1871	0·3064	0·0975	0·1607	0·0548	0·0827
	50	0·5197	0·8542	0·2708	0·4437	0·1402	0·2297
0·300	20	0·0297	0·0681	0·0152	0·0355	0·0091	0·0184
	30	0·0669	0·1531	0·0348	0·0796	0·0180	0·0414
	50	0·1860	0·4256	0·0967	0·2218	0·0501	0·1149
0·500	20	—	—	—	—	0·0040	0·0111
	30	—	—	—	—	0·0091	0·0248
	50	—	—	—	—	0·0253	0·0689
0·700	20	—	—	—	—	—	—
	30	—	—	—	—	—	—
	50	—	—	—	—	—	—
0·900	20	—	—	—	—	—	—
	30	—	—	—	—	—	—
	50	—	—	—	—	—	—

The weight of steam, D, passing through the pipe in one hour is then

$$D = v_d \gamma_d \frac{d^2 \pi}{4} 3600 \quad . \quad . \quad . \quad . \quad . \quad . \quad (145)$$

whence the section of the pipe may be found.

TABLE 30.

pipes of 0·05 0·90 m. diameter and 20 m. long, at velocities of 20, 30 and Fischer and Gutermuth (F).

0·5 354·6 406		0·25 195·5 564·5		0·15 117·5 648		0·072 54·9 705	
S	F	S	F	S	F	S	F
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
0·0030	0·0239	0·0034	0·0135	0·0020	0·0084	—	—
0·0069	0·0537	0·0078	0·0304	0·0046	0·0189	—	—
0·0190	0·1493	0·0225	0·0845	0·0126	0·0526	—	—
0·0052	0·0118	0·0022	0·0068	0·0013	0·0043	0·0008	0·0020
0·0116	0·0260	0·0049	0·0152	0·0029	0·0095	0·0018	0·0046
0·0322	0·0739	0·0136	0·0423	0·0080	0·0263	0·0050	0·0128
0·0026	0·0071	0·0014	0·0041	0·0008	0·0025	0·0003	0·0012
0·0058	0·0159	0·0032	0·0091	0·0019	0·0057	0·0010	0·0028
0·0162	0·0444	0·0089	0·0253	0·0053	0·0158	0·0028	0·0077
—	—	—	—	—	—	0·0003	0·0012
—	—	—	—	—	—	0·0007	0·0019
—	—	—	—	—	—	0·0018	0·0055
—	—	—	—	—	—	0·0002	0·0068
—	—	—	—	—	—	0·0005	0·0015
—	—	—	—	—	—	0·0014	0·0043

For pipes of equal diameter, d, and equal length, l, the velocity of the steam alters only in proportion to the quotient $\sqrt{\frac{z_d}{\gamma_d}}$, for

$$v_d = \sqrt{\frac{1000d}{0{\cdot}0015l}}\sqrt{\frac{z_d}{\gamma_d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (146)$$

If the resistance, z_d, be expressed in percentages of the original pressure (in metres of water), it may be seen that $\frac{z_d}{\gamma}$ gives the same figure *exactly* for all pressures of air and *approximately* for all pressures of steam. The factor $\frac{z_d}{\gamma}$, then remains unaltered for any one particular gas or vapour. For in the case of air, which is generally used far from its point of liquefaction, the weight of 1 cub. m. is proportional to the pressure: 1 cub. m. at a double pressure has double the weight. But with saturated steam the alteration is only approximate: saturated steam of double the pressure has only *almost* double the weight. This approximation is not a very close one, but may be regarded as sufficient for the present purpose, as the following figures show:—

Steam pressure -	92	186	750	1490	2350 mm.
In the proportion -	1 :	2 :	8·15 :	16·2 :	25·54
Weight of 1 cub. m. of steam -	0·0822	0·162	0·600	1·13	1·735 kilos.
In the proportion -	1 :	2 :	7·3 :	13·74 :	21·1

Thus if it is once fixed how much per cent. of the available pressure is to be expended in producing the velocity of the steam, there is found (for equal lengths and with the above-mentioned inaccuracy) for a pipe of each diameter a steam velocity peculiar to it and practically the same for all pressures.

After we have obtained from Table 30 a view of the loss of pressure, which is to be expected with pipes of various diameters, and at different pressures and velocities, we then assume for Table 31 a permissible loss of 0·5 per cent. of the available pressure. The length of the pipe is taken at 20 m., and then, by means of equation (146), the resulting velocities are calculated. In Table 32 are next arranged the weights of steam at different pressures, which pass with these velocities through pipes of 20 – 900 mm. diameter in one hour.

Example.—Steam at atmospheric pressure (weight of 1 cub. m., $\gamma_d = 0·606$ kilo.) passes through a pipe of 0·1 m. diameter and 20 m. long. The loss in pressure is 0·5 per cent., *i.e.*, $z_d = \frac{0·5}{100} 10·3 = 0·0515$. The velocity is then

$$v_d = \sqrt{\frac{1000 \times 0·1}{0·0015 \times 20}} \sqrt{\frac{0·0515}{0·606}} = \sqrt{283} = 16·8 \text{ m. per second.}$$

The weight of steam, which passes through the pipe in one hour, is

$$D = 16·8 \times 0·605 . \frac{3·14 \times 0·1^2}{4} . 3600 = 288 \text{ kilos.}$$

TABLE 31.

Approximate velocity of steam in pipes of 0·025-0·9 m. diameter and 20 m. long, at absolute pressures of 4560-54·91 mm., for a 0·5 per cent. loss of pressure.

Absolute steam pressure	4560	1520	760	634	567	195	54·9 mm.
	Atmospheres.			Vacuum.			
	6	2	1	126	193	564	705 mm.
γ_d	3·26	1·16	0·606	0·511	0·461	0·244	0·0512 kilos.
$\frac{z_d}{\gamma_d}$	0·0908	0·0836	0·0815	0·0822	0·0801	0·0768	0·0697
Bore of the pipe, d.	Velocity of the steam in the pipe in m. per second.						
0·025	8·85	8·38	—	—	—	—	—
0·030	9·47	9·13	—	—	—	—	—
0·035	10·6	9·67	—	—	—	—	—
0·040	10·9	10·6	10·4	—	—	—	—
0·045	11·7	11·0	11·0	—	—	—	—
0·050	12·2	11·8	11·5	—	—	—	—
0·060	13·5	12·9	12·7	—	—	—	—
0·070	14·5	13·4	13·4	13·9	—	—	—
0·080	15·5	14·9	14·7	14·7	14·6	—	—
0·090	16·6	15·9	15·8	15·7	15·5	—	—
0·100	17·3	16·7	16·6	16·1	15·9	15·6	15·1
0·125	19·3	18·6	18·4	18·4	18·2	17·7	17·0
0·150	21·8	21·0	20·6	20·2	19·9	18·4	18·6
0·175	—	—	—	21·9	21·5	21·3	20·1
0·200	—	—	—	23·3	23·0	23·0	21·5
0·225	—	—	—	24·8	24·4	23·7	22·8
0·250	—	—	—	26·1	25·7	25·0	24·1
0·300	—	—	—	28·6	28·3	27·4	26·4
0·350	—	—	—	30·8	30·5	29·6	28·5
0·400	—	—	—	33·1	32·5	31·6	30·5
0·450	—	—	—	35·0	34·6	33·4	32·3
0·500	—	—	—	37·0	36·5	35·1	33·9
0·550	—	—	—	—	—	37·0	35·8
0·600	—	—	—	—	—	39·0	37·0
0·650	—	—	—	—	—	40·3	38·9
0·700	—	—	—	—	—	41·8	40·3
0·750	—	—	—	—	—	—	41·6
0·800	—	—	—	—	—	—	43·1
0·850	—	—	—	—	—	—	44·3
0·900	—	—	—	—	—	—	45·6

TABLE 32.

The weight of steam, *D*, in kilos., which passes in one hour through abs. to 705·09 mm. vacuum, with

Abs. pressure, atmos. / ,, mm. mercury / Vacuum, ,,		6 / 4560 / —	5 / 3800 / —	4 / 3040 / —	3 / 2280 / —	2 / 1520 / —	1·5 / 1140 / —	1 / 760 / —
Bore of the steam pipe, *d*. mm.	Velocity of the steam in the pipe, m. per sec. V_d	Weight of steam, *D*, in kilos., which passes						
25	8.5	50	42	34	26	18	—	—
30	9 0	75	63	51	39	27	—	—
35	9·5	107	90	73	55	38	—	—
40	10·5	155	130	106	81	55	42	—
45	11·0	205	173	140	107	73	56	38
50	11·5	265	223	181	138	95	72	49
60	13	431	363	294	224	153	117	80
70	14	633	533	432	330	225	172	117
80	14·5	855	720	684	446	305	232	159
90	15	1120	943	765	583	398	304	208
100	15·5	1430	1200	977	746	509	388	275
125	17	2590	2170	1760	1340	929	700	478
150	18·5	3810	3320	2610	1990	1360	1040	709
175	20	5670	4750	3850	2940	2020	1530	1050
200	21·5	—	6600	5350	4080	2830	2150	1470
225	23	—	—	7380	5630	3810	2910	1990
250	24	—	—	—	—	4920	3760	2560
300	26·5	—	—	—	—	—	6000	4090
350	28·5	—	—	—	—	—	8750	5980
400	30·5	—	—	—	—	—	—	8350
450	32·5	—	—	—	—	—	—	—
500	34	—	—	—	—	—	—	—
550	35·5	—	—	—	—	—	—	—
600	37·5	—	—	—	—	—	—	—
650	38·5	—	—	—	—	—	—	—
700	40·5	—	—	—	—	—	—	—
750	41·5	—	—	—	—	—	—	—
800	43	—	—	—	—	—	—	—
850	44·5	—	—	—	—	—	—	—
900	46	—	—	—	—	—	—	—

TABLE 32.

pipes of 25-900 mm. diameter and 20 m. long, at pressures of 6 atmos. 0·5 per cent. loss of pressure.

0·834	0·746	0·70	0·5	0·375	0·257	0·195	0·155	0·12	0·072
664	567	525	384	288	195	149	117	92·0	54·9
126	194	234	376	471	564	611	642	668	705

through the pipe in one hour, with 0·5 per cent. loss of pressure.

—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—
133	120	—	—	—	—	—	—	—	—
175	156	147	109	84	—	—	—	—	—
224	200	188	140	107	72	57	46	37	22·5
403	363	337	252	189	133	103	83	66	40
598	537	501	374	285	197	154	123	98	60
888	797	739	554	422	293	226	183	144	89
1240	1120	1040	777	594	411	318	255	202	124
1680	1510	1410	1050	802	555	431	345	274	161
2160	1946	1810	1350	1030	716	554	643	353	216
3450	3100	2890	2150	1650	1140	886	709	563	345
5030	4540	4230	3150	2410	1670	1290	1040	823	505
7050	6340	5910	4410	3370	2330	1690	1450	1070	706
9510	8550	7960	5930	4540	3140	2440	1950	1550	950
12300	11000	10300	7680	5870	4060	3150	2530	2000	1220
—	13900	13000	9680	7400	5140	3980	3190	2530	1550
—	—	—	12200	9320	6450	5000	4010	3180	1930
—	—	—	—	11100	7770	6030	4830	4000	2350
—	—	—	—	13100	9490	7350	5940	4680	2870
—	—	—	—	—	11100	9700	7400	5870	3600
—	—	—	—	—	—	10800	8180	6480	3980
—	—	—	—	—	—	11900	9550	7570	4650
—	—	—	—	—	—	13800	11100	8780	5390

TABLE 33.

The velocities of mixtures of alcohol and water vapours, in pipes of
loss of

Alcohol-water vapour.			Weight of 1 cub. m. of air at the temperature t_d.	Weight of 1 cub. m. of alcohol-water vapour at the temperature t_d.	Diameter,		
Alcohol, per cent. by weight.	Temperature. t_d	Density. γ_d	Kilos.	Kilos.	40	50	60
					Velocities,		
0	100	0·623	1·041	0·648	11·76	13·11	14·35
5	99·5	0·643	1·043	0·670	11·50	12·82	14·08
10	99	0·664	1·044	0·693	11·34	12·64	13·89
15	98·6	0·686	1·045	0·715	11·18	12·46	13·69
20	98·3	0·709	1·046	0·742	10·94	12·19	13·30
25	98	0·735	1·047	0·768	10·82	12·06	13·25
30	97·2	0·763	1·049	0·799	10·58	11·79	12·96
35	96·3	0·792	1·052	0·833	10·34	11·50	12·66
40	95	0·824	1·056	0·870	10·12	11·28	12·36
45	93·8	0·859	1·059	0·909	9·92	11·06	12·12
50	92·4	0·896	1·060	0·950	9·68	10·77	11·84
55	90·9	0·937	1·067	0·999	9·42	10·50	11·53
60	89·5	0·981	1·071	1·050	9·22	10·28	11·29
65	87·8	1·031	1·076	1·109	8·98	10·00	11·00
70	86·3	1·088	1·081	1·176	8·72	9·72	10·68
75	84·5	1·148	1·086	1·247	8·48	9·45	10·83
80	82·7	1·214	1·092	1·326	8·20	9·14	10·00
85	80·5	1·292	1·098	1·418	7·92	8·83	9·70
90	79	1·378	1·103	1·520	7·66	8·54	9·38
95	78·7	1·479	1·104	1·632	7·42	8·27	9·08
100	78·4	1·593	1·105	1·750	7·14	7·96	8·74

Pipes for steam of very low pressure (vacuum) are rarely longer than 20 m. Steam pipes for higher tensions are generally of much greater length. If the pipe is not 20 m. long, but has another length, l_a, the weight of steam, which passes through in one hour, is then found by multiplying the weight given in Table 32 by the factor

$$\sqrt{\frac{20}{l_a}} \qquad (147)$$

TABLE 33.

40-250 mm. bore and 3 m. long, at a pressure of 1·1 atmos. abs. and 0·1 per cent. pressure.

d, of the pipe in mm.									
70	80	90	100	125	150	175	200	225	250
v_d, of the alcohol-water vapour in m. per second.									
15·29	16·36	17·60	18·58	20·58	22·93	24·69	26·28	27·90	29·4
14·95	16·10	17·20	18·17	20·13	22·42	24·15	25·85	27·31	28·74
14·74	15·87	17·01	17·91	19·84	22·11	23·81	25·34	26·93	28·35
14·53	15·65	16·77	17·66	19·56	21·80	23·47	24·96	26·55	27·95
14·22	15·31	16·41	17·28	19·15	21·34	22·97	24·45	25·98	27·35
14·06	15·14	16·23	17·09	18·95	21·10	22·72	24·19	25·69	27·05
13·75	14·81	15·87	16·71	18·51	20·63	22·21	23·64	25·13	26·45
13·44	14·47	15·51	16·34	18·10	20·16	21·72	23·10	24·56	25·85
13·1	14·17	15·18	15·99	17·71	19·74	21·25	22·61	24·13	25·30
12·89	13·89	14·88	15·67	17·36	19·34	20·80	22·17	23·56	24·80
12·57	13·54	14·52	15·26	16·90	18·84	20·28	21·59	22·94	24·15
12·21	13·18	14·12	14·88	16·48	18·37	19·78	21·05	22·37	23·75
11·98	12·81	13·83	14·56	16·13	17·98	19·36	20·60	21·89	23·05
11·67	12·57	13·47	14·17	15·71	17·51	18·85	20·07	21·33	22·45
11·33	12·21	13·08	13·77	15·26	17	18·31	19·49	20·71	21·80
11·00	11·87	12·72	13·39	14·84	16·53	17·80	18·75	20·14	21·20
10·66	11·48	12·3	12·95	14·35	16	17·22	18·32	19·47	20·50
10·29	11·09	11·88	12·55	13·86	15·46	16·63	17·70	18·81	19·80
9·96	10·72	11·49	12·10	13·40	14·96	16·10	17·12	18·19	19·15
9·65	10·39	11·13	11·72	12·88	14·47	15·58	16·58	17·62	18·75
9·28	10·00	10·71	11·28	12·54	13·92	15	15·96	16·96	17·85

If some other loss of pressure, z_a (*not* 0·5 per cent.), is assumed in the pipe, then, in order to correct Table 32, the weight of steam there given must be multiplied by $\sqrt{\frac{z_a}{0\cdot5}}$, in which expression z_a is to be inserted as a percentage.

Example.—If there be 1 per cent. loss of pressure, $z_a=1$; if 5 per cent., $z_a=5$.

In order to obtain the weights of steam for the length, l_a, and the loss of pressure, z_a, the weights in Table 32 must be multiplied by

$$\sqrt{\frac{20}{l_a}\frac{z_a}{0.5}} = \sqrt{\frac{z_a}{l_a}40} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (148)$$

Since, in practice, the weight and the original pressure of the steam to be passed through a pipe in one hour are generally known, the necessary diameter of the pipe can be found in Table 32, 34 or 35 (for lengths of 20 m. and a loss of pressure of 0·5 per cent.). For other lengths and other losses of pressure equation (148) must be used.

B. For Mixtures of Alcohol and Water Vapours.

Table 34 gives the weights of mixtures of the vapours of alcohol and water, which can be conducted in one hour through pipes of different diameters without considerable loss of prossure. In calculating this table it was assumed that the same formulæ hold good for this mixture of vapours as for pure water vapour. But since such vapours are taken as a rule only through short connecting pipes between the different parts of rectifying and distilling apparatus, and since the pressure in such apparatus is always kept as low as possible, a pipe 3 m. long and a loss of pressure of 10 mm. of water ($z = 0·01$) were taken as the basis of the table.

In the apparatus mentioned the pressure is generally about 1·1 atmos. absolute, thus the value for p to be introduced into the calculation is 10,336 + 1033 = 11,369.

The alcohol-water vapours may have any desired composition, the mixtures vary from 1-99·8 per cent. of alcohol by weight. Each of these mixtures has a different specific gravity and boiling point, therefore it was necessary to determine for each the weight of 1 cub. m. at its temperature and at atmospheric pressure.

The temperatures of the various mixtures of vapour of alcohol and water at atmospheric pressure are known; their densities were taken from a paper published by the author. Thus the weight of 1 cub. m. of air at a pressure of 1·1 atmos. and at the temperature of each of the mixtures of vapour (calculated at intervals of 5 per cent.), multiplied by the density of the corresponding mixture of alcohol and water vapours, gives the true weight of 1 cub. m. of alcohol-water vapour at a pressure of 1·1 atmos. absolute.

By means of equation (144)

$$v_d = \sqrt{\frac{1000 z_d d}{0.0015 l \gamma_d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (149)$$

by inserting the values: $l = 3$, $z_d = 0.01$, $\gamma_d = 0.648$ to 1.75, $d = 0.04$ to 0.25, the corresponding velocities of these vapours in pipes of 40-250 mm. bore were found. The results of these calculations are arranged in Table 33.

From the velocities and the densities of the particular mixture of alcohol and water vapours (Table 33) were then readily obtained the weights which pass, at a pressure of 1·1 atmos. abs. and with a loss in pressure of $z_d = 0.01$ m, of water, through pipes 3 m. long of various bores. The results are given in Table 34.

C. For Air.

The loss of pressure of rarefied air in moderately long tubes has not, to the author's knowledge, been investigated. On the other hand, there have been the following researches on the loss of pressure of compressed air in long pipes :—

1. Chief Engineer H. Stockalper at the St. Gotthardt tunnel (1880), with pipes of 200 mm. bore and 4500 m. length, and 150 mm. bore and 542 m. length. Air pressure, 3·6-5·4 atmos. abs. Velocity, 4·7-11·3 m.

2. Prof. A. Devillez and Engineers Cornet and Mahiva at the Colliery Levant du Flénu (1881), with pipes of 125 mm. bore and 981 m. long, and 73 mm. bore and 172 m. long. Air pressure, 3·3-5·3 atmos. abs. Velocity, 2-12·2 m.

3. Profs. Gutermuth and Riedler at the compressed air installation in Paris (1890), with pipes of 300 mm. diameter and 16,502, 8759, 4403 and 3340 m. long. Air pressure, 6·2-8 atmos. abs. Velocity, 2·7-8·6 m.

4. Prof. H. Lorenz at the compressed air installation at Offenbach-on-Maine, on 17th January, 1892, with pipes of 100 mm. bore and 299 m. long. Air pressure, 6·7 atmos. abs. Velocity, 7·8-9·3 m.

Riedler and Gutermuth gave for the loss of pressure (z_l in kilos. per sq. cm.), as the result of their experiments,

$$z_l = \frac{533}{10^{10}} \gamma \frac{l}{d} v_l^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (150)$$

or

$$v_l = \sqrt{\frac{z_l \cdot 10^{10} \cdot d}{533 l \gamma}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (151)$$

TABLE 34.

The weight of mixtures of alcohol and water vapours, in kilos., which at 1·1 atmos. absolute pressure with 0·1 per

Alcohol vapour, per cent. by weight.	Diameter, d, of the pipe in mm.					
	40	50	60	70	80	90
	Weight in kilos. of the mixture of alcohol and					
0	34	57·7	93	134	191	258
5	35	58·3	94	137	194	261
10	35·3	59·6	96	139	197	267
15	36	60·5	97	141	201	272
20	36·5	61·4	101	145	204	276
25	37·3	62·9	102	148	209	282
30	38	63·9	103	151	213	288
35	39	65·2	105	153	217	293
40	40	66·6	108	156	222	300
45	40·5	68	110	161	227	307
50	41·4	69·5	113	163	231	311
55	42·4	71·4	115	167	237	320
60	43·6	73·4	119	173	242	330
65	44·8	75·4	122	177	250	339
70	45·5	77·5	126	181	257	357
75	47·6	80	130	188	266	359
80	48·7	82·7	133	192	273	368
85	50·5	86·1	138	198	282	378
90	52·4	88·8	143	207	292	396
95	54·5	92·2	148	215	304	410
100	56·52	94·8	154	223	317	425

For a loss of pressure of 0·5 per cent. in pipes 20 m. long, the permissible air velocities would be, according to this equation, in pipes of the

Bore	50	60	70	80	90	100	125 mm.
v_i	13·8	14·8	16	17·26	18·17	19·38	22·1 m. per sec.

TABLE 34.

passes in one hour through pipes of 40-250 mm. bore and 3 m. long, cent. loss in pressure (10 mm. of water).

Diameter, d, of the pipe in mm.						
100	125	150	175	200	225	250
water vapours which passes through the pipe in one hour.						
336	587	940	1385	2045	2674	3394
340	594	950	1393	2077	2680	3402
347	606	970	1429	2109	2688	3470
356	617	986	1449	2134	2714	3528
359	627	1000	1472	2145	2756	3585
367	643	1025	1510	2178	2817	3670
374	653	1043	1535	2184	2869	3733
378	666	1061	1564	2198	2922	3802
389	681	1081	1600	2223	2993	3889
399	693	1111	1636	2276	3060	3985
405	707	1186	1668	2317	3117	4052
417	727	1218	1714	2378	3199	4195
428	746	1251	1757	2444	3286	4275
440	767	1287	1809	2509	3381	4397
453	789	1326	1860	2576	3481	4505
467	816	1365	1913	2648	3583	4629
480	836	1400	1963	2721	3691	4770
498	868	1445	2030	2890	3813	4965
514	890	1509	2208	2940	3952	5141
524	924	1558	2230	3050	4111	5400
554	970	1697	2286	3173	4228	5550

Bore	150	175	200	225	250	300 mm.
v_l	24·1	26·19	27·25	28·61	30·29	33·31 m. per sec.

Professor H. Lorenz, who published a re-calculation of the older researches and of his own in the Zeits. d. V. d. I., 1892, pp. 621 and

TABLE 35.

The weight of air, L (at 15° C.), which passes in one hour through pipes of 40·350 mm. diameter and 20 m. long at vacua of 0·740 mm. and 0·5 per cent. loss of pressure.

Diameter of the pipe, d.	Velocity of the air in the pipe, v_l.	Absolute pressure of the air in mm.								
		1520	760	190	150	120	110	55	35	20
		Vacuum in mm.								
		—	0	570	610	640	650	705	725	740
		Weight of air, L, in kilos., which passes through the pipe in one hour.								
mm.	m.									
40	8·3	90	45	11·4	9·2	7·4	6·7	3·3	2·1	1·2
50	9·2	154	77	20	15·7	12·5	10 5	5·7	3·7	2·9
60	10·2	272	136	35	27·5	22	20	10	6·4	3·7
70	11·4	380	190	43	37·5	30	28	14	9	5·0
80	12·8	556	278	70	56·2	45	42	20	13	7·4
90	13·8	766	383	98	76·4	61	56	28	18	10·3
100	14·5	988	494	126	100	79	73	36	23	13
125	16·8	1786	893	228	180	143	132	66	42	24
150	19	2910	1455	380	293	233	213	106	68	40
175	21	4380	2190	570	440	351	322	160	102	60
200	23	6266	3133	798	625	500	462	230	147	84
250	26·6	10788	5394	1368	1080	864	802	400	252	144
300	30	18394	9197	2337	1840	1470	1350	674	430	246
350	33	27574	13772	3515	2750	2200	2090	1040	641	370

835, was led to the following empirical formula, which gives results in excellent agreement with *all* the experiments quoted:—

$$z_l = p_m \beta \frac{273}{T} l v_l^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (152)$$

whence

$$v_l = \sqrt{\frac{z_l T}{p_m \beta \,.\, 273 \,.\, l}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (153)$$

If z_l be expressed as a percentage, x, of p_m, then $z_l = \frac{x}{100} p_m$ and

$$v_l = \sqrt{\frac{\frac{x}{100} p_m T}{p_m \beta \,.\, 273 \,.\, l}} = \sqrt{\frac{xT}{100\beta \,.\, 273 \,.\, l}} \quad . \quad . \quad . \quad . \quad (154)$$

In this equation, if p_a denotes the absolute pressure at the beginning, p_e at the end, then $p_m = \frac{p_a + p_e}{2}$ = the mean absolute pressure; $z_l = p_a - p_e$ = the loss of pressure in kilos. per sq. m. T is the mean absolute temperature of the air; l the length of the pipe in m.; v_l the velocity of the air; d the diameter of the pipe in mm.; β is a factor dependent on the diameter of the pipe.

The values of β, according to Lorenz, calculated for pipes of various diameters, are :—

$$\beta^* = \frac{0.52}{d^{1.30933}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (155)$$

Diameter, d =	50	75	100	125	150
β =	0·003103	0·001824	0·001257	0·000934	0·000736
Diameter, d =	175	200	250	300	350
β =	0·000601	0·0005004	0·000377	0·000297	0·000243

Equation (154) gives, for the same loss of pressure, a somewhat lower velocity of the air as permissible than equation (151). In the want of decisive experiments we shall assume that equation (154) also holds good for air-pipes in which there is a considerably lower pressure than the atmospheric.

The results of the present chapter may be briefly, though somewhat inaccurately, expressed, for the most ordinary cases, as follows :—

The tubes for the evaporation of 100 kilos. of water per hour may be given the following sections :—

For the supply of heating steam at	3·00 atmos. abs.		2·5-3 sq. cm.
,, ,, ,,	1·25	,,	7-12 ,,
For exhaust steam at	1·00 atmos. abs.	- -	6-12 ,,
,, ,,	125 mm. vacuum	- -	8-16 ,,
,, ,,	250 ,,	- -	10-20 ,,
,, ,,	700 ,,	- -	60-100 ,,
For exhausted air at	700 ,,	- -	1-4 ,,

* The values of β given by this formula agree with those given by Prof. Lorenz in the article referred to at the bottom of p. 175, but will not give the velocities tabulated in Table 35. The tabulated values appear to be correct so that β in equation (154) should be taken as $\frac{1}{1000}$ of the values given above [Reviser].

CHAPTER XVIII.

THE DIAMETER OF WATER PIPES.

THE quantity of water, which can flow in a definite time through a system of pipes, depends upon the pressure which produces the movement and on the hindrances (bends, branches, constrictions, roughnesses of wall) which obstruct the flow in the pipe.

It may be assumed that (apart from pumps, pressure and suction pipes, which are not considered here) the pressure, which causes the motion of the water, is provided either *alone* by a water-vessel placed at a high level, in which case the pressure may be that of a column of water 0·5-15 m. high, or *alone* by a vacuum condenser, in which case the pressure is equal to the vacuum measured in metres of water *minus* the height from the point at which the water enters the condenser to the water level. Since the vacuum in the condenser is always lower than the theoretical, the pressure just mentioned (even assuming that the water level is at the height at which the water enters the condenser) is at most 10 m. in practice.

Finally, the pressure causing the flow of water may be due to a water vessel at a high level *and* to the vacuum in the condenser. In this case the maximum pressure of $10 + 15 = 25$ m. is rarely exceeded.

We shall now determine the quantities of water which can flow in one hour through pipes of various diameters with heads of 0·5-25 m. of water. It is necessary to calculate in each case the actual velocity, v_w, with which the water moves.

Let v_w = the velocity of the water in m. per second.
h_w = the total available pressure in m. of water.

Then the velocity theoretically produced at the end of the pipe is

$$v_w = \sqrt{2gh_w} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (156)$$

or

$$h_w = \frac{v_w^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (157)$$

This theoretical velocity is never attained, since in every system of pipes there are several conditions (resistances) which retard the flow of the water. We may assume that of the total available head or pressure of water, h_w, portions, h_1, h_2, h_3, etc., must be used to overcome each of these resistances. These heads are therefore known as "heads of resistance". Each of these pressures, h_1, h_2, h_3, would (if there were no resistance to overcome) impart to the water a corresponding velocity, v_1, v_2, v_3, so that, if v_w be the velocity actually attained and h the head of water theoretically necessary to produce this velocity, the total available pressure, $h_w = h + h_1 + h_2 + h_3 + \ldots$, would produce the velocity, $v_w + v_1 + v_2 + v_3 + \ldots$, *i.e.*,

$$h_w = h + h_1 + h_2 + h_3 = \frac{v_w^2}{2g} + \frac{v_1^2}{2g} + \frac{v_2^2}{2g} + \frac{v_3^2}{2g} \quad . \quad . \quad (158)$$

Now h_1, h_2, h_3 may be written as fractions of the height, h, then

$$h_w = h + \zeta_1 h + \zeta_2 h + \zeta_3 h \quad . \quad . \quad . \quad . \quad . \quad (159)$$

in which h is the head theoretically necessary to produce the actually attained velocity, v_w.

ζ_1, ζ_2, ζ_3 are known as the coefficients of resistance.

Since $h = \frac{v^2}{2g}$, therefore

$$h_w = \frac{v_w^2}{2g} + \zeta_1 \frac{v_w^2}{2g} + \zeta_2 \frac{v_w^2}{2g} + \zeta_3 \frac{v_w^2}{2g} \quad . \quad . \quad . \quad . \quad (160)$$

or

$$h_w = \frac{v_w^2}{2g}(1 + \zeta_1 + \zeta_2 + \zeta_3) \quad . \quad . \quad . \quad . \quad . \quad . \quad (161)$$

Hence the real velocity of water in pipes is

$$v_w = \frac{\sqrt{2gh_w}}{\sqrt{1 + \zeta_1 + \zeta_2 + \zeta_3}} \quad . \quad . \quad . \quad . \quad . \quad (162)$$

The coefficients of resistance are estimated as parts of the height, h :—

$\zeta_1 = 0{\cdot}505$ is the coefficient of resistance for the entry of water from the tank into the pipe. It ranges from 0·08-0·505. If the mouth of the pipe be rounded and made conical, ζ_1 is small, but for safety it will be taken as 0·505.

$\zeta_2 = 0{\cdot}805$ is the coefficient for bends. For right-angled elbows, the radius of the bend of which, $r = 3d$ (d = diameter of the pipe), ζ_2 may be put 0·161. In the following Table 36, five bends are assumed for each pipe, thus $\zeta_2 = 5 \times 0{\cdot}161 = 0{\cdot}805$.

$\zeta_3 = 0{\cdot}6$ denotes the resistance of a tap or valve. If these are almost completely open, ζ_3 may be put 0·6, but as soon as the taps or valves are more or less closed the coefficient of resistance increases enormously.

$\zeta_4 = 1$ is the resistance which arises through the entry of water into a vessel. If the section of the pipe be Q, and that of the vessel Q_1, then the velocity, v, in the pipe becomes $v\frac{Q}{Q_1}$ in the vessel. The resistance head is therefore

$$h_4 = \frac{\left(v - v\frac{Q}{Q_1}\right)^2}{2g} = \left(1 - \frac{Q}{Q_1}\right)^2 \frac{v^2}{2g} \quad . \quad . \quad . \quad . \quad (163)$$

But $h = \frac{v^2}{2g}$ and $h_4 = \zeta_4 h$, therefore

$$\left(1 - \frac{Q}{Q_1}\right)^2 = \zeta_4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (164)$$

If Q_1 be very great in proportion to Q, as is almost always the case, the fraction $\frac{Q}{Q_1}$ becomes very small and $\left(1 - \frac{Q}{Q_1}\right)^2$ differs but little from unity. Thus we shall assume that $\zeta_4 = 1$.

$\zeta_5 = \lambda\frac{l}{d}$ = the coefficient for the friction in the pipe. λ is found by Darcy's formula:

$$\lambda = 0{\cdot}01989 + \frac{0{\cdot}0005078}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (165)$$

This coefficient must be separately found for every diameter and every length of pipe. In the following small table are given the values of λ for diameters from 0·020 to 0·450 m.

According to equation (165):—

For $d=$	20	25	30	35	40	45 mm.
$\lambda=$	0·04528	0·04019	0·03682	0·03439	0·03259	0·03120
For $d=$	50	60	70	80	90	100 mm.
$\lambda=$	0·03004	0·02838	0·02718	0·02624	0·02553	0·02497
For $d=$	125	150	175	200	225	250 mm.
$\lambda=$	0·02394	0·02327	0·02279	0·02231	0·02214	0·02192
For $d=$	300	350	400	450 mm.		
$\lambda=$	0·02155	0·02135	0·02115	0·02101		

On the assumptions made above, the equation for calculating the velocity of water in cylindrical pipes is

$$v_w = \frac{\sqrt{2gh_w}}{\sqrt{1+\zeta_1+\zeta_2+\zeta_3+\zeta_4+\lambda\frac{l}{d}}} \quad . \quad . \quad . \quad . \quad (166)$$

$$v_w = \frac{\sqrt{2gh_w}}{\sqrt{1+0{\cdot}505+5\times0{\cdot}161+0{\cdot}6+1+\lambda\frac{l}{d}}} = \frac{\sqrt{2gh_w}}{\sqrt{3{\cdot}91+\lambda\frac{l}{d}}} \quad (167)$$

This equation has been employed in calculating Table 36, from it was found the velocity, v_w, of water in pipes of 30-225 mm. diameter, for heads of h_w = 0·5-25 m., and lengths of pipe of l = 10-100 m. The quantities of water, W, flowing through the pipe in one hour were then calculated from the velocities.

Since the figures of Table 36 always give the greatest quantity of water flowing through the pipe under the conditions assumed, it is necessary for practical use to add to the diameter of the pipe or to subtract from the quantity of water thus determined, especially in view of the possible occurrence in the pipe of a larger number of bends, branches, alterations of section and valves, and increased roughness of the inner surface.

TABLE 36.

The quantity of water, W, in cub. m., which flows in 1 hour through under heads of water of 0·5-25 m.

Head of water, h_w. m.	Length of pipe, l. m.	Bore of pipe in mm. 30	35	40	45	50	60
		Quantity of water, W, in cub. m. per hour.					
0·5	10	2·0	2·9	4·1	5·5	6·9	10·9
	20	1·5	2·2	3·1	4·2	5·5	8·7
	40	1·4	1·7	2·3	3·2	4·2	6·5
	60	0·9	1·3	1·8	2·6	3·5	5·6
	80	0·8	1·2	1·6	2·3	2·9	4·9
	100	0·7	1·1	1·5	2·1	2·7	4·4
1·0	10	2·8	4·1	5·8	7·8	9·8	15·3
	20	2·2	3·1	4·4	6·0	7·8	12·3
	40	1·6	2·4	3·3	4·5	5·8	9·2
	60	1·3	1·9	2·6	3·7	4·9	7·9
	80	1·2	1·7	2·4	3·1	4·1	7·1
	100	0·9	1·6	2·2	3·0	3·9	6·2
2·0	10	4·3	5·8	8·1	11·0	13·8	21·8
	20	3·1	4·4	6·3	8·5	11·1	17·4
	40	2·3	3·3	4·7	6·3	8·3	13·1
	60	1·8	2·7	3·7	5·3	7·0	11·3
	80	1·6	2·3	3·4	4·6	5·9	10·0
	100	1·5	2·2	3·1	4·2	5·5	8·9
3·0	10	5·0	7·1	9·8	13·5	16·0	26·6
	20	3·8	5·5	7·7	10·4	12·8	21·3
	40	2·8	4·1	5·7	7·8	9·6	16·0
	60	2·2	3·3	4·6	6·5	8·0	13·8
	80	1·9	2·9	4·1	5·6	6·9	12·3
	100	1·6	2·7	3·8	5·2	6·4	10·8
4·0	10	5·7	8·2	11·2	15·6	19·5	30·8
	20	4·3	6·3	8·7	12·0	15·6	24·6
	40	3·2	4·7	6·5	9·0	11·7	18·4
	60	2·6	3·8	5·2	8·0	9·8	16·0
	80	2·2	3·4	4·7	6·6	8·9	14·3
	100	2·1	3·1	4·3	6·0	7·8	12·3

TABLE 36.

pipes of 30-225 mm. diameter and 10, 20, 40, 60, 80, 100 m. long. (5 elbows and 1 valve assumed).

Bore of pipe in mm.								
70	80	90	100	125	150	175	200	225
Quantity of water, *W*, in cub. m. per hour.								
15·7	21·0	27·9	35·7	57·9	84·8	117·1	156·7	203·1
12·6	17·5	23·2	29·6	49·7	75·0	106·4	142·4	184·6
9·7	13·5	18·6	21·7	39·7	60·0	85·7	113·9	147·7
8·3	11·5	15·3	20·7	34·8	55·1	81·9	109·6	142·1
7·3	10·5	13·9	18·6	31·3	49·5	74·5	99·7	129·2
6·5	9·6	12·8	16·3	29·8	45·0	70·2	95·1	121·7
22·3	31·0	39·5	49·1	31·4	120·0	165·7	220·6	288·1
17·8	25·8	32·9	41·8	70·2	106·2	150·6	202·3	261·9
13·7	19·9	26·3	33·3	56·1	84·9	120·5	161·9	209·5
10·7	16·0	21·7	29·2	49·1	78·0	115·9	155·8	201·6
9·4	15·5	19·7	26·3	44·2	70·1	105·4	141·6	183·3
9·4	14·2	18·[illegible]	23·0	42·1	64·3	99·8	133·5	172·8
31·6	42·1	49·7	69·4	115·7	170·4	234·2	315·9	406·6
25·3	35·1	41·4	59·3	99·8	150·7	212·9	287·2	369·7
19·4	27·1	33·1	47·4	79·8	120·5	170·3	229·7	295·7
16·7	23·2	27·3	41·5	69·8	110·8	162·8	221·1	284·6
14·6	21·0	24·8	37·3	62·8	99·4	149·0	201·0	258·7
12·9	19·3	22·8	32·6	59·8	90·4	140·5	189·5	244·0
39·2	52·1	68·4	85·9	141·4	209·1	287·6	386·8	504·8
31·4	43·0	57·0	72·9	121·9	185·1	261·4	351·6	458·0
24·2	33·2	45·6	58·3	97·5	148·0	209·1	281·3	364·4
20·7	28·4	37·6	51·0	85·0	136·0	201·3	270·7	352·6
18·2	25·8	34·2	45·9	76·8	122·1	188·0	248·1	319·6
16·5	23·6	31·6	40·0	73·1	111·0	172·6	232·0	302·2
44·6	45·0	78·8	98·1	163·9	243·5	333·3	447·7	580·9
35·7	37·5	65·7	83·9	141·3	215·6	303·0	407·0	528·1
27·5	28·9	52·5	67·1	113·0	172·5	242·4	325·6	422·5
23·5	24·7	43·3	58·7	98·9	158·4	233·3	313·4	406·6
21·4	22·5	39·4	52·8	89·0	141·2	212·1	284·9	369·6
19·6	20·5	36·1	46·1	84·8	129·3	199·8	256·2	332·6

TABLE 36—(*continued*).

Head of water, h_w. m.	Length of pipe, l. m.	Bore of pipe in mm. 30	35	40	45	50	60
		Quantity of water, W, in cub. m. per hour.					
5·0	10	6·3	8·6	12·9	17·5	22·8	34·0
	20	4·9	6·6	9·9	13·4	17·5	26·1
	40	3·6	4·9	7·4	10·1	13·1	19·6
	60	2·9	3·9	5·9	8·5	11·0	16·7
	80	2·5	3·6	5·4	7·4	9·0	14·9
	100	2·3	3·2	4·9	6·7	8·7	13·1
6·0	10	7·9	10·0	14·2	19·1	25·0	36·0
	20	5·3	7·7	10·9	14·7	19·2	27·7
	40	4·0	5·8	8·1	11·0	14·4	20·7
	60	3·2	4·6	6·5	9·2	12·1	18·0
	80	2·7	4·2	6·0	8·1	10·9	15·7
	100	2·5	3·8	5·4	7·3	9·6	13·7
7·0	10	7·7	10·8	15·3	20·6	27·0	40·2
	20	5·7	8·3	11·8	15·9	20·8	30·9
	40	4·3	6·2	8·8	11·9	15·6	23·2
	60	3·4	5·2	7·1	10·0	13·1	20·1
	80	3·0	4·6	6·5	8·7	11·8	17·6
	100	2·7	4·1	5·9	7·9	10·4	15·4
8·0	10	8·1	11·6	16·3	22·1	28·8	44·9
	20	6·1	8·9	12·6	17·0	22·2	34·5
	40	4·6	6·7	9·4	12·7	16·6	25·9
	60	3·7	5·3	7·5	10·7	14·0	21·5
	80	3·2	4·9	6·9	9·3	12·6	19·7
	100	2·9	4·4	6·3	8·5	11·1	17·2
9·0	10	8·5	12·4	17·4	23·7	32·3	47·7
	20	6·5	9·5	13·4	18·2	24·8	36·7
	40	4·9	7·1	10·0	13·6	18·6	27·5
	60	3·9	5·7	8·0	11·4	15·7	23·8
	80	3·4	4·9	7·3	10·0	14·1	21·2
	100	3·6	4·5	6·7	9·1	12·4	18·7

TABLE 36—(*continued*).

Bore of pipe in mm.								
70	80	90	100	125	150	175	200	225
Quantity of water, *W*, in cub. m. per hour.								
50·0	66·6	87·9	110·1	183·4	272·4	371·4	499·7	645·5
40·0	55·5	73·2	94·1	158·1	241·0	337·6	454·2	586·8
30·8	42·7	58·6	75·2	126·5	192·8	270·1	363·4	469·4
26·4	36·6	48·3	65·8	110·6	177·1	259·7	338·7	451·8
23·2	33·3	43·9	59·2	99·6	159·1	236·3	317·9	410·7
21·0	30·5	40·3	51·7	94·8	144·6	222·8	299·8	387·3
53·1	73·5	98·5	120·6	202·7	294·7	408·5	549·6	708·4
42·4	61·3	81·2	103·1	172·7	260·8	371·4	499·7	644·0
32·7	47·2	65·0	82·5	138·1	208·6	297·1	399·7	515·2
28·0	40·4	62·4	72·4	120·8	191·5	301·3	384·7	495·9
24·6	38·7	48·7	64·9	108·8	172·1	259·9	349·7	450·8
22·2	36·7	47·8	60·9	103·6	156·4	245·1	329·8	424·0
48·4	80·1	104·4	129·6	215·9	316·9	439·0	602·0	763·5
46·7	66·7	87·0	110·7	185·5	280·5	399·1	538·2	694·1
35·9	51·4	71·6	88·7	148·4	224·4	319·3	430·5	635·3
30·8	44·0	57·4	77·6	129·8	206·1	305·1	314·4	534·5
27·1	40·0	53·6	69·8	116·8	185·1	279·4	376·7	485·9
27·9	36·7	47·8	60·9	111·3	168·3	250·5	355·2	458·1
65·0	84·6	112·6	138·8	232·7	339·5	470·4	628·1	818·7
52·0	70·5	93·8	118·6	199·2	302·1	427·7	571·0	744·2
40·0	54·3	75·1	95·1	159·4	241·7	342·1	456·8	595·4
34·3	46·5	61·9	83·0	139·4	222·1	329·3	439·6	573·0
27·7	42·3	56·3	74·7	125·4	195·5	299·3	399·7	520·9
27·3	38·8	52·7	65·2	119·5	183·7	281·6	376·6	490·0
67·0	90·9	117·9	145·7	245·9	362·2	497·1	670·3	865·9
53·6	75·7	98·3	124·6	212·0	320·5	451·9	609·4	787·2
41·2	58·5	78·6	99·7	169·6	256·4	371·5	487·5	629·7
34·7	50·5	64·8	87·2	148·4	235·6	347·9	469·2	606·1
32·1	45·4	57·9	78·5	133·5	211·5	316·3	426·6	551·0
29·4	41·6	54·0	74·7	127·2	192·3	298·2	402·2	519·5

TABLE 36—(*continued*).

Head of water, h_w. m.	Length of pipe, l. m.	Bore of pipe in mm. 30	35	40	45	50	60
		Quantity of water, W, in cub. m. per hour.					
10·0	10	8·9	13·0	18·3	25·1	31·6	48·5
	20	6·9	10·0	14·1	19·3	25·3	38·8
	40	5·1	7·5	10·6	14·5	19·0	29·1
	60	4·1	6·0	8·4	12·2	16·0	25·2
	80	3·6	5·5	7·7	10·6	14·4	22·5
	100	3·0	5·0	7·0	9·6	12·6	19·8
11·0	10	9·4	13·6	19·3	26·0	32·6	51·1
	20	7·2	10·5	14·9	20·0	26·1	40·8
	40	5·4	7·8	11·1	15·0	24·4	38·3
	60	4·3	6·3	8·9	12·6	16·5	26·5
	80	3·8	5·8	8·1	11·0	14·8	23·7
	100	3·5	5·2	7·4	10·0	13·0	20·8
12·0	10	10·0	14·3	19·5	27·3	33·6	53·3
	20	7·5	11·0	15·0	21·1	26·8	42·7
	40	5·6	8·3	11·3	15·8	20·1	32·6
	60	4·3	6·6	9·0	13·2	17·0	27·7
	80	3·9	6·0	8·1	11·6	15·3	24·7
	100	3·7	5·4	7·4	10·5	13·4	21·7
13·0	10	10·2	14·8	20·8	28·2	35·3	55·8
	20	7·8	11·4	16·0	21·7	28·3	44·6
	40	5·9	8·5	12·0	16·3	21·2	33·4
	60	4·7	6·8	9·6	13·6	17·9	29·0
	80	4·2	6·2	8·8	11·9	16·0	25·8
	100	3·8	5·6	8·0	10·8	14·0	22·7
14·0	10	10·6	15·2	20·7	29·2	38·4	59·4
	20	8·2	11·7	16·7	22·4	29·5	45·7
	40	6·1	8·8	12·5	16·8	22·1	34·3
	60	4·9	7·0	10·0	13·5	18·0	27·9
	80	4·4	6·4	9·1	12·3	16·2	26·0
	100	4·0	5·8	8·3	11·2	14·7	22·7

TABLE 36—(*continued*).

Bore of pipe in mm.								
70	80	90	100	125	150	175	200	225
Quantity of water, W, in cub. m. per hour.								
71·4	93·7	120·9	154·4	258·7	391·8	524·7	707·7	913·1
56·3	78·1	103·3	133·1	223·0	337·7	477·0	643·3	830·1
43·4	60·2	82·6	106·4	178·4	270·1	381·6	514·6	730·5
37·2	51·5	68·1	93·1	156·1	249·1	345·3	495·3	639·2
32·6	46·8	61·9	83·8	140·5	222·9	333·9	450·3	581·1
28·2	42·9	56·8	73·2	133·8	202·6	314·8	424·6	547·8
74·3	98·1	130·5	163·0	269·2	391·8	525·9	700·4	954·1
59·4	81·7	108·8	139·6	234·1	355·5	478·1	672·7	867·3
45·7	63·0	87·0	119·6	187·2	284·4	382·5	538·2	693·8
37·2	53·9	71·8	97·7	163·8	261·3	368·1	414·4	667·8
34·4	49·0	65·2	87·8	147·4	234·6	334·7	370·9	607·1
29·8	44·9	59·8	76·7	140·4	213·3	315·5	355·1	572·4
75·6	102·0	136·0	171·2	286·3	416·8	586·1	771·1	1006·0
60·5	85·0	115·3	145·5	245·1	368·9	523·8	701·0	914·6
46·5	66·4	90·6	116·4	216·1	295·1	419·0	560·8	731·6
39·9	56·1	74·8	101·8	171·5	271·1	403·3	539·7	704·2
35·0	51·0	68·0	91·6	154·4	243·4	366·6	490·7	640·2
30·3	46·7	62·3	80·0	147·0	221·3	345·7	462·6	603·6
80·7	107·4	142·8	176·8	293·6	434·8	599·9	807·2	1039·1
64·6	89·5	119·0	151·1	253·9	384·8	545·4	733·8	944·6
49·7	75·9	95·2	120·9	203·1	307·8	436·3	587·1	755·7
31·6	59·0	78·5	105·8	177·7	284·1	419·9	565·1	727·3
37·4	53·7	71·4	95·2	160·0	253·9	381·8	513·6	661·2
32·5	49·2	65·4	83·1	152·3	230·9	359·9	484·3	623·4
83·3	111·7	148·1	183·5	304·8	452·1	619·0	839·5	1078·4
66·6	93·8	123·4	156·8	262·8	400·1	562·7	763·2	980·4
51·3	71·8	98·7	125·4	214·2	320·0	450·2	610·5	784·3
43·9	61·4	81·4	111·7	183·4	294·0	425·6	587·6	754·9
38·6	55·9	74·0	98·8	195·5	263·0	393·9	534·2	686·3
34·9	51·2	67·8	86·2	157·6	240·0	371·4	510·0	647·0

TABLE 36—(*continued*).

Head of water, h_w.	Length of pipe, l.	Bore of pipe in mm.					
		30	35	40	45	50	60
		Quantity of water, W, in cub. m. per hour.					
m	m.						
15·0	10	10·9	15·7	22·3	30·4	39·6	62·1
	20	8·4	12·1	17·1	23·4	30·4	47·7
	40	6·3	9·0	12·9	17·5	22·8	35·8
	60	5·0	7·2	10·4	14·2	18·3	29·2
	80	4·6	6·6	9·3	12·8	16·7	26·2
	100	4·1	6·0	8·5	11·7	15·2	23·9
16·0	10	11·3	16·4	23·3	31·2	41·2	64·1
	20	8·7	12·6	17·9	24·0	31·6	49·3
	40	6·5	9·4	13·4	18·0	23·7	36·9
	60	5·2	7·6	10·8	14·5	19·1	30·0
	80	4·7	6·9	9·7	13·2	17·4	27·1
	100	4·3	6·2	8·9	12·0	15·8	24·7
18·0	10	12·0	17·5	24·6	33·0	42·2	68·0
	20	9·2	13·4	18·9	25·4	32·4	52·3
	40	6·9	10·0	14·2	19·0	24·3	39·2
	60	5·5	8·0	11·4	15·4	26·1	31·8
	80	4·9	7·2	10·2	14·0	17·8	28·8
	100	4·5	6·6	9·3	12·7	16·2	26·2
20·0	10	12·7	18·4	25·9	35·1	45·4	72·0
	20	9·8	14·1	19·9	27·0	34·9	55·4
	40	7·3	10·6	14·9	20·2	26·2	41·5
	60	5·8	8·5	12·0	16·3	18·0	33·6
25·0	10	14·3	20·5	29·0	37·7	48·9	77·4
	20	11·0	15·9	22·3	29·0	39·1	61·9
	40	7·2	11·9	16·7	21·7	27·0	46·4
	60	6·6	9·5	13·4	17·9	24·7	40·2
	80	6·0	8·6	12·1	15·9	21·6	31·1
	100	5·4	7·9	11·0	14·5	19·5	30·9

TABLE 36—(*continued*).

Bore of pipe in mm.								
70	80	90	100	125	150	175	200	225
Quantity of water, *W*, in cub. m. per hour.								
86·7	114·4	153·6	190·9	319·4	467·2	642·8	864·4	1117·8
69·4	96·2	128·0	163·1	273·0	413·4	584·4	785·8	1016·2
53·4	74·1	102·4	130·5	218·4	330·7	467·5	618·6	812·9
45·8	63·5	84·4	114·2	191·1	303·8	457·6	605·0	782·4
40·2	57·7	76·8	102·7	171·9	272·0	409·0	550·0	711·3
36·5	52·9	70·4	89·7	163·8	248·0	385·7	518·6	670·7
91·0	119·0	161·4	196·7	327·4	485·1	661·7	888·0	1149·3
72·8	99·4	134·5	168·1	282·2	429·3	601·7	807·3	1044·8
56·1	76·6	107·6	134·5	225·7	343·4	481·3	645·7	835·8
48·0	65·6	88·7	117·7	197·5	315·5	463·3	621·6	804·5
42·2	59·6	80·7	105·9	177·8	282·6	423·3	565·1	731·3
38·3	54·7	73·9	92·4	169·3	257·6	397·1	532·8	689·7
94·5	127·6	172·8	208·3	345·8	515·3	703·1	951·4	1243·7
75·6	106·3	144·0	178·0	298·1	451·6	639·1	864·9	1130·7
58·2	81·9	115·2	142·4	238·5	361·3	511·3	691·9	904·5
49·9	70·1	95·0	124·6	208·7	331·9	492·1	666·0	870·6
42·8	63·8	86·6	111·5	187·8	297·8	447·4	605·4	791·5
39·7	58·4	79·2	97·9	178·8	270·9	421·8	559·8	746·2
99·6	132·5	177·2	219·9	363·8	535·0	743·8	1001·2	1291·0
79·7	110·5	147·7	187·9	313·6	477·0	676·1	910·2	1173·6
61·4	85·1	118·1	150·3	250·8	381·6	531·9	728·1	938·9
52·6	72·9	97·4	131·5	219·5	340·1	520·6	700·8	903·7
111·8	149·7	197·8	244·2	407·2	587·7	833·3	1106·9	1459·4
89·5	124·8	164·8	210·5	351·1	534·3	757·5	1006·3	1326·8
68·9	96·1	131·9	168·4	280·9	427·4	666·0	905·0	1261·4
59·0	82·3	97·9	147·3	245·8	392·0	621·6	852·3	1123·8
53·7	74·8	88·9	132·6	221·2	352·6	583·3	774·8	1021·6
49·2	68·6	90·6	126·0	210·6	320·5	499·9	664·1	875·6

CHAPTER XIX.

THE LOSS OF HEAT FROM APPARATUS AND PIPES TO THE SURROUNDING AIR AND MEANS FOR PREVENTING THE ESCAPE.

A. The Loss of Heat.

1. *According to E Péclet's Equations.*

E. PÉCLET, in his classic work, *Traité de la chaleur*, has laid down the principles for calculating the loss of heat from hot bodies. We ought not, however, to omit the many later researches and methods of calculation; we shall therefore give the losses of heat according to Péclet and also according to more recent and simpler estimations. Unfortunately the results of the two methods of calculation differ considerably, Péclet's equations giving too low numbers, the more recent equations too high figures. The *observed* losses of heat, although they also are not all in agreement, lie approximately in the mean of those calculated according to the two formulæ.

According to Péclet, the total hourly loss of heat, M, expressed in calories, from 1 sq. m. of hot surface is composed of two parts, *viz.* :—

(*a*) The loss due to *radiation*, R, which only depends upon the material and the nature of the radiating surface, in addition to the temperature of the air, θ, and the difference in temperature, t, between the hot body and the surrounding air. The influence of the material and nature of the surface is expressed by the coefficient, k, which is for :—

Copper - - - - - - - -	0·16
Wrought iron - - - - - -	2·77
Cast iron - - - - - - -	3·36

According to Péclet's empirical equation,

$$R = 124{\cdot}72ka^{\theta}(a^{t} - 1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (168)$$

in which $a = 1{\cdot}0077$.

(*b*) The loss caused by *contact* with the surrounding air, A. In this case the shape of the body, in addition to the difference in temperature, has a considerable influence upon the loss, which influence is expressed by the coefficient, k_1.

According to Péclet

$$A = 0{\cdot}552k_1 t^{1{\cdot}233} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (169)$$

The total loss of heat from the body is therefore, for 1 sq. m., one hour and the difference in temperature, t,

$$M = R + A = 124{\cdot}72ka^{\theta}(a^{t} - 1) + 0{\cdot}552k_1 t^{1{\cdot}233} \quad . \quad (170)$$

The coefficient, k_1, was determined by Péclet for many forms of surface; it is different for flat plane surfaces, for horizontal and vertical cylinders, and also depends on the diameter of the cylinder.

In Table 37 are given the following values, calculated according to Péclet's data:—

(*a*) The loss of heat by radiation, R, from copper, wrought and cast iron, for 1 sq. m., one hour, and for temperature differences of 20°-180° C.

(*b*) The loss of heat by conduction, A, for 1 sq. m. and one hour:—

- (*a*) From horizontal pipes of 20-1000 mm. diameter, and differences in temperature of 20°-180° C.
- (β) From vertical cylinders of 1-3 m. diameter, 1-5 m. high, for temperatnre differences of 20°-150° C.
- (γ) From plane surfaces of 1·5 m. height and differences in temperature of 20°-180° C.

(*c*) The coefficient, k_1, for horizontal pipes, with differences in temperature of 20°-180° C.

(*d*) The coefficient, k_1, for vertical cylindrical surfaces of 1-3 m. diameter, and 1-5 m. high.

(*e*) The coefficient, k_1, for vertical plane surfaces.

From Table 37 the calculated loss of heat (per sq. m. per hour) can be read off for the most usual cases. For this purpose the loss by radiation, R, for the particular material and the prevailing difference in temperature, is added to the loss by conduction, A,

TABLE 37.

Loss of heat by radiation, R, by conduction, A (also the coefficients, k and cast iron, at temperature differences of 20°-180° C.,

	Temperature Difference.							
	20°	30°	40°	50°	60°	70°	80°	90°
	(a) Loss of heat by radiation, R, per 1 sq. m., from copper,							
	Sheet copper ($k = 0·16$).							
$R=$	3·7	5·8	8·0	10·4	13·9	15·9	19	22·2
	Wrought iron ($k = 2·77$).							
$R=$	64	100	138·5	181	226	275	328	384
	Cast iron ($k = 3·36$).							
$R=$	78	121	168	219	274	334	396	466
Diameter of the pipe, mm.	(b) (a) Loss of heat by							
20	130	215	306	404	505	610	716	832
30	101	168	241	316	396	479	562	754
40	88	145	207	272	340	412	483	561
50	79·4	131	186	246	307	372	436	505
60	74	121	173	228	285	345	404	470
70	70	115	164	216	270	328	384	444
80	66·6	109·8	156·6	205·8	258	312	367	426
90	65	107·5	153	202	252	305	360	415
100	62·6	103	147	193	242	293	345	399
150	57	94	133	176	220	266	313	364
200	54	89	127	167	210	249	298	344
300	51	84	120	158	197·8	239	282	326
400	49·9	82	117	156	194	234	276	319
500	48·6	81	115	151	190	230	271	313
600	48·4	80	113·7	148	187	227	267	309
800	47·7	78·7	112	147	185	223	263	305
1000	47	76·7	111	146	183	221	260	298
Height of the cylinder, mm.	(b) (β) Loss of heat by							
	Diameter of the cylinder = 1 m.							
1000	59	96	138	182	228	275	323	375
2000	52	86	123	162	202	245	289	334
3000	50	82	117	154	194	235	275	333
4000	48·8	81	116	152	191	227	267	309
5000	48·4	80	113·7	148	187	222	261	299

TABLE 37.

and k_1) from plane and cylindrical surfaces of sheet copper, wrought in calories per sq. m., per hour, according to E. Péclet.

Temperature Difference.								
100°	110°	120°	130°	140°	150°	160°	170°	180°
wrought iron and cast iron, at temperature differences of 20°-180° C.								
Sheet copper ($k = 0·16$).								
25·7	29·7	33·8	38·3	43	48	54	60	67
Wrought iron ($k = 2·77$).								
447	506	585	662	746	836	939	1045	1159
Cast iron ($k = 3·36$).								
541	622	709	803	904	1014	1139	1269	1406
conduction, A, from horizontal pipes.								
948	1065	1185	1309	1432	1561	1691	1822	1955
742	837	931	1028	1125	1226	1328	1431	1535
638	717	800	883	966	1053	1140	1229	1318
586	648	724	798	873	952	1031	1112	1192
536	601	671	740	810	883	957	1030	1105
507	567	636	706	768	838	907	978	1048
484	544	606	669	733	798	864	931	999
477	534	595	655	717	782	847	913	979
454	511	570	629	688	750	812	875	939
414	465	517	572	626	683	739	796	853
393	441	493	544	595	649	703	758	812
371	417	465	513	562	612	662	714	766
363	408	454	502	550	599	648	698	750
357	400	446	493	540	588	636	686	736
352	396	440	486	532	580	628	677	726
347	390	434	479	525	572	619	667	716
342	383	430	475	519	566	613	633	709
conduction, A, from vertical cylinders.								
Diameter of the cylinder = 1 m.								
428	480	535	591	646	705	—	—	—
381	427	477	526	575	627	—	—	—
364	408	457	504	551	601	—	—	—
352	396	440	477	532	580	—	—	—
344	385	432	486	516	569	—	—	—

TABLE 37—(*continued*).

Height of the cylinder. mm.	Temperature Difference. 20°	30°	40°	50°	60°	70°	80°	90°
	Diameter of the cylinder = 1·5 m.							
1000	59	95	137	180	226	273	320	371
2000	51	86	121	159	199	242	286	330
3000	49	82	115	151	191	231	272	315
4000	48·6	81	114	149	189	229	270	312
5000	48	79	112·5	147	185	225	265	306
	Diameter of the cylinder = 2 m.							
1000	58	94	136	179	224	270	317	368
2000	50	84	121	159	199	240	283	328
3000	48·8	82	116	152	191	225	271	308
4000	48·6	79·5	113	148	187	222	265	299
5000	47	76·7	111	146	183	221	260	298
	Diameter of the cylinder = 2·5 m.							
1000	56	91	132	173	217	262	307	357
2000	51	84	120	158	197·8	239	282	326
3000	48·6	81	115	151	190	230	271	313
4000	48	79	113	147	186	224	264	307
5000	47	76·7	111	146	183	221	260	298
	Diametor of the cylinder = 3 m.							
1000	55	91	131	172	216	260	305	355
2000	51	84	120	157	197	238	280	324
3000	48·6	81	114	150	189	229	270	312
4000	47·7	78·7	112	147	185	223	263	305
5000	47	76·7	111	146	183	221	260	298
	(*b*) (γ) Loss of heat by conduction.							
1000	53·2	53·2	87·8	125·3	206	253	294	349
2000	48·6	81	115	151	190	230	271	313
3000	47·0	76·7	111	146	183	221	260	298
4000	46·4	76·1	108·5	142·6	178·3	219	255	284
5000	45·1	75	107	140·5	176·3	213	251	290

(*c*) Value of the coefficient, k_1, for horizontal pipes.
d = diameter in mm.

d =	20	25	30	40	50	60	mm.
k_1 =	5·87	5·11	4·61	3·96	3·58	3·32	
d =	70	80	90	100	150	200	mm.
k_1 =	3·15	3·0	2·94	2·82	2·567	2·44	
d =	300	400	600	800	900	1000	mm.
k_1 =	2·3	2·25	2·21	2·18	2·15	2·13	

TABLE 37—(*continued*).

Temperature Difference.								
100°	110°	120°	130°	140°	150°	160°	170°	180°
Diameter of the cylinder = 1·5 m.								
424	475	530	585	640	698	—	—	—
377	420	470	522	570	617	—	—	—
358	401	448	495	546	591	—	—	—
355	398	444	490	537	585	—	—	—
348	392	436	481	527	575	—	—	—
Diameter of the cylinder = 2 m.								
420	470	525	580	633	690	—	—	—
373	419	467	516	565	615	—	—	—
350	395	438	484	530	577	—	—	—
344	385	432	477	521	569	—	—	—
342	383	430	475	519	566	—	—	—
Diameter of the cylinder = 2·5 m.								
405	456	509	562	615	670	—	—	—
371	417	465	513	562	612	—	—	—
357	400	466	493	540	588	—	—	—
348	392	436	482	528	575	—	—	—
342	382	430	475	519	566	—	—	—
Diameter of the cylinder = 3 m.								
403	452	505	560	612	667	—	—	—
369	415	463	510	560	609	—	—	—
355	398	444	490	537	585	—	—	—
347	390	434	479	525	572	—	—	—
342	383	430	475	579	566	—	—	—
A, from vertical plane surfaces.								
388	426	484	535	586	638	691	745	800
363	408	454	502	550	599	648	698	750
342	383	430	475	519	566	613	660	709
336	379	420	463	508	553	599	645	692
331	369	414	451	501	545	590	637	682

(*d*) Value of the coefficient, k_1, for vertical cylinders.

h = height. d = diameter.

	h = 1000	2000	3000	4000	5000 mm.
d = 1000	k_1 = 2·65	2·36	2·26	2·22	2·18
d = 1500	k_1 = 2·62	2·33	2·24	2·20	2·16
d = 2000	k_1 = 2·60	2·31	2·22	2·17	2·13
d = 2500	k_1 = 2·52	2·30	2·21	2·16	2·13
d = 3000	k_1 = 2·51	2·29	2·20	2·15	2·13

(*e*) Value of the coefficient, k_1, for vertical plane surfaces.

h = height in mm.

h = 1000	2000	3000	4000	5000 mm.
k_1 = 2·4	2·21	2·13	2·08	2·05

which depends on the form of the body and its position at the present difference in temperature.

Example.—A horizontal cast-iron pipe of 200 mm. external diameter loses, with a temperature difference of 100° C.,

$M = R + A = 541 + 393 = 934$ calories per sq. m. per hour.

These *calculated* losses of heat probably approximate to the truth, but it is still necessary to state what values have been obtained by more recent experiments conducted both on a large and small scale. It may be assumed *a priori*, that experiments with larger objects in larger rooms will show somewhat greater losses of heat, since they, being generally undertaken for practical purposes, do not so completely exclude all the subsidiary conditions (*e.g.*, the rapid motion of the air about the warm body under the experiment), as Péclet's purely laboratory experiments did. We have endeavoured to collect the accounts of researches on loss of heat dispersed throughout the literature of the subject. The results of the search are collected in Table 38; it should be remarked that these experiments do not all appear to be of equal value, since some were certainly not carried out with regard to all the circumstances to be considered.

In Table 38 are given the quantities of condensed water found in the different experiments, and thence are calculated the calories given out per sq. m. per hour. Then in the next column is given the loss of heat *calculated* for the particular case by means of Péclet's formulæ.

Comparison of these figures shows that in reality hot surfaces lose about 25 per cent. more heat than Péclet's formula indicates, which is without doubt explained by the ever-present air currents, which, as is well known, considerably facilitate the loss of heat to the air. The irregularity of the results of the experiments is due to the same cause and to the variable quantity of air in the steam.

It is not possible to arrange in one table the losses of heat from *all* these hot bodies of such various shapes and sizes. The loss must generally be determined as the product of the calculated exterior surface and the loss from unit surface, obtained from Table 37 or 39.

For the most ordinary apparatus—horizontal pipes and vertical cylinders of cast-iron, wrought-iron and copper—the losses of heat per hour calculated by Péclet's equations are given in Table 39, for pipes of 20-1000 mm. diameter per running metre and for vertical

cylinders of 1·5 m. height per 1 sq. m. of surface, for temperature differences of 30°-160° C.

In order to find the loss of heat really to be expected, the figures of Table 39 must be multiplied by about 1·275, *i.e.*, increased by about 25 per cent.

2. *According to more Modern Formulæ.*

The second, more modern, and somewhat simplified formula for the determination of the loss of heat, M, from warm bodies to the surrounding air, runs as before,

$$M = R + A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (171)$$

The loss by radiation is here, according to Dulong and Petit,

$$R = 125k(1{\cdot}0077^{t_1} - 1{\cdot}0077^{t_2}) \quad . \quad . \quad . \quad . \quad (172)$$

The coefficient of radiation, k, according to Péclet, for copper $= 0{\cdot}16$, wrought iron $= 2{\cdot}77$, cast iron $= 3{\cdot}36$; t_1 is the temperature of the hot space, t_2, of the cold space.

The loss by conduction is

$$A = 0{\cdot}55b(t_1 - t_2)^{1{\cdot}233} \quad . \quad . \quad . \quad . \quad . \quad . \quad (173)$$

in which b is the coefficient of conduction, which is, according to Valerins, for air at rest, 4, for air in motion, 5-6.

Thus the formula for the loss of heat from hot bodies to the surrounding air becomes

$$M = 125k(1{\cdot}0077^{t_1} - 1{\cdot}0077^{t_2}) + 0{\cdot}55b(t_1 - t_2)^{1{\cdot}233} \quad . \quad . \quad (174)$$

By means of this equation the loss of heat from cast-iron, wrought-iron, and copper surfaces, to the surrounding air, per hour and per sq. m., has been calculated for differences in temperature of 20°-180° C. The results are given in Table 40.

These figures (Table 40) will be found to be considerably higher than those calculated by means of Péclet's formula (Table 39), and even greater than the losses experimentally determined. As is often the case, the truth lies in the mean.

In the compilation of experimental results (Table 38), the values calculated by both formulæ are introduced, in order to facilitate comparison.

The loss of heat from multiple effect evaporators is greater than would be due to their simple surface. Let C_I, C_{II}, C_{III}, C_{IV} calories

[*Continued on p.* 202.

TABLE 38.

Compilation of the results of experiments, on the loss of heat, by Ordway, Gutermuth, Pasquay, Russner and Paul Müller.

1	2	3	4	5	6	7	8	9	10	11	12
Author.	Internal diameter = d External " = D Length = l mm.	External surface of the pipe. sq. m.	Pressure of the steam in the pipe. Atm. abs.	Internal temperature. ° C.	External temperature. ° C.	Steam condensed per hour. Kilos.	Steam condensed per hour per 1 sq. m. of surface. Kilos.	Loss of heat per 1 sq. m. in 1 hour. Cal.	Loss calculated according to Péclet. Cal.	Loss calculated by equation (174). Cal.	Loss of heat, in calories, when covered with
J. M. Ordway, Report 14. Boston Institute of Technology, 1883	$d=50$ $D=59{\cdot}7$ $l=304{\cdot}8$	0·057	4	150	15	—	Naked 3·176	1594	1628	2060	Felt 363
1885, Gutermuth, Zeits. d. V. d. I., 1887, No. 33, p. 653.	Cast iron $d=150$ $D=174$ $l=3000$	1·677	2·45	139	16·2	5·45			Average.		
		1·677	2·60	140	18·3	5·45					Kieselguhr 561
		1·677	2·30	137	15·5	5·49					
		1·677	2·50	139	18·2	5·73	Naked 3·28	1672	1230	1700	
		1·677	2·37	138	15·8	5·37					
		1·677	2·50	139	18·2	5·59					Cork 495
		1·677	2·53	139	23·2	5·25					
		1·677	2·60	140	19·2	5·46					
	Cast iron $d=75$ $D=83$ $l=330000$	97·5	3	144	20 ?	98	Cov'd 1·0				? 506
		97·5	4	152	20 ?	107·6	1·04				552
		97·5	5	159	20 ?	115	1·18				585
		97·5	6	165·3	20 ?	120	1·23				605
	Cast iron $d=140$ $D=168$ $l=323000$	184	3	144	20 ?	159	0·864				437
		184	4	152	20 ?	168	0·92				460
		184	5	159	20 ?	186·6	1·014				503
		184	6	165·3	20 ?	205	1·114				546
	Cast iron $d=75$ $D=88$ $l=330000$ *plus* $d=140$ $D=168$ $l=323000$	Total, 281·5	3	144	20 ?	262	0·929				470
			4	152	20 ?	312	1·109				555
			5	159	20 ?	323	1·14				565
			6	165·3	20 ?	319	1·13				556
			3	144	20 ?	253	0·9				455
			4	152	20 ?	300	1·067				583
			5	159	20 ?	301	1·067				529
			6	165·3	20 ?	317	1·11				546

TABLE 38—(*continued*).

1	2	3	4	5	6	7	8	9	10	11	12
Author.	Internal diameter = d External " = D Length = l	External surface of the pipe.	Pressure of the steam in the pipe.	Internal temperature.	External temperature.	Steam condensed per hour.	Steam condensed per hour per 1 sq. m. of surface.	Loss of heat per 1 sq. m. in 1 hour.	Loss calculated according to Péclet.	Loss calculated by equation (174).	Loss of heat, in calories, when covered with
	mm.	sq. m.	Atm abs.	° C.	° C.	Kilos.	Kilos.	Cal.	Cal.	Cal.	
Pasquay, Private Communication, 1895 ( ).	Cast iron					Naked	Naked	Naked	Naked		Kieselguhr
				115	15	2·332	3·332	1230	954	1431	309
	d = 140		·1·7	145	14·5	3·547	3·547	1791	1368	2052	
	D = 160-173			139	21	3·06	3·06	1561	1221	1710	
	l = 1870			135	15	3·145	3·145	1613	1221	1824	
				135	10	4·08	4·08	2093	1299	1935	
				129	25	2·769	2·769	1431	1148	1720	
				129	29	3·061	3·061	1581	954	1431	
				122	22	2·433	2·433	1267	954	1431	
J. Russner. Jahresb. d. tech. Staatsanstalt Muhlhausen. Oct., 1891.	d = 120 D = ? l = ?	Wrought iron 1	1·0	99·3	10·8	1·97	1·97	1058	805		
	d = ? D = 88·5 l = 3600	1	1	99·3	20	1·676	1·676	900	688		
P. Muller, Aug. 24, 1895. Pamphlet.	Cast iron	4	3·6	139·8	30·3		2·98	1635	1080	1612	
	d = ?		1·7	115·5	37·5		2·54	1038	756	1050	
	D = 159		1·7	115·1	39·8		2·48	958	650	990	
	l = 8008		1·2	106·6	36·6		2·34	871·5	594	907	
			3·6	140·3	34·2		2·66	1432	1020	1590	
			4·5	148·2	41·6		2·93	1567	1030	1590	
			3·6	140·1	34·8		2·68	1538	1020	1525	
			4·5	148	42·8		3·00	1584	1030	1550	
			4·5	148·4	36·4		2·76	1489	1072	1650	
			5·5	154·6	42·5		2·99	1663	1100	1640	
			1·2	105							
			1·7	115							
			3·6	140							
			5·5	155							

TABLE 39.

(*a*) Loss of heat, in calories, from cast-iron (*C*), wrought-
hour, according

(*b*) Loss of heat from vertical cylinders, 1·5 m.
The real loss is about 25 per cent.

Bore of pipe, *d*. mm.	External diameter of pipe, d_a. mm.	Cooling surface per 1 m. of length. sq. m.	Material.	Temperature Difference. 30°	40°	50°	60°	70°
							(*a*) Loss of heat,	
20	26	0·081	*W*	—	—	—	—	—
20	23	0·075	*K*	—	—	—	—	—
30	38	0·120	*W*	—	—	—	—	—
30	33	0·103	*K*	—	—	—	—	—
40	44·5	0·140	*W*	—	—	—	78	95
40	43	0·135	*K*	—	—	—	45	51
50	54	0·169	*W*	—	—	—	100	110
50	54	0·169	*K*	—	—	—	51	72
60	66	0·207	*W*	—	—	—	100	121
60	64	0·201	*K*	—	—	—	57	72
70	76	0·238	*W*	—	—	—	117	142
70	74	0·232	*K*	—	—	—	64	78
80	100	0·314	*C*	—	—	—	162	135
80	89	0·279	*W*	—	—	—	197	162
80	85	0·267	*K*	—	—	—	71	86
90	110	0·345	*C*	—	—	—	176	214
90	98	0·307	*W*	—	—	—	145	175
90	95	0·300	*K*	—	—	—	76	97
100	120	0·377	*C*	—	—	—	190	232
100	108	0·339	*W*	—	—	—	166	192
100	105	0·330	*K*	—	—	—	83	100
125	145	0·455	*C*	—	136	175	225	273
125	133	0·417	*W*	—	113	150	189	228
125	131	0·411	*K*	—	57	78	100	118
150	172	0·050	*C*	—	162	210	264	320
150	159	0·499	*W*	—	136	177	222	270
150	157	0·493	*K*	—	70	90	110	130
200	223	0·700	*C*	—	210	284	350	420
200	210	0·659	*W*	—	174	229	287	346
200	208	0·653	*K*	—	86	114	144	174
250	276	0·867	*C*	—	258	337	424	511
250	260	0·817	*W*	—	218	287	358	433
250	258	0·810	*K*	—	113	250	188	228

Table 39.

iron (W) and copper (K) pipes per running metre in one
to E. Péclet.*
high, per sq. m. per hour, according to E. Péclet.
greater than that calculated here.

Temperature Difference.								
80°	90°	100°	110°	120°	130°	140°	150°	160°
in calories, per running m. in 1 hour.								
76	94	102	113	129	143	160	177	193
48	60	65	70	80	85	95	105	112
96	115	130	144	165	185	205	225	250
53	71	81	85	95	105	110	120	135
110	127	149	165	190	210	235	257	281
64	75	95	100	105	118	130	141	153
124	143	170	190	217	245	268	293	328
75	86	90	110	125	138	150	163	180
150	168	200	220	250	280	310	340	395
85	97	112	125	138	154	165	185	198
167	195	224	225	286	309	356	396	433
90	105	120	135	152	166	185	201	217
231	171	318	355	403	448	500	553	610
192	224	258	294	340	368	408	450	500
103	118	135	152	170	190	207	226	243
254	297	349	388	438	490	546	607	670
205	235	276	305	350	390	430	477	525
112	129	150	165	184	195	225	244	265
276	322	377	422	477	533	593	659	727
227	264	311	344	391	438	483	537	591
118	138	168	178	198	217	240	265	280
322	377	434	494	558	625	696	772	854
267	310	367	413	468	515	585	643	710
141	161	188	211	225	251	280	310	335
379	442	510	580	707	733	815	907	1004
319	372	431	483	577	616	688	758	839
160	190	210	240	270	300	325	360	390
511	588	700	770	875	980	1092	1211	1330
410	477	574	623	706	792	877	976	1082
214	234	275	305	345	376	410	456	490
607	705	814	924	1048	1178	1308	1466	1612
513	600	689	777	888	995	1107	1225	1353
273	313	356	400	446	495	542	592	643

TABLE 39—(*continued*).

Bore of pipe, d. mm.	External diameter of pipe, d_a. mm.	Cooling surface per 1 m. of length. sq. m.	Material.	Temperature Difference. 30°	40°	50°	60°	70°
							(a) Loss of heat,	
300	332	1·043	*C*	205	295	378	471	575
300	310	0·974	*W*	177	250	329	409	498
300	308	0·967	*K*	87	124	163	203	247
400	410	1·288	*W*	233	326	441	537	651
400	408	1·282	*K*	113	150	215	266	322
500	510	1·60	*W*	289	404	531	665	808
500	509	1·60	*K*	154	197	257	324	394
600	612	1·92	*W*	345	480	628	792	969
700	712	2·23	*W*	404	559	733	918	1115
800	813	2·55	*W*	448	642	841	1057	1275
900	913	2·87	*W*	505	723	947	1190	1435
1000	1013	3·18	*W*	556	791	1040	1299	1578
		Height. m.					(b) Loss of heat	
		1	*C*	216	305	399	500	607
			W	195	275	361	452	548
			K	101	145	191	240	290
		2	*C*	207	289	378	473	576
			W	186	259	340	425	517
			K	92	129	170	211	260
		3	*C*	203	283	370	465	565
			W	182	253	332	418	506
			K	88	124	162	204	247
		4	*C*	201	282	367	463	563
			W	181	252	330	415	494
			K	87	123	160	202	245
		5	*C*	200	280	365	460	560
			W	179	250	328	411	500
			K	85	121	158	200	241

be the losses of heat from the separate vessels. It is evident that heat lost from one vessel cannot produce evaporation in the following vessels.

TABLE 39—(*continued*).

Temperature Difference.								
80°	90°	100°	110°	120°	130°	140°	150°	160°
in calories, per running m. in 1 hour.								
702	820	947	1077	1213	1469	1517	1683	1865
588	689	793	895	1038	1129	1268	1404	1553
292	356	375	433	496	544	589	640	694
773	900	1037	1170	1330	1490	1658	1837	2032
380	439	494	565	659	688	764	834	905
960	1015	1286	1350	1649	1848	2057	2272	2520
464	535	612	688	768	849	932	1017	1104
1148	1357	1636	1722	1978	2213	2463	2718	2818
1322	1540	1774	2007	2279	2551	2845	3146	3639
1505	1746	2014	2269	2601	2907	3238	3595	3978
1693	1932	2252	2615	2927	3272	3715	4047	4477
1762	2162	2501	2820	3226	3612	4017	4458	4931
from vertical cylinders per sq. m. per hour.								
716	832	965	1097	1242	—	—	—	—
648	755	871	981	1115	—	—	—	—
340	395	450	505	564	—	—	—	—
682	796	918	1042	1180	—	—	—	—
614	714	824	926	1055	—	—	—	—
305	352	403	450	505	—	—	—	—
668	781	899	1023	1157	—	—	—	—
600	699	805	907	1033	—	—	—	—
291	337	384	431	481	—	—	—	—
666	778	896	1020	1152	—	—	—	—
598	696	802	904	1029	—	—	—	—
289	334	381	428	478	—	—	—	—
665	772	889	1014	1145	—	—	—	—
593	690	795	898	1021	—	—	—	—
284	328	374	422	470	—	—	—	—

In the double effect the first vessel loses C_{I} calories, and since these C_{I} calories cannot evaporate anything in the second vessel, as much again is lost, *i.e.*, altogether $2C_{I}$ calories. The second vessel in its turn loses C_{II} calories.

Thus there are lost:—

In the double effect: $2C_I + C_{II}$.
In the triple effect: $3C_I + 2C_{II} + C_{III}$.
In the quadruple effect: $4C_I + 3C_{II} + 2C_{III} + C_V$.

TABLE 40.

Difference in temperature. °C.	Cast-iron.	Wrought-iron.	Copper.	Difference in temperature. °C.	Cast-iron.	Wrought-iron.	Copper.
Loss of heat in calories per sq. m. per hour at the respective differences in temperature.				Loss of heat in calories per sq. m. per hour at the respective differences in temperature.			
20	200	192	133	110	1612	1550	986
30	324	312	210	120	1824	1652	1134
40	456	440	292	130	2052	1968	1252
50	590	570	384	140	2246	2156	1386
60	741	710	475	150	2485	2380	1496
70	907	877	552	160	2725	2610	1625
80	1074	1034	686	170	2945	2820	1747
90	1248	1200	794	180	3240	3100	1880
100	1431	1380	901				

In vertical evaporators the cooling surface per sq. m. of heating surface ranges from 0·12-0·36 sq. m., as a rule it is 0·16-0·2 sq. m.

Example.—In a quadruple effect evaporator, with vessels of equal size, the cooling surface = 0·18 sq. m. per sq. m. of heating surface. The temperatures are:—

	I.	II.	III.	IV.
In vessel	100°	95°	85°	60°
Thus the temperature differences are	80°	75°	65°	40°

If the vessels are of wrought iron, the loss of heat in each, per 1 sq. m. of heating surface, is (Table 39)

	0·18 × 600	0·18 × 550	0·18 × 460	0·18 × 253,
i.e.,	108	99	83	45·5 calories.

The whole loss of heat is thus

$4 \times 108 + 3 \times 99 + 2 \times 83 + 45{\cdot}5 = 432 + 297 + 166 + 45{\cdot}5 = 940{\cdot}5$ calories. Therefore the average loss per 1 sq. m. of heating surface in one hour is $\frac{940{\cdot}5}{4} = 235$ calories, which is equal to about 2·3 per cent. of the efficiency.

In an unprotected quadruple-effect evaporator of	300	400	600	800	sq. m.
The loss of heat is about	70,500	94,000	141,000	188,000	calories
Or about	130	195	260	345	kilos. of steam
Or about	22	33	45	58	kilos. of coal

per hour. Rather more than less.

The loss of heat from a large apparatus is thus not inconsiderable, and it is very advisable to protect from such losses.

B. Means for Preventing Loss of Heat and their Efficacy.

The results obtained in different experiments, which are in tolerable agreement, show that the best protection against loss of heat is afforded by porous substances, which contain air. The order of efficiency, the best first, is as follows: silk, hair, wool, cotton, straw, turf, cork, wood, ashes, kieselguhr, sawdust, powdered coke, slag wool, mixtures of clay, lime and gypsum, with or without hair. The coating should not be too thick or the surface is unduly increased; a larger and cooler surface may easily lose more heat than a smaller and hotter surface. The coating should be light, incombustible and fairly resistant to external injury. The conductivities of the various protective materials, as determined by Pasquay, appear to be reliable; silk waste is the best non-conducting material.

Pasquay found the following conductivities for heat:—

Silk	0·045-0·048
Cow-hair felt	0·057
Cork shavings	0·073
Chopped turf	0·073-0·0997
Kieselguhr	0·077-0·144
Leroy's mixture	0·089-0·125
Knoch's mixture	0·090-0·240
Slag wool	0·101
Grünzweig and Hartmann's (Kieselguhr)	0·122
Einsiedel's mixture	0·139

The coefficient of radiation for the protective mass was taken as 3·65.

Pasquay also found (*Wärmeschutz im Dampfbetrieb*, 1895) the following amounts of condensed steam in a naked and covered pipe, other conditions being the same. The temperature of the steam was 135° C.; of the air, 13·5°-16° C. (mean, 15°).

The pipe condensed per sq. m. of surface in one hour:—

Naked - - - - - - -	2·972-3·087	kilos. of steam.
When covered with a cushion of silk 25 mm. thick - - -	0·446	"
When covered 55 mm. thick with cork shavings - - - -	0·467	"
When covered with kieselguhr -	0·640-0·895	"
When covered with Leroy's mixture 25 mm. thick - - - -	0·672-0·871	"
When covered with Knoch's mixture 25 mm. thick - - - -	0·845-1·216	"
When covered with Klehmet's mixture - - - - - -	1·396	"

It is to be observed that the composition of the compound non-conducting materials has considerable influence on their efficiency, and that the composition is in reality not always the same. Price also influences the choice of a non-conducting material.

By using the best protective coating, in the most favourable case about 80-85 per cent. of the loss which occurs from a naked pipe may be avoided.

Johannes Russner proposes for steam pipes a double covering of tin-plate, fitting tight, which is said to be still better than silk. This covering appears to be rather expensive. In this case the width of the space between the pipe and its jacket is important, it should not be too small or too large; about 10 mm. is stated to be suitable.

CHAPTER XX.

CONDENSERS.

THE appliances by means of which vapours (or gases) are liquefied or condensed are known as condensers. Sometimes the vapours or gases are to be condensed at atmospheric pressure, but more frequently it is desired to produce and maintain a vacuum by means of the condensation. In the latter case the condensation must naturally be effected in a space shut off from the air. The condensation is accomplished almost without exception in the cases under consideration by the withdrawal of heat, for which purpose cold water is generally used, cold air more rarely, since the former is the cheapest and most convenient means. It may be used in two ways: either the cooling water is injected directly into the vapour to be condensed, or the vapour is conducted over surfaces cooled by water or air. Thus there are obtained:—

A. Jet-condensers.
B. Surface-condensers.

The former are cheaper and are therefore always used, unless it is required to separate the vapours of valuable liquids (alcohol, ether, benzene, etc.) or to obtain pure condensed water.

Of the jet-condensers, which are employed to create a vacuum and must therefore be connected to an air-pump, two different kinds may be distinguished, namely:—

(*a*) The so-called *wet* condensers, from which the air-pump extracts the condensed vapours and injected water together with the air and uncondensed vapours. The principle of opposite currents between vapour and cooling water may be utilised in these condensers, but is not of great service. Wet condensers are generally arranged for parallel currents.

(*b*) The so-called *dry* condensers, from which the air-pump extracts only the air and uncondensed vapour, whilst the condensed vapour and injected water are carried off automatically in another way. The principle of opposite or counter-currents is almost always applied in this class, and with great effect, thus they are also called dry counter-current condensers.[1]

Surface-condensers, since they generally require a large surface, are almost always tubular; they are constructed of one or several long pipes or of many short tubes. The vapour may then pass through, and the cooling water outside, the tubes, but the opposite arrangement is also used. In both cases the whole mass of the water may flow slowly, generally upwards (opposite currents), in a closed space over the condensing surface. Thus these condensers are called *closed surface-condensers*. In many cases it is not only necessary to liquefy the vapours in the condenser, but also to cool the liquid. A cooling surface must then be attached to the condensing surface; this apparatus is then known as a *cooler*. If the vapour is passed through the tubes and the cooling water allowed to flow down outside exposed to the air, the apparatus is known as an *open surface-condenser*.

A. Jet-Condensers.

1. *General.*

When a definite weight of steam at a determined pressure is admitted into a condenser, perfectly closed and quite empty, and sufficient cold water is injected, almost the whole of the steam is converted into water and the injected or cooling water becomes considerably hotter by the exchange of heat. After the condensation there remain in the condenser: warm water, and over it, an absolutely empty space, in which the pressure would be zero (*i.e.*, a vacuum of 760 mm.) if the space were not immediately filled by:—

(*a*) The vapour, evolved by the warm water. Its pressure, which depends on the temperature of the water, is always known.

(*b*) Air, which is always introduced into the condenser along with the steam and cooling water.

[1] It will be seen that the differentiation of jet-condensers into "wet" and "dry" in no way corresponds to the true meaning of the words. These expressions have been once introduced and are now almost universally employed in interested circles. We might propose to call "dry" condensers *fall-pipe condensers*.

If, as a matter of reality, no air at all entered the condenser, after the condensation there would be in the condenser only water and vapour at a pressure corresponding to the temperatnre of the water. Since, however, air is *always* introduced by the steam and water, to this vapour pressure is to be added the pressure of the air introduced. The pressure in the condenser is then the *sum* of the pressures of air and vapour.

Warm water, which has been used for condensing, then artificially coolod and again led into the condenser, contains little air, but still always some quantity.

In a closed vessel, partially filled with hot water, in which a considerable air pressure is produced by artificial means, the water would still evolve steam of a pressure corresponding to its temperature, which would increase by its own amount the pressure already existing.

The air-pumps are used to exhaust as rapidly and completely as possible the air introduced by steam and water, so that there may be in the condenser only the pressure of the steam, which depends on the temperature of the water.

The pressure in the condenser should be as low as possible, for as it decreases the boiling point also falls and the evaporative capacity of the heating surface in the vacuum increases.

There can be no intention of exhausting, by means of the air-pump, the vapour formod from the water together with the air, in order to increase the vacuum, since the volume of this vapour is so great that it cannot be dealt with by pumps of reasonable size. If it were desired to exhaust steam from the condenser with the air-pump, and thus to form fresh vapour from the water, which process would cool the warm water and so produce a higher vacuum, the air-pump would have to be of quite impossible dimensions.

Example.—In order to condense 100 kilos. of steam, under certain circumstances, 3030 kilos. of water are required, which become heated from 15°-35° C.

In order to cool these 3030 kilos. of water through 5° C. (to 30°) it would be necessary to deprive them of 15,150 calories, *i.e.*, to evaporate $\frac{15,150}{580} = 26{\cdot}1$ kilos. Now 1 kilo. of steam at 30°-35° C. has a volume on the average of 28,750 litres, thus 26·1 kilos. measure 750,375 litres. Such great volumes can naturally not be pumped out in a short time.

It is therefore necessary to restrict tho operation to removing the air alone from the condenser as completely as possible.

Since the pressure in the condenser is always the *sum* of the pressures of air and steam, it follows that the pressure of the air is found if that of the steam be deducted from the total pressure. The pressure of the steam is, however, dependent on the temperature of the injected water when warmed by the condensed steam, since the two are in contact.

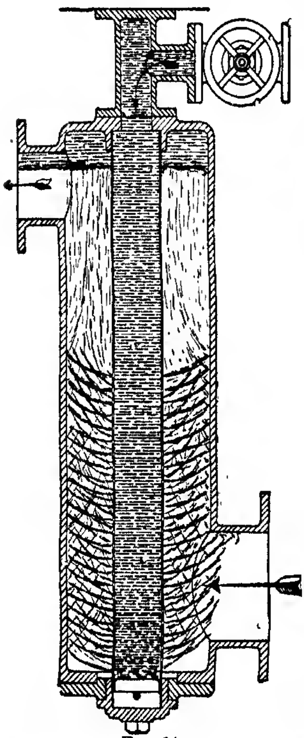

FIG. 14.
Parallel Current Jet-Condenser.

The temperature of the water at different parts of the same condenser is different, so must also be the pressures of the steam and air. The total pressure cannot be the same in all parts of the condenser, because *currents* of air and steam must be produced, but this total pressure must always be somewhat lower than the pressure in the evaporating apparatus, the vapours of which are to be liquefied in the condenser, since the friction of the vapour in the pipes between the evaporator and condenser naturally absorbs a certain amount of pressure.

There must be a somewhat higher pressure in the evaporator than in the condenser, in order to impart their velocity to the exhausted vapours. This difference of pressure will be the less, the shorter the connecting pipe and the slower the movement of the steam in it. On this subject see Chapter XVII.

The higher the temperature of the water in the condenser at the place where the air is exhausted, the higher is also the corresponding vapour pressure at this point. With a fixed total pressure in the condenser, the pressure of the air must be lower (*i.e.*, a definite weight will occupy a proportionately larger volume, which is to be removed

from the condenser) the higher the temperature the water with which it is last in contact.

Thus it follows that, other things being equal, the volume of air to be extracted is least when it is directly or indirectly in contact with *cold* water at its removal from the condenser. This is the case in opposite current and surface-condensers, whilst in parallel current condensers the warm water goes into the pump *in common* with the air and steam.

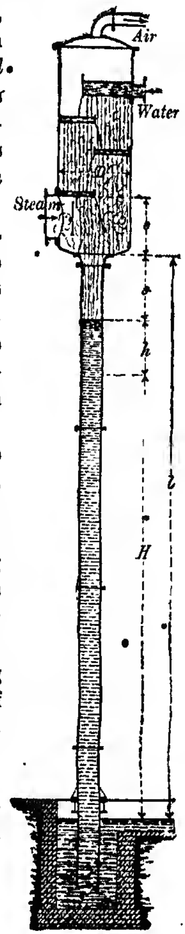

Fig. 15.
Fall-pipe Condenser.

The amount of cooling water used in a condenser must always be so great that the temperature of the waste water is somewhat lower than corresponds to the vacuum, since only then can the vacuum in the condenser be maintained somewhat higher than in the evaporator (*i.e.*, the pressure somewhat lower), which we found to be necessary.

In wet (parallel current) jet-condensers the steam enters the closed condenser at the top, together with the water in the finest spray, and both move downwards with diverse velocities. The steam then gives up its heat to the cooling water and is liquefied, and the cooling water takes up this heat and becomes warmer. The velocity of the steam diminishes to zero in its downward path, the velocity of the water increasing downwards in accordance with the laws of falling bodies. Air, water and uncondensed gases collect at the lower part of the condenser and are exhausted by the air-pump.

Wet condensers are constructed in many different ways. Fig. 14 indicates *one* construction, which is quite practical and permits of the necessary injected water being pumped direct from a well.

Opposite currents may also be arranged in a wet condenser, by admitting the steam below and exhausting the air above, by which means the latter, since it is

last in contact with cold water, may be removed colder, which is in itself an advantage. However, the air in the pump cylinder, or even earlier, is in contact with the warm water, above which is steam of corresponding pressure. Thus an advantage of this construction can hardly be recognised, for the air is intimately mixed with the water and very rapidly acquires its temperature, when the condition of things is then the same as if air and water were exhausted by the *same passage*. The pressure in the wet air-pump, which is still in question, is always dependent on the temperature of the water pumped out.

In dry (counter-current, fall-pipe) condensers the steam enters below and the cooling water in fine spray above. The steam rises with decreasing velocity, the cooling water falls. It is endeavoured to arrange that the cooling water, when it leaves, shall be as nearly as possible at the temperature of the entering steam and the air as nearly as possible at that of the cold water. It is often assumed that the temperature of the steam is the same throughout the condenser, which cannot, strictly speaking, be the case. From the bottom of the condenser the injected water and condensed steam flow away spontaneously through a vertical pipe at least 10·7 m. long. In the most favourable case the pressure in this condenser corresponds to the temperature of the cooling water as it enters.

Dry condensers also may be constructed in different ways. Fig. 15 shows, with details omitted, an ordinary design, which is quite clear without further explanation.

We shall next consider separately the factors which affect the dimensions of jet-condensers, and then use the results in determining these dimensions.

2. *The Necessary Quantity of Cooling Water.*

The quantity of cooling water required in each case depends in particular on its *original* temperature, on *that at which it is to leave* the condenser, and, finally, on the *total heat* of the steam, which depends on the vacuum to be produced.

Let D = the weight of steam to be condensed, in kilos.,

c = the total heat of 1 kilo. of this steam,

W = the weight of the cooling water in kilos.,

t_a = original temperature of this water in ° C.,

t_e = the final temperature of the waste water after the condensation.

Then $$Dc + Wt_a = (W + D)t_e \quad . \quad . \quad . \quad . \quad . \quad . \quad (175)$$

Thus the weight of cooling water,

$$W = \frac{D(c - t_e)}{t_e - t_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (176)$$

Example.—$D = 100$ kilos. of steam are to be condensed by water at $t_a = 10°$, so that the waste water is at $t_e = 40°$. How much cooling water is required?

At 40° C. 1 kilo. of steam has $c = 618{\cdot}7$ calories, therefore

$$W = \frac{D(c - t_e)}{t_e - t_a} = \frac{100(618{\cdot}7 - 40)}{40 - 10} = 1929 \text{ kilos.}$$

Thus in this case $W = 1929$ kilos. of cooling water are necessary.

It is occasionally convenient to have these data at hand, accordingly Table 41 has been drawn up, giving the number of kilos. of water required to condense 1 kilo. of steam under various conditions —water injected at temperatures of 5°-40° C., and waste water at 20°-60° C. The heat of the steam is taken throughout at $c = 630$ calories, whilst in reality it varies somewhat in each case.

3. *The Diameter of the Water Supply Pipe.*

The diameter of the pipe, which conveys the water to the condenser, depends on the quantity to be supplied in unit time and on the pressure with which it is injected into the condenser. The quantities of water necessary in each case may be taken from Table 41, the available pressure depends on the special conditions of each installation and may vary greatly. If the water tank (or well) is at the same level as the condenser, the whole excess of the pressure of the atmosphere over the pressure in the condenser is available for drawing the water into the condenser. If there is a vacuum in the condenser of 700 mm. of mercury, corresponding to a water column of $H = 9{\cdot}525$ m., then the head of water in this case is also $h_w = H = 9{\cdot}525$ m. If the water-tank is at the height h_h above the condenser, then this difference in height is to be added to the vacuum expressed as a head of water. The total head is then $h_w = H + h_h$. If the water is at a lower level than the condenser, *viz.*, at the distance h_l below it, then the pressure of the water is equal to the difference of these heights: $h_w = H - h_l$. The heights h_h and h_l must always be measured from the point where the water enters the condenser.

TABLE 41.

The weight of cooling water, W, required to condense 1 kilo. of steam.

Temperature of the injected water, t_i. ° C.	Temperature of the waste water, t_e, in ° C.								
	20°	25°	30°	35°	40°	45°	50°	55°	60°
	Weight of injected water, in kilos., required for 1 kilo. of steam.								
5	44·3	30	23·8	19·7	16·7	14·5	12·7	11·4	10·3
6	43·2	31·5	24·7	20·5	17·2	14·9	13	11·6	10·5
7	46·5	33·3	25·6	21·3	17·8	15·2	13·3	11·8	10·7
8	50·5	35·3	27	22	18·3	15·7	13·7	12·13	10·9
9	55	37·5	28·3	23	18·9	16·1	14	12·4	11·1
10	60·5	40	29·3	24	19·6	16·4	14·4	12·7	11·3
11	66·2	42·9	31·3	24·6	20	17·1	14·8	13	11·5
12	75·6	46·2	33	25·6	20·9	17·6	15·1	13·25	11·8
13	86·4	50	35	26·5	21·3	18·1	15·4	13·6	12
14	101	55	37·2	28·1	22·5	19	16	14	12·3
15	121	60	39·6	29·5	23·4	19·7	16·4	14·25	12·6
16	152	66	42·5	31·1	24·1	20	16·9	14·6	12·85
17	202	75	45·6	33	25·4	20·7	17·4	15	13·15
18	303	86	49·6	34·5	26·6	21·5	18	15·4	13·4
19	—	100	54·1	36·5	27·8	22·3	18·5	16	13·8
20	—	120	59·5	39·5	29·3	23·2	19·1	16·3	14·1
21	—	150	65	42·1	30·8	24·1	19·8	17	14·5
22	—	200	74·4	45·4	32·4	25·1	20·6	17·3	14·8
23	—	—	84·4	49·5	34·4	26·4	21·3	17·8	15·3
24	—	—	99·2	53·6	36·5	27·6	22·1	18·4	15·7
25	—	—	119	59	38·5	29·3	23	19	16
26	—	—	149	65·6	42	30·5	23·9	19·6	16·4
27	—	—	—	74·3	45	32·2	25	20·5	17·1
28	—	—	—	84·3	49	34·1	26·14	20·7	17·7
29	—	—	—	98·3	53·2	36·2	27·4	21·5	18·2
30	—	—	—	147	58·5	38·6	28·75	22·4	19·2
31	—	—	—	197	65	41·4	30·3	23·3	19·5
32	—	—	—	—	73	44·6	32	24·1	20·2
33	—	—	—	—	97·5	48·3	33·8	25·4	20·5
34	—	—	—	—	117	53	35·9	26·7	21·7
35	—	—	—	—	149	58	38·3	28	22·6
36	—	—	—	—	—	—	41	29·4	23·5
37	—	—	—	—	—	—	44·2	31·1	24·6
38	—	—	—	—	—	—	48	33	25·7
39	—	—	—	—	—	—	52·5	35	27
40	—	—	—	—	—	—	57·5	37·3	28·3

If it is desired to avoid forcing the water into the condenser by means of a pump, the apparatus must never be arranged so that $H = h_t$, for a certain excess of pressure is required to overcome the resistance to the movement of the water and to give the water a definite velocity. This excess of head should never be made less than 3 m., and more would be better.

The dimensions of the water supply pipe for the different cases are to be found in Chapter XVIII. and Table 36.

4. *The Waste-Water Pipe (Fall-Pipe) of the Dry Condenser* (Fig. 15).

The fall-pipe of the dry condenser is used to conduct away continuously the condensed steam and the water used to condense it. Since there is a more or less complete vacuum in the condenser, the pressure of the external atmosphere will keep the water in the fall-pipe at a corresponding height, just as it supports the mercury in the barometer.

The pressure of the atmosphere is equal to that of a column of water 10·336 m. high at its maximum density, *i.e.*, at 4° C.; it is 1·0336 kilo. per sq. cm. Since, however, there is never a *complete* vacuum in the condenser, the height at which the column of waste water is kept by the atmosphere is always less. If b be the vacuum in the condenser measured in mm. of mercury, and the temperature of the water 4° C., then the height of the column of water in the fall-pipe is, in metres,

$$H = 10{\cdot}336\frac{b}{760} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (177)$$

Now the waste water is always somewhat warmer than 4° C., hence its specific gravity is less and its volume greater; the column of water must accordingly be higher in proportion.

According to Volkmann (1881), the volume of water V_w, when it is unity at 4° C., is :—

At	4°	30°	40°	50°	60°	70° C.
V_w =	1·0	1·00425	1·007700	1·01197	1·01694	1·02261
At		80°	100° C.			
V_w =		1·02891	1·04323			

TABLE 42.

The height of the water barometer at vaqua of 570-750

Vacuum, mm. mercury	570	611	642
Temperature °C.	65	60	55
Water barometer, mm. at 4° C.	7793	8310	8734
Water volumes at above temperatures	1·01966	1·01695	1·01441
Water barometer, mm., at above temperatures	7945	8450	8856

The velocity of fall of the water, v_w, and the quantity

Diameter of the pipe, mm.		100	125	150
The head, $h = 0·10$ m. The length of the fall-pipe, $l = 10117+100+500=10717$ mm.	$v_w =$	0·63	0·66	0·695
	$W=$	17·8	29·3	44·2
The head, $h = 0·20$ m. The length of the fall-pipe, $l = 10117+200+500=10817$ mm.	$v_w =$	0·89	0·93	0·98
	$W=$	25·2	40·8	62·65
The head, $h = 0·30$ m. The length of the fall-pipe, $l = 10117+300+500=10917$ mm.	$v_w =$	1·09	1·10	1·21
	$W=$	30·8	48·2	76·9
The head, $h = 0·40$ m. The length of the fall-pipe, $l = 10117+400+500=11017$ mm.	$v_w =$	1·26	1·33	1·40
	$W=$	35·0	58·5	89·1

The height of the water barometer, $H = 10·117$

Thus the height of the column of water when at rest is, more accurately, for each vacuum and each temperature,

$$H = 10·336\frac{b}{760}V_w = 0·0136bV_w \quad . \quad . \quad . \quad . \quad (178).$$

Now the fall-pipe must convey a certain quantity of water in

TABLE 42.

mm. of mercury and at the corresponding temperatures.

668	705	718	728	736	742	750	
50	40	35	30	25	20	10	
9085	9592	9768	9902	10016	10100	10212	
1·011877	1·007627	1·00593	1·00425	1·00300	1·00173	1·00090	
9184	9665	8817	9944	10046	10117	10212	

of water, W, flowing away, in cub. m. per hour.

175	200	225	250	300	350	400	450
0·70	0·74	0·75	0·761	0·785	0·81	0·81	0·815
60·5	83·7	103·5	134·4	199·5	280·5	366·2	466·5
1·00	1·04	1·06	1·08	1·11	1·13	1·14	1·15
86·4	117·5	145·0	190·8	282·2	391·3	575·4	658·8
1·25	1·28	1·30	1·32	1·36	1·38	1·40	1·41
108·0	144·3	177·8	234·1	355·9	477·9	633·0	807·0
1·44	1·47	1·50	1·53	1·57	1·59	1·61	1·63
124·4	166·2	205·2	270·3	399·0	552·4	727·9	933·0

m.; the addition for safety, $s = 0·5$ m.

unit time, therefore the water must attain a certain velocity of fall, which can only be imparted to it by a certain head, h.

This head, h, is that column of water, by which the water must stand higher in the fall-pipe than the difference between the external atmospheric pressure and the absolute pressure in the condenser. It is designed in the first place to overcome the resistances offered to the downward flow of the water, and, in the second, to impart the necessary velocity to the water.

If this head of water, h, be assumed for a definite case, the velocity of the fall of the water, and hence the quantity of water, which flows through a pipe of known section in a certain time, are found from well-known formulæ [Chapter XVIII., Equation (162)]. Or, inversely, a certain velocity of fall may be required, and the head, h, necessary to create this velocity may be calculated; since we have adopted the plan of always calculating the efficiency of apparatus of known dimensions, the former course is taken here.

Let (compare Fig. 15)

H = the height of the water in the fall-pipe maintained by the vacuum,

h = the head of pressure, then $H + h$ = the length of pipe traversed by the water in metres, *i.e.*, the theoretical height of the fall-pipe,

v_w = the velocity of fall of the water in m. per sec.,

d = the diameter of the pipe in m.,

ζ_1 = the coefficient for the resistance of the water on entering the fall-pipe = 0·505 (see p. 180),

λ = the coefficient for the friction of the water against the walls of the pipe (see p. 180),

then the following equation holds good:—

$$v_w = \frac{\sqrt{2gh}}{\sqrt{1 + \zeta_1 + \lambda\frac{H + h}{d}}} \quad . \quad . \quad . \quad . \quad . \quad (179)$$

$H + h$, the length of the pipe traversed by the water, we may assume for purposes of calculation, with a slight error, to be always 10 m., we may then, by inserting various values for h, determine the resulting velocity of fall, v_w, for all diameters of the pipe, d, to be considered.

In Table 42 may be found the velocities of fall calculated from equation (179), and thence the *quantities* of water flowing in one hour through the fall-pipe, for pipes of diameter d = 100-450 mm., and for heads, h, of 0·100-0·400 m.

The waste water thus always stands in the pipe at the height $H + h$ above the lower level of the water. However, this position of the water is not steady, but rises and falls in consequence of slight variations in the vacuum and in the water supply. Safety also demands that there shall be a certain space, s, above the water in the pipe, so that

the water may never collect in the condenser. Thus the fall-pipe must have *at least* the height, $l = H + h + s$. The length, s, may be chosen as desired; it has been taken as 0·5 m.

With these assumptions there are given in Table 42, for various degrees of vacuum, pressure heads and diameters of pipe, the lengths of the fall-pipe, l, and the quantities of waste water, W, per hour. If the length of the waste pipe be increased its diameter may be decreased, and *vice versâ*. In making the choice of a diameter of pipe for a definite quantity of waste water, a high vacuum (750 mm.) in the condenser will naturally be assumed.

The mean atmospheric pressure at the level of the sea is 760 mm. of mercury. At inland places, which always lie higher, it is less, but may there even reach 780 mm.

The vacuum in the condenser will rarely be higher than 740 mm., but it would be well to calculate for a vacuum of at least 750 mm.

In order to facilitate the entry of water into the fall-pipe, it should commence with a conical portion connected to the convex (downwards) bottom of the condenser. The angle enclosed by the sides of the cone should be 30°.

5. *The Distribution of the Water in the Condenser.*

After determining the weight of water required to condense a definite weight of steam, it is necessary to calculate the dimensions of the appliances for distributing the water in the condenser.

There are two principal methods used for distributing the water:—

(*a*) The production of a falling sheet (veil) of water by *overflow* over a straight or circular edge (sill).

(*b*) The production of water jets or drops by means of flat *plates*, provided with a rim and perforated by holes, by means of perforated pipes, roses, etc.

(*a*) *Overflows.*—The following equation may be used to determine the quantity of water which passes over an overflow in one hour:—

$$W = \tfrac{2}{3}\mu bh\sqrt{2gh}\,3600 \times 1000 \quad . \quad . \quad . \quad . \quad . \quad (180)$$

in which

W = the quantity of water flowing over in litres per hour,

μ = a coefficient of contraction, which we shall take as 0·6, excluding the not very considerable alterations due to

shape and inclination of the edge by selecting an average section,

g = acceleration of gravity = 9·81 m.,

h = the head in metres,

b = the width of the overflow (sill) in metres.

If the constants in equation (180) be replaced by their numerical values we obtain

$$W = 6{,}400{,}000\, b \sqrt{h^3} \text{ (approx.)} \quad . \quad . \quad . \quad . \quad (181)$$

By means of this equation the necessary dimensions may be calculated for any case, but in order to avoid this calculation the quantities of water, W, in cub. m. per hour which pass over sills of b = 0·5-5 m. in width, with heads, h, of 0·005-0·050 m., are given in Table 43.

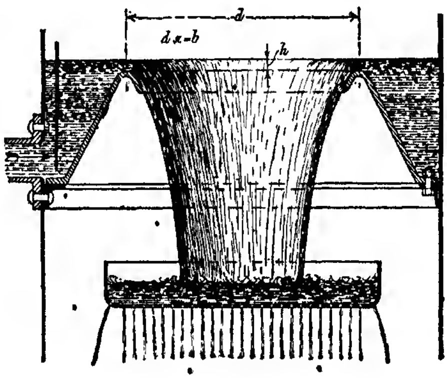

FIG. 16.

Example.—If the width of the edge of the overflow (*i.e.*, the length of the sill) be b = 3 m., the head h = 0·020 m., then the quantity of water flowing per hour is

$$W = 6{,}400{,}000 \times 3 \times \sqrt{(0{\cdot}02)^3} = 54{,}200 \text{ litres.}$$

(*b*) *Sieves.*—The quantity of water, in litres, which flows in one hour through a hole of diameter d decimetres in the bottom of a vessel, in which the water stands at the constant height, h, without regard to all the contractions which diminish the rate of flow, is

$$W = 10\frac{d^2\pi}{4}\sqrt{2gh}\, 3600 \text{ litres} \quad . \quad . \quad . \quad . \quad . \quad (182)$$

TABLE 43.

The quantity of water, in cub. m., which flows in one hour over sills 0·5-5 m. wide, with heads of 5-50 mm.

Width of overflow, b.	Head, h, in mm.							
	5	10	15	20	25	30	40	50
m.	Quantity of water flowing over, in cub. m. per hour.							
0·5	1·1	3·2	6·3	9·0	12·6	16·6	25·6	35·6
0·6	1·3	3·8	7·6	10·8	15·2	19·9	30·7	42·7
0·7	1·5	4·4	8·8	12·7	17·7	23·2	35·8	49·8
0·8	1·7	5·2	10·1	14·5	20·3	26·6	41·0	57·0
0·9	2·0	5·7	11·4	16·3	22·8	29·9	46·1	64·1
1·0	2·2	6·4	12·6	18·1	25·3	33·2	51·2	71·2
1·1	2·4	7·0	13 9	19·9	27·9	36·5	56·3	78·4
1·2	2·6	7·6	15·2	21·7	30·4	39·9	61·5	85·5
1·3	2·9	8·3	16·4	23·5	32·9	43·2	66·7	92·6
1·4	3·1	8·9	17·7	25·4	35·5	46·5	71·7	98·7
1·5	3·3	9·6	19·0	27·2	38·0	49·8	76·8	106·9
1·6	3·5	10·5	20·2	29·0	40·6	53·2	82·0	114·0
1·7	3·7	10·8	21·5	30·8	43·1	56·5	87·1	121·1
1·8	4·0	11·5	22·8	32·6	45·6	59·8	92·2	128·3
1·9	4·2	12·1	24·0	34·4	48·2	63·1	97·4	135·4
2·0	4·4	12·8	25·3	36·2	50·7	66·5	102·5	142·5
2·1	4·6	13·4	26·6	38·1	53·2	69·8	107·6	149·6
2·2	4·9	14·1	27·8	39·9	55·8	73·1	112·7	156·8
2·3	5·1	14·7	29·1	41·7	58·3	76·5	117·9	163·9
2·4	5·3	15·3	30·4	43·5	60·9	79·8	123·0	171·0
2·5	5·5	16·0	31·6	45·3	63·4	82·5	128·1	178·2
2·6	5·8	16·6	32·9	47·1	65·9	85·2	133·3	185·3
2·7	6·0	17·3	34·2	48·1	68·5	89·2	138·4	191·4
2·8	6·2	17·9	35·4	49·2	71·0	93·1	143·5	199·5
2·9	6·4	18·5	36·7	52·6	73·6	96·4	148·6	205·7
3·0	6·6	19·2	38·0	54·2	76·1	99·7	153·7	213·8
3·1	6·9	20·1	39·2	56·2	78·6	103·1	158·9	220·9
3·2	7·1	21·0	40·5	58·0	81·2	106·4	164·0	228·0
3·3	7·3	21·1	42·6	59·8	83·7	109·7	169·1	235·2
3·4	7·5	21·6	43·0	60·8	86·2	113·0	174·2	242·3
3·5	7·8	22·4	44·3	63·5	88·8	116·4	179·4	249·4
3·6	8·0	23·0	45·6	65·3	91·3	119·7	184·5	256·6
3·7	8·2	23·7	46·8	67·1	93·9	123·0	189·6	263·7

TABLE 43—(*continued*).

Width of overflow, *b*.	Head, *h*, in mm.							
	5	10	15	20	25	30	40	50
m.	Quantity of water flowing over, in cub. m. per hour.							
3·8	8·4	24·3	48·1	68·9	96·4	126·3	194·8	270·8
3·9	8·7	24·9	49·4	70·7	98·9	129·6	199·9	277·9
4·0	8·9	25·6	50·6	72·5	101·5	133·0	205·0	285·1
4·1	9·1	26·2	51·9	74·3	104·0	136·3	210·1	292·2
4·2	9·3	26·9	53·2	76·2	106·5	139·6	215·3	299·3
4·3	9·5	27·5	54·4	78·0	109·1	143·0	220·4	306·5
4·4	9·8	28·1	55·7	79·8	111·6	146·3	225·5	313·6
4·5	10·0	28·8	57·0	81·6	114·1	149·6	230·6	320·7
4·6	10·2	29·4	58·2	83·4	116·7	153·0	235·8	327·8
4·7	10·4	30·1	59·5	85·2	119·2	156·3	240·9	335·0
4·8	10·7	30·7	60·8	87·0	121·8	159·6	246·0	342·1
4·9	10·9	31·3	62·1	88·9	124·3	162·3	251·1	348·2
5·0	11·1	32·0	63·3	90·7	126·9	165·1	256·3	356·4

This theoretical amount of flow is, however, diminished by the shape of the opening, the form of the edges of the orifice, the roughness of the walls of the hole and the thickness of the bottom, to such an extent that in reality only a fraction of the theoretical quantity of water can flow through the hole. The holes to be considered here are such as are bored without any great care in the sieve-plate. The amount of flow is also affected in high degree by the violent motion in which the water is kept, before its escape, by the supply of fresh water falling into the sieve.

Thus since it cannot be assumed that the quantities of water, even when calculated by well-known formulæ with regard to the contractions, are realised in practice, we have determined by direct observation the quantities of water which flow through holes of 3, 4, 5, 6, 7 and 8 mm. in diameter from vessels which are kept constantly filled with water to heights of 10, 15, 30, 40, 50 and 200 mm. It was fonnd that the real amounts of flow were very different in each case from those calculated without regard to all the disturbing influences—to

TABLE 44.

(*a*) The volume of water, in litres, which runs from a sprinkler in one hour through holes 2-10 mm. in diameter, with the water at heights of h = 10-200 mm. (Taken at 15 per cent. less than the calculated.)

(*b*) The number of holes of 2-10 mm. diameter required to pass 4-300 cub. m. of water per hour, when h = 10 mm.

Height of the water on the sieve, h. mm.	Diameter of the holes in mm.								
	2	3	4	5	6	7	8	9	10
	(*a*) The volume of water, in litres, flowing through one hole in one hour.								
10	4·75	9	17	27	38	52	68	86	106
15	5·2	11	20	31	47	64	83	105	130
30	7·46	16	29	45	65	87	100	149	184
40	8·5	18	34	53	77	104	136	172	213
50	9·67	24	38	59	86	120	153	196	242
200	19·88	42·4	76	119	171	227	300	402	497
Hourly flow of water. cub. m.	(*b*) The necessary number of holes, n, when the water stands at the height, h = 10 mm.								
4	842	423	235	150	105	77	59	46	38
6	1263	634	353	226	157	115	88	70	56
8	1684	846	470	301	210	154	118	93	75
10	2105	1057	588	376	262	192	147	116	94
15	3158	1585	882	564	393	289	220	175	141
20	4210	2214	1176	752	524	382	294	232	148
25	5264	2643	1470	940	655	481	367	291	236
30	6315	3171	1764	1126	786	576	441	348	282
35	7368	3699	2058	1316	917	672	514	406	329
40	8420	4228	2352	1504	1048	768	588	464	376
50	10527	5285	2940	1880	1309	962	734	582	472
60	12630	6342	3528	2256	1572	1152	882	696	564
70	14735	7399	4116	2632	1834	1344	1029	812	658

TABLE 44—(*continued*).

Hourly flow of water. cub. m.	Diameter of the holes in mm.								
	2	3	4	5	6	7	8	9	10
	(*b*) The necessary number of holes, n, when the water stands at the height, $h = 10$ mm.								
80	16840	8456	4704	3008	2096	1536	1176	928	752
90	18947	9513	5292	3384	2357	1730	1322	1046	848
100	21053	10570	5880	3759	2618	1923	1468	1163	943
125	26362	13212	7350	4699	3272	2404	1832	1454	1179
150	31580	15850	6820	5639	3927	2885	2202	1745	1415
175	36889	18497	10290	6579	4581	3366	2566	2036	1651
200	42106	21140	11760	7518	5236	3846	2936	2326	1886
225	47415	23782	13230	8458	5890	4327	3300	2617	2122
250	52733	26425	14700	9398	6545	4808	3670	2908	2358
275	57942	29062	16170	10338	7199	4289	4034	3199	2594
300	63160	31710	17640	11278	7954	5770	4404	3490	2830

such an extent that they were 1-30 per cent. less. The mean difference in the flow from that calculated *without* regard to the contraction was 8·3 per cent. less.

In Table 44 are given the probable amounts of flow, as shown by the experiments, through holes of 2-10 mm. diameter in one hour, when the water stands upon the sieve at heights of 10-200 mm.

Since it is always known how much water per hour is to be sprayed into the condenser, the number of holes required in the sieve can be at once calculated by the aid of this table. The sieve naturally passes the more water, the greater the height at which it stands on the sieve, so that the height of the water itself regulates the varying supplies of water required in working every condenser.

Table 44 also gives the number of holes, n, of 2-10 mm. diameter, necessary to transmit 4-300 cub m. of water per hour, when the water stands at a height of 10 mm. If the water stands at any other height, h_a, in metres, the necessary number of holes in the sieve is then

$$n_a = n \cdot \frac{\sqrt{0{\cdot}010}}{\sqrt{h_a}} = \frac{0{\cdot}1\, n}{\sqrt{h_a}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (183)$$

Accordingly, if n holes are necessary to pass a certain volume of water, when the height of the water is 10 mm, the number of holes, n_a, required to pass the same quantity of water, when it stands at some other height, h_a, is

$h_a =$	15	30	40	50	200 mm.
$n_a =$	$0{\cdot}82n$	$0{\cdot}58n$	$0{\cdot}5n$	$0{\cdot}447n$	$0{\cdot}224n$

6. *The Diameter of the Steam Pipe.*

The weight of steam, D, to be condensed in a certain time is known in each case, as also the desired vacuum. The diameter of the pipe conveying the steam can therefore be found from Table 32 (Chapter XVII.). It is there assumed, in calculating the bore of the pipe, that it is 20 m. long, and that the loss of pressure is 0·5 per cent. If the pipe leading from the evaporator to the condenser has another length, l_a, the weight of steam passing with 0·5 per cent. loss of pressure is obtained by multiplying that given in Table 32 by $\sqrt{\frac{20}{l_a}}$. If a greater loss of pressure is allowed in order that a narrower pipe may be used, the weight of steam passing through the pipe with z_a per cent. loss of pressure is obtained by multiplying that given in Table 32 by $\sqrt{\frac{z_a}{0{\cdot}5}}$.

For another length, l_a, and another loss of pressure, z_a, the weight of steam passing through the pipe in one hour is obtained by multiplying the weight in Table 32 by $\sqrt{40\frac{z_a}{l_a}}$.

Example.—Through a pipe 20 m. long and 200 mm. in diameter, at a vacuum of 750 mm., and with 0·5 per cent. loss of pressure, 124 kilos. of steam pass in one hour. Through a similar pipe, $l_a = 30$ m. long, and with 5 per cent. loss of pressure allowed, pass

$$D = 124\sqrt{\frac{40 z_a}{l_a}} = 124\sqrt{\frac{5 \times 40}{30}} = 318{\cdot}47 \text{ kilos. of steam.}$$

7. *The Diameter of the Air Pipe.*

The diameter of the pipe leading from the condenser to the air-pump is determined by the hourly weight of air to be exhausted, which we assume (somewhat extravagantly, see Chapter XXIII.) to be 0·25

kilo. per 1000 kilos. of injected water. Table 35 gives the weight of air passed through pipes of various diameters, 20 m. long, with 0·5 per cent. loss of pressure, in one hour. For any other length, l_a, and another loss of pressure, z_a, the weights given in Table 35 are to be multiplied by $\sqrt{\frac{40 z_a}{l_a}}$ in order to obtain the weights of air conveyed under these conditions.

8. *The Heating of the Injected Water.*

The injected water is heated through the medium of its surface by the steam, with which it comes into direct contact. The greater the surface of a quantity of water in proportion to its volume, the more rapidly will it be heated by the surrounding steam. With regard to this point, the division of the water in the jet-condenser may be effected in four different ways:—

The cooling water may flow over surfaces across which passes the steam to be condensed.

It may fall down in plane or curved sheets, which are in contact with the steam on both sides.

It may fall in jets into the steam in the condenser.

It may be sprinkled into the condenser in the form of drops.

The ratio of the surface of the water to its volume depends on the thickness of the sheets of flowing or falling water and on the diameter of the jets or drops. The following short Table 45 has been arranged in order to form an idea of these conditions. The ratio is given of the surface (o) in sq. mm. to the volume in cub. mm. (i) for thicknesses (δ) or diameters (δ) of 2-10 mm.

Of the conditions considered here, assumed by the water in the condenser, the ratio of the surface to the volume $\left(\frac{o}{i}\right)$ is the least in the case of water flowing over surfaces and the greatest in the case of spherical drops. Thus water divided into drops will *ceteris paribus* most rapidly acquire the temperature of the surrounding steam in a condenser. Regarded from this point of view, it would be best to spray the water into the condenser in the smallest drops possible; but this is not easily effected, since it is difficult to divide water up into uniform drops.

TABLE 45.

The surface and volume and their ratio, of flowing and falling sheets, jets and drops of water.

Thickness or diameter, δ		2	3	4	5	6	7	8	9	10
Surface of sphere	o	12·56	23·27	50·2	78·5	113·08	153·92	201·04	254·47	314·16
Volume of sphere	i	4·1887	14·137	85·51	65·43	113·08	1·796	268·07	381·8	523·58
Surface of jet	o	12·56	28·27	50·2	78·5	113·08	153·92	201·04	254·4	314·16
Volume of jet	i	6·28	21·2	50·2	98·15	169·6	269·3	401	572	785
Sheet (flowing)	$\frac{o}{i}$	0·5	0·333	0·25	0·2	0·1667	0·1429	0·125	0·111	0·1
Sheet (falling)	$\frac{o}{i}$	1·0	0·667	0·5	0·4	0·333	0·2859	0·25	0·222	0·2
Jet	$\frac{o}{i}$	2	1·333	1·0	0·80	0·666	0·5718	0·5	0·4447	0·4
Drop	$\frac{o}{i}$	3	2	1·5	1·2	1·00	0·855	0·75	0·666	0·6
Sheet (flowing)	$\frac{i}{o}$	2	3	4	5	6	7	8	9	10
Sheet (falling)	$\frac{i}{o}$	1	1·5	2	2·5	3	3·5	4	4·5	5
Jet	$\frac{i}{o}$	0·5	0·75	1	1·25	1·5	1·75	2	2·25	2·5
Drop	$\frac{i}{o}$	0·333	0·50	0·666	0·883	1	1·17	1·333	1·5	1·666

All methods of distributing water are employed in condensers; thus it is important to consider each, and to see what time each requires in order that the injected water may be heated from its original low temperature to the desired higher temperature.

In most cases heat is transferred to liquids by means of movements, circulations and currents, naturally or artificially produced in them; but in this case, in which the water falls free, such movements cannot be assumed, since, apart from the friction exerted by the steam on its surface, and the motions due to the vibrating opening of the orifices, only gravity acts upon the particles of water. This force, on account of the complete uniformity of its action on all parts, cannot cause internal movements. Thus the heat is transferred from the exterior to the interior of the masses of water principally by *conduction.*

The conductivity of water for heat is very low. According to several concordant researches its coefficient, $\lambda = 0{\cdot}093$ gram-calories (*i.e.*, per 1 sq. cm., 1 minute, 10 mm. thickness of the water layer and 1° C. difference in temperature on the two sides of the mass of water) or

$$\lambda = \frac{0{\cdot}093 \times 10{,}000 \times 10}{60 \times 1000} = 0{\cdot}155 \text{ calories}$$ (*i.e.*, per 1 sq. m., 1 second 1 mm. thickness and 1° difference in temperature); or in other words, through a layer of water 1 sq. m. in surface and 1 mm. thick, the two surfaces of which are kept constantly at a difference in temperature of 1° C., 0·155 calories pass in 1 second.

It will further be assumed that the quantity of heat passing through a layer of water in the condition of equilibrium is directly proportional to the section (Q in sq. m.), the time (z, in seconds), the constant difference of temperature (θ_a in °C.), and inversely proportional to the thickness of the layer of water to be penetrated (η in mm.). Thus *in the condition of equilibrium,*

$$C = \frac{Q\lambda z\theta_a}{\eta} \text{ calories} \quad . \quad . \quad . \quad . \quad . \quad . \quad (184)$$

However, in warming water, which is falling in a condenser in the form of sheets, jets or drops, we have not to do with a condition of equilibrium, but with the initial period of the heating, in which the heat penetrates the water from outside by conduction. In this period it is true that the temperature difference between the steam and the last layer just reached by the heat wave is constant $= \theta_a$, but the resistance, which the thickness of the sheet of water opposes to the

penetration of the heat, is zero at the commencement of the heating (at the surface) and increases with the depth, η, to which the heat has penetrated. The thickness of the sheet of water is on the average only $\frac{\eta}{2}$. The quantity of heat, which all the more or less heated layers *together* have taken up, is equal to the weight of these layers multiplied by the average increase in temperature of all layers (if $\sigma_f = 1$).

The equation for the initial period of the heating has thus the following form:—

$$C = \frac{Q\lambda z_t \theta_a}{\frac{\eta}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (185)$$

Now the heat does not advance from the surface into the interior in such a manner that the thin layer first in contact with the steam *completely* acquires its temperature, and then a second, third, etc., acquire the same temperature. The process is that the layer of contact first acquires a small increase in temperature, which gradually rises, but during this rise in temperature the first layer is already communicating heat to the second, this to the third, and so on. Whilst the heat advances in succession from one layer to the following colder layers, the already heated layers are becoming hotter and hotter at the same time. The law is: *As the distance from the surface of contact (between the two substances which are becoming equal in temperature) increases in arithmetical progression, the temperature decreases in geometrical progression.*

The decrease in temperature from layer to layer follows the same law as the decrease in the temperature difference from moment to moment in heating by steam, as explained in Chapter I.

At the commencement of heating water by conduction, after the layer of contact has almost attained the temperature of the steam, the temperatures of the following layers increase at first rapidly, then very slowly.

The average *rise* in temperature of the mass of the water at the commencement of heating may be determined, as in Chapter I., by equation (8), but it may also be found in a finite manner, with tolerable accuracy, just as the mean temperature difference was there found.

If the whole difference in temperature between steam and water

at first be θ_a, then, after a certain time, when the heat has penetrated the water to some distance, and assuming that the sections of the layers remain of equal size, the difference in temperature

Between the steam and the first layer $= x\theta_a$.

„ first and second layers $= x(\theta_a - x\theta_a) = x\theta_a(1 - x)$.

„ second and third layers $= x\{(\theta_a - x\theta_a) - x\theta_a(1 - x)\}$.
$= x\theta_a(1 - x)^2$.

„ last but one and the last layer $= x\theta_a(1 - x)^{n-1}$.

If, as in Chapter I., we represent by θ_e the difference in temperature between the last, or nth, layer, which is just warmed, and the first layer, which is not warmed at all, then from the above considerations, just as before,

$$x = 1 - \sqrt[n]{\frac{\theta_e}{\theta_a}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (186)$$

We may now, just as before with the *differences* in temperature, sum the *increases* in temperature of the single layers, and divide by the number of layers, in order to obtain the average increase in temperature. The increases in temperature of the single layers are:—

Of the first layer - - θ_a.

„ second layer - $\theta_a - x\theta_a = \theta_a(1 - x)$.

„ third „ - $\theta_a(1 - x)^2$.

„ nth „ - $\theta_a(1 - x)^{n-1}$.

The sum

$$S_e = \theta_a\{1 + (1 - x) + (1 - x)^2 + (1 - x)^3 + \ldots + (1 - x)^{n-1}\}.$$

Thus the mean increase in temperature of the water is

$$t_m = \frac{\theta_a - \theta_e}{n\left(1 - \sqrt[n]{\frac{\theta_e}{\theta_a}}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (187)$$

If we now express, as before, θ_e as a fraction of θ_a, then $\frac{\theta_e}{\theta_a}$ is always a proper fraction. The value of $\frac{\theta_e}{\theta_a}$ must, in fact, with an infinite number of layers, almost become zero. We assume its value, on account of the finite nature of our calculation, as in Chapter I., to be $0{\cdot}01 = 1$ per cent. The inaccuracy is not of much importance.

The average, or mean, increase in temperature, t_{cm}, of the 100 ideal parallel and equal layers in the sheet of water is, assuming that the whole difference in temperature at the beginning is θ_a and at the end is $\theta_e = 0{\cdot}01\theta_a$, according to Table 1, $t_{cm} = 0{\cdot}215\theta_a$.

The quantity of heat which the water has absorbed, when it is heated to the depth, η, in mm., is therefore

$$C = 0{\cdot}215\theta_a Q\eta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (188)$$

Now, in order to obtain an expression for the time, $z_{,,}$ during which the quantity of heat, C, has penetrated through the surface (or section), Q, at the constant difference in temperature, θ_a, into a sheet of water to the depth, η, the expressions (185) and (188) are put equal to one another. We obtain

$$2Q\frac{\lambda}{\eta}z_{,}\theta_a = 0{\cdot}215\theta_a Q\eta \quad . \quad . \quad . \quad . \quad . \quad . \quad (189)$$

$$2\lambda z_{,} = 0{\cdot}215\eta^2;$$

or, since

$$\lambda = 0{\cdot}155,$$

$$z_{,} = 0{\cdot}694\eta^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (190)$$

and

$$\eta = \sqrt{\frac{z_{,}}{0{\cdot}694}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (191)$$

Equation (190) gives the time, $z_{,,}$ in seconds, in which a sheet of water, η mm. thick, heated by steam on one side, acquires the temperature of the steam on the heated side and is just beginning to get warmer on the other side.

From equation (191) the thickness, η, of the sheet which is heated in this manner in the time, $z_{,,}$ may be calculated. It is seen very plainly from equations (190) and (191) that the steam rapidly heats the external layers of the water with which it is in contact, and that the heat then proceeds only slowly (at a speed inversely as the square of the thickness) into the interior of the body of water.

The principal quantity of heat, which is conducted in a definite time into the water, remains in and near the outer layers. Little heat is transmitted to the interior, and this little only after the lapse of time.

From these considerations follow the conditions for a rapid heating of water to a high temperature by direct contact with steam :—

1. The surface of the water must be very great.
2. The surface must rapidly change.
3. The period of contact between steam and water must be as long as possible.

In order to express these statements precisely in figures, Table 46 is added. It gives the depth in mm. to which the heat penetrates in 0·1-1·2 seconds into a sheet of water in contact with steam on one side, the number of calories which are taken up in this time, and to what fraction of the total difference in temperature, θ_a, the total quantity of water, 1·7 mm. thick, would be heated if the heat were supposed to be uniformly distributed throughout. These values are given for sheets, jets and spheres.

It is clearly seen from Table 46, that the quantity of heat which enters in no way increases proportionately with the time, but that much more heat is taken up by the water at the first contact than later.

If the heat has entered a *sheet* of water from one surface and has warmed it (decreasingly) only to the depth η, of the whole thickness, δ, then, as we have seen, the quantity of heat which has entered is as great as if the volume, $Q\eta$, of a portion of the sheet had received the increase in temperature, $0{\cdot}215\theta_a$, or as if the *whole* sheet of thickness, δ, had attained the increase in temperature of

$$t_{\eta} = \frac{\eta}{\delta} 0{\cdot}215\theta_a \text{ in } {}^\circ\text{C.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (192)$$

In a jet (cylinder) of diameter, δ, which is heated from its surface, the heat spreads as in a sheet. But since the volumes of the cylindrical layers decrease from outside inwards, and also the temperatures of the layers, we obtain the following equation, if t_{cc} be the hypothetical increase in temperature of the whole jet:—

$$t_{cc} = \frac{\delta^2\pi}{4} = 0{\cdot}215\theta_a\eta(\delta - 2 \times 0{\cdot}2\eta)\pi \quad . \quad . \quad . \quad . \quad (193)$$

or

$$t_{cc} = \frac{0{\cdot}86\theta_a\eta(\delta - 0{\cdot}4\eta)}{\delta^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (194)$$

In drops (spheres) something similar takes place. The average increase in temperature, t_{ck}, is found by multiplying the volume of the heated hollow sphere by its mean increase in temperature and dividing by the volume of the whole drop. The volume heated is equal to the thickness of the heated hollow sphere multiplied by the central surface of that sphere.

$$t_{ck}\frac{\delta^3\pi}{6} = 0{\cdot}215\theta_a\eta(\delta - 2 \times 0{\cdot}2\eta)^2\pi \quad . \quad . \quad . \quad . \quad . \quad (195)$$

$$t_{ek}\delta^3 = 6 \times 0{\cdot}215\theta_a\eta(\delta - 2 \times 0{\cdot}2\eta)^2$$

$$t_{ek} = \frac{1{\cdot}29\theta_a\eta(\delta - 0{\cdot}40\eta)^2}{\delta^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (196)$$

Table 46 gives, in column 3, the depth, η, to which, according to equation (191), the heat would penetrate in $z_t = 0{\cdot}1\text{-}1{\cdot}2$ seconds into a sheet of water warmed on one side, and in column 4 the quantity of heat in calories which enters in this time through 1 sq. m. of the water surface with a temperature difference of $\theta_a = 1°$ C. Columns 6-12 give, for sheets of water, jets and drops of $\delta = 1\text{-}7$ mm. thickness or diameter respectively, the mean increase in temperature of the whole mass in the times given, for each 1° difference in temperature.

It is clearly seen from this Table 46 that the greatest transference of heat takes place at the moment of contact of water and steam, and that it then becomes much slower, since the difficulty experienced by the heat in entering the water increases with the depth.

It is not maintained that this method of consideration, and the conclusions drawn therefrom, lead to infallible figures to be at once applied in construction. They appear, however, to approach very nearly to the truth and to give very valuable indications.

9. *The Volumes occupied by 1 kilo. of Air at Various Pressures below 1 Atmosphere and at Various Temperatures.*

In determining the dimensions of condenser and air-pump, it is necessary to know the volume occupied by 1 kilo. of air under diminished pressure and at various temperatures. Table 47 gives these volumes for most ordinary cases. It has been calculated in the following manner:—

Let γ_l = the weight of 1 cub. m. of air in kilos.,

a_l = the volume of 1 kilo. of air in cub. m.,

t_l = the temperature of the air in ° C.,

T = the absolute temperature,

$= \frac{1}{a} + t_l$, in which a is the coefficient of expansion of air.

According to Dronke, for air under very low pressures $\frac{1}{a} = 274{\cdot}6$. Therefore $T = 274{\cdot}6 + t_l$,

p = the mean atmospheric pressure = 10,336 kilos. per sq. m., when the barometer stands at 760 mm.,

R = a constant, which for air is 29·27.

TABLE 46.

The heating of sheets, jets and drops of water by direct contact with steam.

The depth, η, to which the heat penetrates in the time, z_t (column 3).

The fraction of the original difference in temperature, through which the whole mass of the water is warmed in the times, z_t = 0·1-1·2 seconds ($t_{mt}\theta_a$ for $\theta_a = 1$).

Period of heating. z_t secs.	Height of fall in the time, z_t. h mm.	The distance to which the heat penetrates in the time, z_t. η mm.	Heat which passes through 1 sq. m. in z_t seconds at 1° temperature difference. Calories.	Sheet (S). Jet (J). Drops (D).	Thickness or diameter, δ, in mm., of the sheets, jets, or drops. 1	2	3	4	5	6	7
					Mean increase in temperature, t_{mt} of the mass of water for $\theta_a = 1$.						
0·1	49·05	0·38	0·085	S	0·085	0·043	0·028	0·022	0·017	0·014	0·012
				J	0·272	0·148	0·102	0·079	0·061	0·052	0·043
				D	0·358	0·204	0·138	0·113	0·089	0·078	0·062
0·2	196·2	0·532	0·116	S	0·115	0·058	0·038	0·029	0·023	0·019	0·017
				J	—	0·205	0·142	0·109	0·088	0·074	0·064
				D	—	0·270	0·204	0·151	0·121	0·106	0·092
0·285	400	0·640	0·138	S	0·138	0·069	0·046	0·034	0·028	0·023	0·020
				J	—	0·240	0·156	0·129	0·104	0·088	0·076
				D	—	0·312	0·230	0·179	0·143	0·126	0·102
0·30	441	0·660	0·141	S	0·141	0·070	0·047	0·035	0·028	0·024	0·020
				J	—	0·247	0·172	0·133	0·105	0·090	0·078
				D	—	0·319	0·236	0·184	0·147	0·128	0·105
0·35	598	0·710	0·153	S	0·153	0·077	0·051	0·039	0·031	0·026	0·022
				J	—	0·261	0·184	0·142	0·115	0·091	0·083
				D	—	0·334	0·251	0·196	0·157	0·139	0·113
0·40	785	0·756	0·164	S	0·164	0·082	0·055	0·041	0·033	0·028	0·023
				J	—	0·276	0·195	0·150	0·120	0·104	0·090
				D	—	0·351	0·265	0·206	0·166	0·147	0·119
0·45	993	0·808	0·173	S	0·173	0·087	0·058	0·044	0·035	0·029	0·025
				J	—	0·293	0·220	0·160	0·135	0·110	0·095
				D	—	0·360	0·276	0·217	0·176	0·156	0·125
0·50	1226	0·848	0·183	S	0·183	0·092	0·061	0·046	0·037	0·031	0·026
				J	—	0·314	0·222	0·175	0·140	0·118	0·101
				D	—	0·375	0·288	0·227	0·184	0·163	0·130

TABLE 46—(*continued*).

Period of heating. z_t secs.	Height of fall in the time, z_t. h mm	The distance to which the heat penetrates in the time, z_t. η mm.	Heat which passes through 1 sq. m. in z_t seconds at 1° temperature difference. Calories.	Sheet (S). Jet (J). Drops (D).	Thickness or diameter, δ, in mm., of the sheets, jets or drops. 1	2	3	4	5	6	7
					Mean increase in temperature, t_{me}, of the mass of water for $\theta_a = 1$.						
0·60	1766	0·930	0·200	S	0·200	0·100	0·067	0·050	0·040	0·034	0·029
				J	—	0·325	0·233	0·182	0·150	0·125	0·108
				D	—	0·396	0·308	0·244	0·200	0·176	0·143
0·70	2403	1·0	0·217	S	0·217	0·109	0·073	0·055	0·044	0·037	0·031
				J	—	0·344	0·248	0·194	0·158	0·134	0·116
				D	—	0·412	0·314	0·259	0·212	0·188	0·152
0·80	3139	1·070	0·231	S	—	0·116	0·077	0·058	0·046	0·039	0·033
				J	—	—	0·263	0·199	0·170	0·140	0·123
				D	—	—	0·338	0·272	0·223	0·199	0·161
0·90	3971	1·41	0·245	S	—	0·123	0·082	0·062	0·049	0·041	0·035
				J	—	—	0·277	0·216	0·177	0·151	0·135
				D	—	—	0·351	0·286	0·234	0·210	0·170
1·0	4905	1·20	0·259	S	—	0·129	0·086	0·065	0·052	0·043	0·037
				J	—	—	0·290	0·227	0·190	0·160	0·137
				D	—	—	0·364	0·299	0·245	0·219	0·178
1·1	5935	1·26	0·271	S	—	0·136	0·090	0·068	0·054	0·045	0·039
				J	—	—	0·304	0·240	0·199	0·170	0·147
				D	—	—	0·374	0·306	0·254	0·228	0·187
1·2	6953	1·315	0·283	S	—	0·142	0·091	0·071	0·057	0·046	0·041
				J	—	—	0·311	0·245	0·201	0·171	0·150
				D	—	—	0·384	0·314	0·263	0·236	0·192

Then the law is

$$\frac{a_l p}{T} = R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (197)$$

The volume of 1 kilo. of air at the pressure, p, and the temperature, t_l, is therefore

$$a_l = \frac{1}{\gamma_l} = \frac{29{\cdot}27(274{\cdot}6 + t_l)}{p} \quad . \quad . \quad . \quad . \quad . \quad (198)$$

TABLE 47.

The volumes, in cub m., of 1 kilo. of air, at absolute pressures of $b =$ temperatures

Temperature t_l	Vacuum.										
	757·39	755	753	750	748	745	743	740	735	730	725
	Absolute pressure, b.										
	2·61	5	7	10	12	15	17	20	25	30	35
	Volumes, a_l, in cub m, of 1 kilo. of air.										
5	170·35	120 16	85·53	60·08	50·07	40·06	35·34	30·05	24·02	20·02	17·16
10	174·46	122·31	87·37	61·16	50·97	40·79	35·97	30·58	24·46	20·39	17·47
15	178·58	124·45	88·90	62·23	51·86	41·51	36·60	31·11	24·88	20·47	17·77
20	182·69	126·60	90·44	63·31	52·76	42·25	37·24	31·66	25·31	21·10	18·09
25	186·81	128·74	91·97	64·38	53·65	42·97	37·87	32·20	25·73	21·45	18·39
30	190·93	130·91	93·51	65·45	54·55	43·70	38·50	32·73	26·16	21·81	18·70
35	195·04	133·06	95·04	66·52	55·44	44·42	39 14	33·27	26·58	22·16	19·00
40	199·16	135·21	96 58	67·60	56·34	45·14	39·77	33·80	27·02	22·53	19·31
45	203·27	137·36	98·11	68·67	57·24	45·87	40·40	34·34	27·44	22·88	19·61
50	207·39	139·51	99·65	69·75	58·13	46·60	41·03	34·88	27·87	23·25	19·93
55	211·51	141·67	101·67	70·81	59·02	47·32	41·67	35·42	28·29	23·60	20·23
60	215·63	143·8	102·72	71·90	60·12	48·05	42·30	35·92	28·75	23·96	20·54

When the barometer is at b mm. of mercury, the absolute pressure on 1 sq. m. is

$$p = \frac{10,336b}{760} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (199)$$

Thus the volume of 1 kilo. of air is

$$a_t = \frac{2·15(274·6 + t_l)}{b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (200)$$

Table 47 has been calculated by inserting the various values for b and t_l.

TABLE 47.

2·61-210 mm. of mercury, *i.e.*, at vacua of 757·39-550 mm., and at from 5°-60° C.

Vacuum.											Temperature. t_l
720	715	710	705	700	695	690	685	680	675	670	
Absolute pressure, b.											
40	45	50	55	60	67	70	75	80	85	90	
Volumes, a_t, in cub. m., of 1 kilo. of air.											
15·01	13·34	12·00	10·92	10·00	9·27	8·58	8·01	7·51	7·07	6·67	5
15·29	13·59	12·23	11·12	10·19	9·44	8·74	8·15	7·64	7·19	6·79	10
15·55	13·82	12·43	11·32	10·36	9·60	8·89	8·29	7·78	7·32	6·91	15
15·82	14·06	12·65	11·51	10·55	9·77	9·04	8·44	7·91	7·44	7·03	20
16·08	14·29	12·85	11·70	10·55	9·93	9·20	8·58	8·04	7·57	7·15	25
16·36	14·54	13·08	11·90	10·91	10·00	9·35	8·72	8·18	7·70	7·27	30
16·62	14·77	13·28	12·08	11·08	10·26	9·50	8·87	8·31	7·82	7·39	35
16·89	15·01	13·51	12·30	11·28	10·43	9·66	9·04	8·44	7·95	7·51	40
17·15	15·24	13·71	12·48	11·44	10·59	9·81	9·15	8·58	8·07	7·63	45
17·43	15·49	13·94	12·68	11·63	10·76	9·97	9·31	8·77	8·20	7·75	50
17·69	15·72	14·14	12·87	11·79	10·92	10·12	9·45	8·84	8·33	7·87	55
17·97	15·97	14·37	13·07	11·98	11·09	10·27	9·58	8·98	8·46	7·99	60

10. *The Time of Fall of the Injected Water.*

In Table 48 are given the distances through which drops of water fall in 0·05-1·7 secs., when gravity alone acts on them, without the interference of currents of steam or gas. It is seen that water, when it falls free, passes through condensers even 4 m. high in 0·9 sec., and remains a still shorter time in lower condensers.

If the current of steam moves downwards in the same direction as the water (wet condensers), the time of fall is somewhat further decreased, but if the steam moves upwards against the falling water (dry counter-

TABLE 47—*(continued)*.

Temperature. t_l	Vacuum.											
	665	660	655	650	645	640	635	630	625	620	615	610
	Absolute pressure, b.											
	95	100	105	110	115	120	125	130	135	140	145	150
	Volumes, a', in cub. m., of 1 kilo. of air.											
5	6·32	6·01	5·72	5·46	5·22	5·00	4·80	4·62	4·45	4·29	4·14	4·00
10	6·44	6·12	5·825	5·56	5·32	5·09	4·89	4·70	4·53	4·37	4·22	4·08
15	6·55	6·22	5·92	5·66	5·41	5·18	4·97	4·78	4·61	4·44	4·29	4·15
20	6·67	6·33	6·03	5·75	5·50	5·27	5·06	4·87	4·69	4·52	4·36	4·22
25	6·78	6·44	6·13	5·85	5·60	5·36	5·15	4·95	4·77	4·60	4·44	4·29
30	6·88	6·546	6·24	5·95	5·69	5·45	5·23	5·03	4·85	4·68	4·51	4·36
35	7·00	6·66	6·33	6·05	5·79	5·54	5·32	5·11	4·93	4·75	4·58	4·44
40	7·11	6·76	6·44	6·15	5·88	5·63	5·41	5·20	5·01	4·83	4·66	4·51
45	7·22	6·87	6·54	6·24	5·97	5·72	5·50	5·28	5·08	4·90	4·73	4·58
50	7·34	6·98	6·65	6·34	6·07	5·80	5·58	5·36	5·17	4·98	4·80	4·65
55	7·45	7·08	6·75	6·44	6·17	5·89	5·67	5·44	5·24	5·06	4·88	4·72
60	7·57	7·19	6·85	6·53	6·25	5·98	5·74	5·53	5·33	5·14	4·95	4·79

current condensers), the time is somewhat longer. In any case large drops of water can experience but a slight and insufficient heating in this short time, as Table 46 shows. Since the distances fallen through in the first moments are much smaller than those in the succeeding moments, steps or catch-plates, placed at short distances apart, and continually bringing the water again to rest after brief intervals of falling, serve to lengthen considerably the time of fall.

By the aid of the preceding separated considerations of the requirements of jet-condensers, we can now determine their principal dimensions for the most usual cases; this is done in Tables 49 and 51. The principles upon which these tables have been calculated must first be briefly indicated.

TABLE 47—(*continued*).

Vacuum.												Temperature.
605	600	595	590	585	580	575	570	565	560	555	550	
Absolute pressure, b.												
155	160	165	170	175	180	185	190	195	200	205	210	
Volumes, a_l, in cub. m., of 1 kilo. of air.												t_l
3·87	3·75	3·64	3·53	3·43	3·33	3·24	3·16	3·08	3·00	2·93	2·86	**5**
3·94	3·82	3·70	3·60	3·49	3·39	3·30	3·22	3·14	3·06	2·98	2·91	**10**
4·01	3·89	3·77	3·66	3·56	3·45	3·36	3·27	3·18	3·10	3·03	2·97	**15**
4·08	3·95	3·83	3·72	3·62	3·52	3 42	3·33	3·24	3·16	3·08	3·01	**20**
4·15	4·02	3·90	3·79	3·68	3·57	3·48	3·39	3·30	3·22	3·14	3·06	**25**
4·22	4·09	3·97	3·85	3·74	3·63	3·53	3·44	3·35	3·27	3·19	3·12	**30**
4·29	4·15	4·03	3·91	3·80	3·69	3·59	3·49	3·40	3·32	3·24	3·17	**35**
4·36	4·22	4·09	3·97	3·86	3·75	3·65	3·55	3·46	3·37	3·29	3·22	**40**
4·43	4·29	4·16	4·04	3·92	3·81	3·70	3·61	3·52	3·43	3·34	3·27	**45**
4·50	4·36	4·23	4 10	3·98	3·87	3·77	3·67	3·58	3·49	3·40	3·22	**50**
4·57	4·42	4·29	4·16	4·05	3·93	3·82	3·73	3·63	3 54	3·45	3·37	**55**
4·64	4 49	4·35	4·23	4·11	3·99	3·88	3·78	3·68	3·60	3·50	3·42	**60**

11. *The Dimensions of Wet (Parallel-Current) Jet-Condensers.*

Wet condensers are used with advantage in connection with evaporators of small and medium capacity, evaporating 100-3000 kilos. per hour, for which limits Table 49 has been calculated (Fig. 14, p. 210).

The wet parallel-current condenser is a closed vessel, which is entered at the top by the steam to be condensed and the cooling water, and from which the liquefied vapours, the heated cooling water and the uncondensed gases are together exhausted by means of a "wet" air-pump. The diameter and height of the condenser and the diameter of the pipes, by which the steam and water enter and the water leaves, are to be calculated.

TABLE 48.

Distance in mm. traversed in a free fall during 0·05-1·7 seconds.

Time, z_t. sec.	Height of fall. mm.	Time, z_t. sec.	Height of fall. mm.	Time, z_t. sec.	Height of fall. mm.	Time, z_t. sec.	Height of fall. mm.
0·05	12·5	0·30	441·45	0·775	2943	1·25	7663
0·06	17·62	0·325	517·4	0·80	3139	1·275	7947
0·07	23·8	0·35	597·9	0·825	3335	1·30	8289
0·08	31·36	0·375	699	0·85	3541	1·325	8604
0·09	39·69	0·40	784·8	0·875	3751	1·35	8936
0·10	49·05	0·425	884·9	0·90	3971	1·375	9260
0·11	59·35	0·45	993·2	0·925	4193	1·40	9613
0·12	70·6	0·475	1105·4	0·95	4414	1·425	9947
0·13	82·8	0·50	1226·3	0·975	4658	1·45	10000
0·14	96·1	0·525	1350·4	1·00	4905	1·475	10657
0·15	110·4	0·55	1483·7	1·025	5169	1·50	10996
0·16	125·5	0·575	1629·9	1·05	5507	1·525	11417
0·17	141·7	0·60	1765·8	1·075	5659	1·55	11823
0·18	158·9	0·625	1926	1·10	5935	1·575	12132
0·19	177·1	0·65	2069	1·125	6188	1·60	12544
0·20	196·2	0·675	2232	1·15	6483	1·625	12936
0·225	247·9	0·70	2403	1·175	6771	1·650	13343
0·25	306·5	0·725	2575	1·20	6953	1·675	13750
0·275	370·4	0·75	2756	1·225	7350	1·70	14161

This species of condenser is called "wet," since it is always connected with a "wet" air-pump, *i.e.*, an air-pump which exhausts the water together with the air.

"Dry" condensers are so called because they are connected with a "dry" air-pump, *i.e.*, a pump which extracts only air, without water. The waste water of dry condensers generally passes away by its own weight by means of a fall-pipe (Fig. 15, see observations on p. 208).

A wet condenser should never be connected with a dry air-pump, which cannot take the waste water.

The diameter of the steam-pipe leading to the condenser may be found by means of Table 32, in which is given the weight of steam passing in one hour through pipes 20 m. long with a loss of pressure of 0·5

per cent. In settling the conditions for Table 49 we have, however, assumed that the resistance in the pipe between evaporator and condenser may take 2 per cent. of the absolute pressure. In this case double the quantity of steam passes through the same pipe, and for the desired capacity the pipe will be narrower and therefore cheaper. This condition is taken because in reality the assumed high vacuum (705 mm.) is not always maintained, and since, in order to meet fluctuations in working, condensers are generally made very large in proportion to the work required of them Steam-pipes of very much smaller diameter are frequently found.

The difference in temperature between steam and cooling water, when they enter at the top, ranges between about 55°-30° C.

The temperature difference at the end (bottom) is 35°-20° C., since the waste water should never be allowed to become very warm. The temperature difference at the bottom accordingly is to that at the top in the ratio $\frac{35}{55}$ or $\frac{20}{30}$, *i.e.*, at the mean, is about 0·66 of the difference at the top. The cooling water is therefore only heated through about $\frac{1}{3}$ of the original difference in temperature between steam and water, or $t_{*} = 0{\cdot}33\theta_a$, for which the following times are sufficient, according to Table 46, for drops of

δ = 1	2	3	4	mm. diameter
z = 0·1	0·3	0·6	1·1	seconds.

In order that the drops may be in the condenser during these times, the following heights of free fall are necessary:—

$$h = 49 \quad 441 \quad 1765 \quad 5935 \text{ mm.}$$

When the water is very finely divided, a very short time suffices to warm it; for drops of 1-2½ mm. diameter, condensers 1000 mm. high, without steps, are approximately sufficient. Much larger drops cannot be sufficiently heated by similar condensers of great height. Experience shows that in practice, when the water is well divided, good results are obtained with these dimensions. If thicker masses of water are intended, one step is, in general, sufficient.

The free section of the wet condenser need not be much greater than that of the steam pipe, if the latter has the proper dimensions; but it may be larger without harm, since the velocity of the steam diminishes in the condenser, from its entrance downwards, to zero, and is on the average about half as large as at its entrance.

The section of the condenser is generally diminished by the pipe through which the water is injected, and also by the jets and drops of water. Since the friction of the great number of particles of water against the current of steam is not inconsiderable, it is well to enlarge the section of the condenser correspondingly, in order to prevent loss of pressure. For condensers without steps we adopt a section about 20 per cent. greater than that of the steam pipe of liberal dimensions. If there are one or two steps in the condenser, the section must be at least double that of the pipe by which the steam enters.

The mean pressure, which the current of steam exerts on the falling drops in their direction of motion, increasing their acceleration and thus decreasing the time during which they are falling through the condenser, is calculated only at about one-quarter of that which the entrant velocity of the steam would exert; this is because the drops, by their velocity of fall, themselves diminish the influence of this pressure. Even if the velocity of the steam on entering the top of the condenser were 30 m. per second, it would only slightly shorten the time of fall of small drops of 2 mm. diameter, and this all the less when the drops, thrown violently about, touch the walls and are retarded.

The internal height of condensers without steps, from the steam entrance to the water exit, is therefore taken for small apparatus at not less than 1000 mm., and somewhat greater for larger apparatus, since in the latter the water is not perhaps quite so thoroughly divided. This height is also sufficient when one step is introduced. With two steps the total height may be 1·25 times as great.

The diameter of the water-pipe. The limits of the temperature of the steam to be condensed are about 40°-45° C., the limits of the initial temperature of the injected water are about 8°-25° C. Thus we find from Table 41 that the condensation of the steam rarely requires more, and generally much less, cooling water than 45 times the weight of the steam.

The water may be conveyed to the condenser from a tank at a more or less high level in such a manner that the natural suction of the vacuum in the condenser, together with the hydrostatic pressure from the condenser to the tank, causes the velocity of the water in the supply pipe. The suction of the condenser alone may also draw the water direct from a vessel, well or tank at a lower level (Chapter XVIII.).

In the former case the pressure which moves the water is con-

siderable, being equal to the vacuum (measured in metres of water column) *plus* the hydrostatic pressure. In the latter case it is very small, being equal to the vacuum *minus* the distance from the water level to the point at which the water enters the condenser. It is not advisable to employ a lower pressure than 3 m., since, otherwise, variations in the level of the water and in the vacuum may be dangerous, although it is always possible to work with a very slight excess of pressure, even only 200-300 mm. In that case, however, very wide supply pipes must be used, and there arises the danger that the supply of water to the condenser may be stopped by any accident. With a vacuum of 680 mm. of mercury (9·248 m. of water) the *greatest* permissible normal depth of the water level below the water entrance into the condenser would be 9·248 − 3·0 = 6·248 m.

In Table 49 are given, by the aid of Table 36, the diameters of the water supply pipe for the four cases of an excess pressure of 1, 3, 6, and 9 m., and under the assumption that the largest quantity of water mentioned (45 times the weight of the steam) is to be introduced into the condenser.

The spraying of the water in the condenser is generally accomplished by means of perforated pipes or plates. The holes in the pipes and plates should be small, since the water always passes through them at a considerable velocity, on account of the tolerable excess of pressure. The number of holes has been calculated for diameters of 2 and 3 mm.

If the injector pipes are vertical and enter from below, too many holes are no disadvantage, since, when a number of them remain unused, the water is still well divided.

The injector pipe must be closed at the end in the condenser, so that the water may remain in it under at least a part of the excess of pressure. The water will then be thrown, with a certain velocity, from the small holes on to the condenser wall, where it is broken up into fine drops. A portion of the water will doubtless flow down the condenser wall, by which its surface is diminished, but since the water flows down much more slowly on the wall than when it falls free, the disadvantage of the smaller surface is to a great extent counterbalanced by the longer contact with the steam.

The outlet pipe of the condenser leads directly to the air-pump. It must be wide enough to carry off air and water together. The lower part of the section of this pipe, which is required for the *water*, is determined on the permissible assumption that it has a velocity of

TABLE 49.

The dimensions of wet (parallel-current) jet-condensers with vacuum of

Steam to be condensed in one hour, in kilos.	100
The necessary cooling water, in litres } weight of steam × 15	1500
" × 45	4500
Diameter of the condenser, without steps	160
Height " " "	1000
Diameter of the steam inlet, for 705 mm. vacuum and 2 per cent. loss of pressure	150
Diameter of the water inlet, at 1 m. excess pressure	40
" " " at 3 m. "	35
" " " at 6 m. "	30
" " " at 9 m. "	25
" " connection to the air-pump	75
Diameter of the separate air-pipe to the pump, if one were used	40
Diameter of the internal pipe of the injector	50
Number of holes in the injector pipe (+ 20 per cent.):—	
Holes 2 mm. diameter, 0·5 m. pressure (30 litres per hole per hour)	180
Holes 3 mm. diameter, 0·5 m. pressure (68 litres per hole per hour)	80

0·5 m. per second, corresponding to a pressure-head of about 25 mm. The upper part of the section is for the air, and is obtained from Table 35; the section of the pipe there given for the quantity of air is added to that necessary for the water. It is assumed that 1000 litres of cooling water contain 0·25 kilos. of air.

Example.—For the condensation of 1000 kilos. of steam per hour, the diameter of the steam pipe, at a vacuum of 705 mm., is 350 mm. by Table 32, if a loss in pressure of 2 per cent. is permitted; the section of the condenser without steps should be 20 per cent. greater, hence its diameter is 400 mm.

The height of the condenser we take at 1400 mm.

The maximum quantity of water is, according to our assumption, 45 × 1000 = 45,000 kilos. per hour. The supply pipe must, therefore, by Table 36, be 80 mm. in diameter for a length of 20 m. with 3 m. excess of pressure.

Through a hole, 2 mm. in diameter, 25 litres pass in one hour at 0·5 m. excess pressure, according to Table 44. The perforated pipe must therefore have, in the

TABLE 49.

out steps, for condensing 100-3000 kilos. of steam per hour at a 705 mm.

200	300	500	1000	1500	2000	3000
3000	4500	7500	15000	22500	30000	45000
9000	13500	22500	45000	C7500	90000	135000
185	215	280	400	440	500	555
1000	1200	1300	1400	1500	1600	1800
175	200	250	350	400	450	500
55	60	75	100	125	140	165
45	55	60	80	95	115	125
40	45	55	70	80	95	115
30	40	50	65	75	85	100
90	110	150	190	235	270	325
45	50	60	75	80	90	100
60	80	90	100	125	160	200
360	580	900	1800	2700	3600	5400
160	250	400	780	1200	1600	2400

present case, $\frac{45,000}{25} = 1800$ holes. On account of possible stoppages we take 2000 holes.

The injector pipe is taken at 100 mm. diameter.

The weight of air to be exhausted in one hour is $\frac{4500 \times 0{\cdot}25}{1000} = 11{\cdot}25$ kilos., and at a vacuum of 705 mm., according to Table 35, the air suction pipe (if such were used) must have a diameter of 65 mm., *i.e.*, a section of 0·33 sq. dcm.

The pipe leading from the condenser to the air-pump must have this section for the air—0·33 sq. dcm.—and also that required for the water, which is, for a velocity of 0·5 m. per second, $\frac{45,000}{3600 \times 5} = 2{\cdot}5$ sq. dcm. The connection to the air-pump has therefore a section of $0{\cdot}33 + 2{\cdot}5 = 2{\cdot}83$ sq. dcm., equal to a diameter of 190 mm.

12. *The Dimensions of the Dry (Counter-current) Fall-pipe Jet-Condenser.*

The "dry" jet-condensers, which are almost always constructed to work with counter-currents, are closed vessels, which the steam to he condensed enters at the bottom, and the well-sprayed cooling water at the top. The heated water flows away spontaneously together with the condensed steam by means of a fall-pipe (barometer tube) at the bottom, whilst the air and gases are exhausted cold at the top. Dry condensers are often used for small and medium capacities, for large almost invariably. Their chief dimensions are given in Table 51 for an hourly condensation of 300-12,000 kilos. (See Fig. 15, p. 211.)

If the cooling water has in the condenser a free fall of

$h =$	1	2	3	4	5 m.
its theoretical time of fall, $z_s =$	0·46	0·64	0·79	0·91	1·015 seconds.

In these times a jet of water of thickness δ mm. takes up such an amount of heat (according to Table 46) from the surrounding steam that it is heated through the following fractions of the original temperature difference, θ_a:—

If $\delta = 1$, the heating is	$0{\cdot}460\theta_a$	—	—	—	—
$\delta = 2$, ,,	$0{\cdot}300\theta_a$	$0{\cdot}335\theta_a$	—	—	—
$\delta = 3$, ,,	$0{\cdot}225\theta_a$	$0{\cdot}225\theta_a$	$0{\cdot}247\theta_a$	$0{\cdot}278\theta_a$	$0{\cdot}290\theta_a$;
$\delta = 4$, ,,	$0{\cdot}163\theta_a$	$0{\cdot}188\theta_a$	$0{\cdot}193\theta_a$	$0{\cdot}217\theta_a$	$0{\cdot}227\theta_a$.

Example.—If a jet of water of thickness $\delta = 3$ mm., at a temperature of 10° C., falls through 4 m. in steam of 55° C., it is heated through $(55 - 10)\ 0{\cdot}278 = 12{\cdot}5°$ C., and thus has finally the temperature $10 + 12{\cdot}5 = 22{\cdot}5°$ C.

From the above figures it may be gathered that, although the increases of temperature just given may not be exact, a condenser, in which the water fell straight to the bottom without stops, must be very high, and the water very finely divided, if it is to be heated nearly to the temperature of the steam. A very fine spray of water is not easily obtained and necessitates a slowly rising current of steam. Therefore dry condensers without steps must be of great height and diameter.

The water may be made much hotter if it is allowed to fall through the same total height in several short stages, by each of which it ...
...ve, in the

given a fresh surface. This is made clear by the example below. For since the velocity of fall is the least at the beginning, the period during which the water is in the condenser increases with the number of steps, as also does the number of changes of surface.

Example.—If a jet of water, $\delta = 3$ mm. in diameter, at 10° C., falls down five steps, of 800 mm. each, through steam at 55° C., the heating is:—

At the end of the first fall (Table 46): $(55 - 10)\ 0{\cdot}200 = 9{\cdot}0°$;
the temperature of the jet is then $10 + 9{\cdot}0 = 19{\cdot}0°$.
After the second fall: $(55 - 19{\cdot}0)\ 0{\cdot}200 = 7{\cdot}2°$;
the temperature of the jet is then $19{\cdot}0 + 7{\cdot}2 = 26{\cdot}2°$.
After the third fall: $(55 - 26{\cdot}2)\ 0{\cdot}200 = 5{\cdot}76°$;
the temperature of the jet is then $26{\cdot}2 + 5{\cdot}76 = 31{\cdot}96°$.
After the fourth fall: $(55 - 31{\cdot}96)\ 0{\cdot}200 = 4{\cdot}61°$;
the temperature of the jet is then $31{\cdot}96 + 4{\cdot}61 = 36{\cdot}57°$.
After the fifth fall: $(55 - 36{\cdot}57)\ 0{\cdot}200 = 3{\cdot}69°$;
the temperature of the jet is then $36{\cdot}57 + 3{\cdot}69 = 40{\cdot}26°$.

In a straight fall without steps the heating would only be through 22·51°.

The determination of the number and the height of the steps is accomplished by the method in the following paragraph, in which it is assumed that the temperature of the steam to be condensed remains the same from bottom to top of the condenser. This assumption is not quite accurate, for the pressure in the counter-current condenser must be somewhat less at the top than below, because only so would there be a current of steam towards the top. The pressure at the bottom is due almost alone to the steam, at the top to the air almost entirely; between the extremes the pressure of the air diminishes towards the bottom, that of the steam towards the top, consequently the *temperature* of the steam also must diminish towards the top. But these differences are not very considerable at the places where condensation is still really taking place (which condition we are considering here), therefore we neglect them for the sake of simplicity. In what follows it is assumed that all the steps are of equal height.

If the whole temperature difference between steam and cooling water be θ_a, and this be diminished below the top step by the fraction, $a\theta_a$, by absorption of heat by the water from the steam, then, of the residual difference, $\theta_a - a\theta_a$, a fraction, $a(\theta_a - a\theta_a) = a\theta_a(1 - a)$, is removed below the second step. Below the third step the remaining temperature difference, $\theta_a - a\theta_a - a\theta_a(1 - a) = \theta_a(1 - a) - a\theta_a(1 - a) = \theta_a(1 - a)^2$, is diminished by $a\theta_a(1 - a)^2$, and by the last (lowest or nth) step by the fraction, $a\theta_a(1 - a)^{n-1}$.

The sum of all these intervals of temperature would be, in the most favourable case, equal to the whole temperature difference, θ_a, but is, in reality, only a more or less large part of the whole difference. It is naturally endeavoured to make the temperature of the waste water approximate as nearly as possible to that of the steam.

Let p be a percentage and $\frac{p\theta_a}{100}$ the portion of the original temperature difference removed, *i.e.*, the sum of all the separate intervals of temperature given above, then

$$\frac{p}{100}\theta_a = a\theta_a\{1 + (1 - a) + (1 - a)^2 + (1 - a)^3 + \ldots (1 - a)^{n-1}\};$$

or, summing the geometrical progression,

$$\frac{p}{100}\theta_a = \frac{a\theta_a\{(1-a)^n - 1\}}{(1-a) - 1}$$

or

$$\frac{p}{100} = 1 - (1 - a)^n. \quad . \quad . \quad . \quad . \quad . \quad . \quad (201)$$

If the increase in temperature of the water, a, in the highest step is known, and also the number of steps, then this equation gives the fraction of the *whole* difference in temperature which is removed by all the steps, *i.e.*, by how much the temperature of the water approaches that of the steam.

The value of a depends on the time during which the water drops are exposed to the action of the steam, which time is obtained directly from the height of fall of the drop.

Table 50 gives, by the aid of equations (110) and (194) and Tables 46 and 48, figures which show by what fraction the original temperature difference, θ_a, is diminished in condensers with 1-8 steps of equal heights of 200-1000 mm., when the water falls in jets of 2·7 mm. thickness. The table shows to what extent the temperature of the waste water increases with the smallness of the drops and the number and height of the steps.

In reality there are in the condenser not only jets of every size but also drops and sheets of water. A very fine water-dust is formed, which is heated, and then unites with the other water, because of the currents of steam and the fall, or is carried to the wall This circumstance, and also the presence of sheets of water moving in the condenser, from which drops are *thrown off*, in conjunction with the

inaccuracy of the formulæ which have been given to represent the process of heating, often cause the water to be heated to a greater extent in actual practice than would be expected from Table 50. This table is to be regarded as giving only a general picture of what occurs, without being an exact representation of fact.

Experience shows that with 5-6 steps, and a total height of 2500-3000 mm., very warm waste water may be obtained, even when the water is injected in jets of 5-6 or even 8 mm. diameter. A finer spray of water and more steps improve the action.

The maximum velocity of the steam at the bottom of a condenser without steps should be that velocity which exerts a pressure on a falling drop equal to *double* its weight (Chapter XV.). If there are steps in the condenser, the greatest velocity should only be somewhat greater than that which exerts a pressure equal to the *single* weight of a drop.

Thus, according to Table 23, the greatest velocities for steam at 40° C. (706 mm. vacuum) would be:—

For drops of diameter	0·1	0·25	0·5	1	2	3	4	5	mm.
In condensers without steps	9·2	14·6	20·6	29·2	42	50·5	58·5	65·3	m.
In condensers with steps	6·5	10·3	14·59	20·6	29·2	35·3	42	46·2	m.

In the author's opinion, founded on observations made on condensers, these calculated velocities are too low. In order to exert the pressures mentioned the velocities must be about 1·33-1·5 times as great. Also in all condensers it is a question not only of drops, but also of jets of water, upon which the current of steam has much less action. The majority of the drops, however small, are heated by the current of steam and then unite with the other water or are thrown against the walls and thus prevented from being carried forward. Finally, in almost all condensers a portion of the steam (10-15 per cent.) is condensed *before* it comes to the vertical rise.

On all these grounds, according to experience, the first and lowest contraction of a condenser without steps may have such a section that steam of 705 mm. vacuum attains in it a velocity of about 65 m. per second. In a condenser with steps the velocity may be 55 m. per second. If there is a lower vacuum in the condenser, the volume

TABLE 50.

The fractions by which the original difference in temperature, θ_a, between steam and water is diminished in dry counter-current condensers with 1-8 steps, each 200-1000 mm. in height. The water is in jets of δ = 2-7mm. diameter.

($t_e\theta_a$ when $\theta_a = 1$.)

Number of equal steps. n	Height of each step.	Time of fall through one step. z_s	Height of the condenser. h	Diameter of the water jets, δ, in mm. 2	3	4	5	6	7
1	200	0·20	200	0·205	0·142	0·109	0·088	0·074	0·064
2	,,	,,	400	0·368	0·264	0·199	0·158	0·143	0·124
3	,,	,,	600	0·498	0·368	0·293	0·229	0·220	0·178
4	,,	,,	800	0·600	0·459	0·359	0·293	0·266	0·233
6	,,	,,	1200	0·748	0·600	0·500	0·408	0·378	0·324
8	,,	,,	1600	0·841	0·706	0·580	0·500	0·462	0·418
1	300	0·25	300	0·225	0·150	0·120	0·097	0·082	0·071
2	,,	,,	600	0·400	0·298	0·242	0·185	0·157	0·137
3	,,	,,	900	0·535	0·386	0·340	0·264	0·227	0·198
4	,,	,,	1200	0·630	0·479	0·427	0·336	0·290	0·245
6	,,	,,	1800	0·784	0·623	0·564	0·460	0·403	0·357
8	,,	,,	2400	0·871	0·730	0·672	0·559	0·496	0·445
1	400	0·285	400	0·240	0·156	0·129	0·104	0·088	0·076
2	,,	,,	800	0·423	0·288	0·242	0·198	0·168	0·146
3	,,	,,	1200	0·562	0·388	0·340	0·281	0·242	0·211
4	,,	,,	1600	0·668	0·493	0·426	0·357	0·308	0·271
6	,,	,,	2400	0·808	0·695	0·565	0·483	0·426	0·378
8	,,	,,	3200	0·890	0·743	0·671	0·587	0·521	0·469
1	600	0·35	600	0·261	0·184	0·142	0·115	0·091	0·083
2	,,	,,	1200	0·436	0·335	0·264	0·237	0·174	0·159
3	,,	,,	1800	0·596	0·457	0·369	0·307	0·249	0·229
4	,,	,,	2400	0·682	0·558	0·458	0·387	0·318	0·293
6	,,	,,	3600	0·837	0·705	0·602	0·590	0·436	0·406
8	,,	,,	4800	0·899	0·805	0·706	0·624	0·535	0·500
1	800	0·41	800	0·277	0·196	0·151	0·121	0·105	0·091
2	,,	,,	1600	0·476	0·352	0·279	0·229	0·199	0·174
3	,,	,,	2400	0·622	0·481	0·388	0·321	0·283	0·249
4	,,	,,	3200	0·727	0·580	0·480	0·404	0·358	0·318
6	,,	,,	4800	0·857	0·731	0·625	0·531	0·456	0·425
8	,,	,,	6400	0·927	0·824	0·730	0·645	0·588	0·534

TABLE 50—(*continued*).

Number of equal steps. n	Height of each step.	Time of fall through one step. z_1	Height of the condenser. h	Diameter of the water jets, δ, in mm.					
				2	3	4	5	6	7
1	1000	0·46	1000	0·294	0·221	0·161	0·136	0·116	0·096
2	,,	,,	2000	0·502	0·393	0·297	0·254	0·200	0·183
3	,,	,,	3000	0·651	0·527	0·410	0·355	0·297	0·262
4	,,	,,	4000	0·752	0·632	0·505	0·443	0·376	0·333
6	,,	,,	6000	0·878	0·776	0·652	0·584	0·505	0·455
8	,,	,,	8000	0·939	0·865	0·756	0·691	0·611	0·555

of the steam will be lower, and the velocity, and hence also the danger of carrying drops away with the steam, less.

Since about 10 per cent. of the steam to be condensed is already liquefied *before* it enters the lowest narrow section, this section may be based upon a velocity of 70 m. for the whole quantity of steam.

1 kilo. of steam at a vacuum of 705 mm. has a volume 19,500 litres, therefore 1000 kilos. of steam at 70 m. velocity require, without steps, a section of

$$\frac{19500 \times 1000}{3600 \times 700} = 7{\cdot}5 \text{ sq. dcm. (approx.).}$$

In condensers with steps the velocity may reach 55 m., therefore 1000 kilos. of steam at 705 mm. vacuum require a section of

$$\frac{19500 \times 1000}{3600 \times 550} = 10 \text{ sq. dcm. (approx.).}$$

Since, however, only half the section of a condenser is left free for the passage of steam by reason of the inserted plates, sieves and divisions, the whole section of the condenser without steps should be 15 sq. dcm. for 1000 kilos. of steam, and the section of the condenser with steps 20 sq. dcm., from which the diameter may be obtained.

For the smaller capacities, to condense 1000-2000 kilos. per hour, the diameters, as determined by this rule, must be somewhat increased, in order to allow for the greater friction, the inaccuracies

TABLE 51.

The dimensions of (dry counter-current) fall-pipe jet-condensers, with at a vacuum

Steam to be condensed in one hour in kilos.			300	500	1000	1500
The necessary quantity of cooling water	Weight of steam × 10,	litres	3000	5000	10000	15000
	Weight of steam × 40,	litres	12000	20000	40000	60000
Condenser without steps	Diameter	mm.	400	450	550	650
	Height measured to the sieve	mm.	At least 3000 mm.—			
Condenser with steps	Diameter	mm.	500	550	600	700
	Height measured to the sieve	mm.	2400	2400	2400	2800
Diameter	of the steam inlet, for 705 mm. vacuum, 2 per cent. loss of pressure	mm.	200	250	350	400
	of the water inlet with a head of 3 m.	mm.	50	60	80	90
	„ „ „ „ „ 6 m.	mm.	45	55	70	80
	„ „ „ „ „ 9 m.	mm.	40	50	65	75
	„ air-pipe (at 15° C.)	mm.	50	60	80	90
	„ fall-pipe, when 10,700 mm. high	mm.	90	105	145	175
	„ „ „ 11,020 mm. „	mm.	75	85	110	125
Number of holes in the perforated plate, with a head of 10 mm. of water, + 10 % for obstructions	5 mm. diameter		125	210	415	620
	6 mm. „		90	145	290	435
	7 mm. „		70	110	215	320

and contractions. The diameters in Table 51 are determined in this manner.

If the diameter of the condenser, Δ dcm., is fixed, then the height of the lowest stage, e_u, for condensing the weight of steam, D, in one hour is *at least*

$$e_u = \frac{10D}{1000\,\Delta}\,\text{dcm.}$$

Accordingly,

For D =	1000	2000	5000	10,000	kilos. of steam,
and Δ =	600	775	1175	1600	mm.
e_u =	170	255	440	630	mm.

But, on account of the vortex and friction occurring at this place, the height of the lowest stage should be increased to about

$$e_u = 220 \quad 330 \quad 550 \quad 700 \text{ mm.}$$

The succeeding upper steps may then be put nearer and nearer together. There may be 3-4 whole stops or 6-8 half stops.

TABLE 51.

and without steps for condensing 300-12,000 kilos. of steam per hour of 705 mm.

2000	3000	4000	5000	6000	7000	8000	9000	10000	11000	12000
20000	30000	40000	50000	60000	70000	80000	90000	100000	110000	120000
80000	120000	160000	200000	240000	280000	320000	360000	400000	440000	480000
700	775	900	1000	1100	1200	1275	1350	1400	1450	1550
Holes in perforated plate not larger than 2 mm. diameter.										
775	900	1050	1175	1250	1350	1450	1550	1600	1675	1750
2800	2800	3200	3200	3200	3200	3600	3600	3600	3600	3600
450	500	575	650	700	750	800	850	900	950	1000
105	125	135	155	170	180	190	205	215	225	230
90	110	120	135	145	155	165	175	185	190	195
85	100	115	125	135	145	150	160	170	175	185
100	115	125	135	145	155	160	165	175	180	190
200	235	280	300	330	350	380	400	420	440	460
150	175	190	215	225	250	275	285	300	315	325
825	1240	1660	2070	2480	2895	3300	3720	4135	4550	4960
580	865	1150	1440	1730	2090	2305	2595	2880	3165	3455
420	635	845	1060	1270	1480	1690	1905	2115	2335	2545

The diameter of the steam pipe is obtained as with wet-condensers. It is determined by means of Table 32.

The diameter of the water pipe may also be determined as before. The limits of the temperatures of the steam are about 35°-60° C., of the water about 8°-30° C., and consequently, according to Table 41, 10-40 kilos. of water are required to condense 1 kilo. of steam. The diameter of the water supply pipe is then obtained from Table 36, if the available pressure is known or assumed in each case. In Table 51 the diameters are given for heads of 3, 6 and 9 m.

The water is sprayed in the condenser in many different ways. If the water is distributed by means of an overflow (sill), or an overflow is used as a preliminary, Table 43 serves to fix the dimensions. The width or circumference of the overflow (length of the sill) is generally known from the diameter of the condenser. Table 43 then gives the depth of the layer of water running over. The sheet of water so formed naturally diminishes in thickness during its fall.

When the water is distributed through a perforated plate, by

assumption of the diameter of the holes, the number may be at once obtained from Table 44, and then from the size of the plate the distances between the holes can be determined.

In calculating the number of holes, n, in the sieve, their diameter must be taken according to discretion. The smaller they are, the more thoroughly is the water divided, but they are the more readily stopped up.

The number of holes is determined for the smallest probable consumption of water, assuming a suitable height for the water (10 mm. in Tables 44 and 51). An increased head of water causes the flow of an increased quantity of water sprayed to the same extent.

The perforated plates have naturally a high rim, in order to make possible a large pressure.

In Table 51 the number of holes is given for the minimum quantity of water, a head of 10 mm. and holes of 5, 6 and 7 mm. diameter.

The section of the air-pipe follows from the weight of air to be hourly exhausted, which is taken at 0·25 kilo. per 1000 kilos. of water, calculating from the greatest consumption of water. Table 35 gives the necessary measurements.

The diameter of the fall-pipe or barometer pipe is obtained from the maximum quantity of injected water, to which is to be added the weight of the condensed steam. It is found in Table 42.

In Table 51 the diameter of this waste pipe is given for two heights—10·7 and 11·02 m.

It hardly appears to be necessary to calculate an example, which would be merely repetition, in view of the example calculated of a wet condenser.

The loss of heat from the warm condenser walls is an advantage, but it is insignificant compared with the weight of steam hourly condensed.

Example.—The condenser for condensing 1000 kilos. of steam per hour has a surface of 7 sq. m. (Table 51). It therefore loses in one hour, if its average temperature is 55° C. and that of the atmosphere 10° C., 7 × 505 = 3535 calories (Table 39). Thus it condenses about 6 kilos. of steam per hour on the inner wall, which is equal to 0·6 per cent. of the total condensation.

The surface of the cold water, on the perforated plate and in the feed-box inside the condenser, does not condense steam, which should always be completely liquefied below the plate, but it serves to cool

the air. For this purpose the jets and sheets of water formed above the perforated plate are also useful.

B. Surface-Condensers (Coolers).

Surface-condensers are designed to condense vapours from the most diverse sources, and generally also to cool the condensed liquid (hence they are often known as coolers), without the cooling medium —generally cold water, more rarely air—coming into direct contact with the substance. The exchange of heat takes place through a metal wall.

The space in which condensation occurs may be under the pressure of an atmosphere or under a lower pressure (vacuum).

There are at present no certain observations to show that the vapours of different liquids have different coefficients of transmission of heat (which might perhaps depend on the specific gravity of the vapour). Thus it must for the present be assumed that these coefficients are the same for all vapours, and also that they do not alter for different pressures. It may be left an open question whether the coefficient is not in fact less at very low pressures.

Surface-condensers may be formed from systems of tubes, through which the vapours pass, whilst the water flows outside, or the water may pass through the tubes and the vapours outside. They may be made from coils, bundles of pipes, and cylindrical or plane surfaces, which are cooled by water or air on one side, whilst the other is in contact with the vapour.

If water is used as the condensing agent, it may rise *en masse* abont the surfaces or flow down in a thin layer over them.

If the air is used as the cooling agent, it is forced through pipes round which moves the liquid to be cooled.

Thus this species of condenser may be separated into :—

1. Enclosed surface-condensers cooled by water.
2. Enclosed surface condensers cooled by air.
3. Open surface-condensers.

1. *Enclosed Surface-Condensers with Water Cooling (Coolers).*

Figs. 17, 18 and 19 show typical forms of these condensers.

(a) *The Mean Temperature Differences, θ_{mc} and θ_{mk}.*

If there are not particular reasons for another arrangement, this species of apparatus is naturally constructed for opposite currents, *i.e.*, in vertical condensers the steam enters at the top and the water below. Generally the *vapour* passes *through* and the *water around* the tubes: occasionally, however, for convenience in cleaning the tubes, the

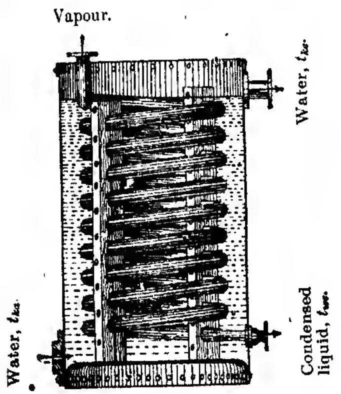

Fig. 17.

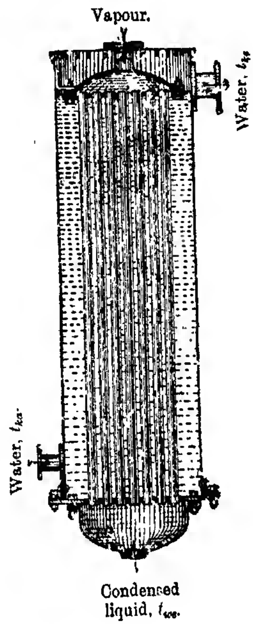

Fig. 18.

vapour is sent *round* and the *water through* them. This latter arrangement influences the exchange of heat only in so far as it generally diminishes the velocity of the steam and increases that of the water.

From what was said in Chapter I. it is evident that two periods must be distinguished in condensers which also cool, *viz.*, the period during which the vapour is condensed and the period during which the condensed liquid is cooled.

If the vapour brought no air with it, it would retain the same temperature to the end of the first period in which the condensation occurs, since its pressure would remain almost the same. In proportion as it advanced over the cooling surface, its quantity, and hence its velocity, would gradually diminish until both became zero, but it would remain at a constant temperature so long as it existed. If then all the vapour had disappeared at a certain place in the condenser, the remaining space would be filled with air at a pressure equal to that of the vapour. The spaces filled with vapour and air would be

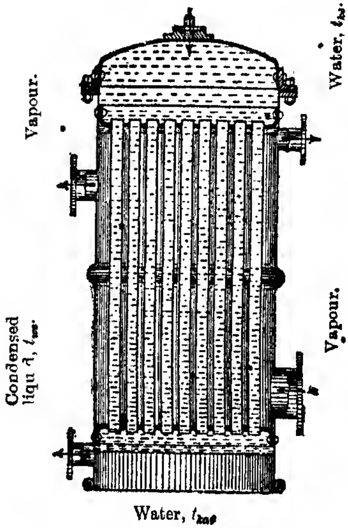

FIG. 19.

marked off with tolerable sharpness, and this would also be the case if the condensation occurred *in vacuo*. In reality, however, the vapour always contains more or less air, which increases in pressure the more the quantity of the vapour is diminished by condensation. Thus there is a gradual transformation from the space in which there is *only* vapour to that in which there is *only* air, through a space in which the two are mixed.

This air, which is introduced by the vapours to be condensed, must be conducted away, either into the atmosphere or to the air-

pump. Thus condensers or coolers must be provided with a pipe, which leads the air from their interior into the open or to the air-pump. This pipe must not be obstructed by liquid, since the variations in the pressure and amount of air introduced into the condenser would cause currents backwards and forwards in this pipe in order to equalise the pressure. The presence of liquid in the pipe would prevent the free movement of the air and might cause irregularities in working.

Since condensation, *i.e.*, the production of liquid from the vapour, commences immediately the vapour enters the condenser, its walls are at once covered by liquid flowing downwards, the quantity and velocity of which increase towards the bottom. This liquid forms an obstacle to the transfer of heat which cannot well be disregarded. The liquid flowing down has not the temperature of the vapour nor that of the cooling medium (water); its temperature lies between the two. At that place in the condenser at which condensation is practically finished, the condensed liquid is always cooler than the vapour from which it was formed. Unfortunately, in the lack of suitable experiments, it is not accurately known what relation its temperature bears to those of the vapour and cooling water.

For this reason, and because we wish to avoid other arbitrary assumptions, and finally also because this condition has only a slight influence on the estimation of the size of the cooling surface, we shall assume in what follows (though incorrectly) that the liquid condensed has at the end of the condensation the temperature of the vapour, and that in the following period it is cooled from the temperature of the vapour to the desired lower temperature.

The transfer of heat is universally assumed to be directly proportional to the difference in temperature between the two substances engaged in the process. Therefore, in the first place, we must determine the *mean temperature difference* between vapour and cooling water and then that between the condensed liquid and the water.

We know, from Chapter I., that the mean difference in temperature is in most cases not equal to the arithmetic mean of the initial and final differences, but is (equation 10):

$$\theta_m = \frac{\theta_a\left(1 - \frac{p}{100}\right)}{\log\frac{100}{p}},$$

in which θ_a denotes the greatest and p the least difference in temperature, the latter expressed as a percentage of the former.

Example.—If the greatest difference, $\theta_a = 60°$, the least difference $= 6°$, then $p = \frac{6 \times 100}{60} = 10$ per cent.

In Table 1 are found the values of θ_m calculated for the case in which $\theta_a = 1$, and for $p = 1 - 100$ per cent.

Example.—For $\theta_a = 60°$ and $p = 10$, Table 1 gives $\theta_m = 0{\cdot}391 \times 60 = 23{\cdot}46°$.

In order to determine the cooling surfaces, it is necessary to know the mean temperature difference for each of the two periods *singly*, *i.e.*, for the period of condensation of the vapour and for that of cooling the condensed liquid. It would, however be inconvenient to calculate this specially every time. Table 52 is therefore given, in which the mean differences are given for a large number of cases—for steam at atmospheric pressure at the temperature of 100° C., for steam of lower pressure at vacua of 611 and 705 mm. (temperatures of 60° and 40° C.), and also for alcohol vapour at 80° C., always cooling by water.

The *cooling water* may have various original temperatures, those of $t_{ka} = 2{\cdot}5°$, 5°, 10°, 15° and 20° C. are considered in the Table. The water may also flow away at various temperatures; the final temperatures, $t_{ke} = 20°$, 30°, 40°, 50°, 60°, 70° and 80° C., are given in Table 52. Finally, the condensed liquid is obtained at different temperatures; the cases are considered in which it leaves 2°, 5°, 10°, 15,° 20° and 25° C. hotter than the cooling water.

In Table 52 the mean difference in temperature between vapour and cooling water in the first period (condensation) is represented by θ_{mc}, the mean difference between condensed liquid and cooling water in the second period (cooling) is represented by θ_{mk}.

Example.—The steam to be condensed is at 100°, the cooling water is originally at 10° and is to flow away at 60°. The condensed liquid is required to be at 15° C.

According to our assumption, the steam is only to be condensed in the first period, not cooled. 1 kilo. of steam at 100° C. has a total heat of 637 calories, of which 537 must be withdrawn in condensation. The condensed steam, the liquid, has still 100 calories; therefore, in order to cool it down to 15° C., 85 units of heat must still be removed (in all 537 + 85 = 622 calories). In the

cooling period, therefore, $\frac{85}{637 - 15} = \frac{85}{622}$ of the total heat is to be removed, and in the condensing period $\frac{537}{622}$ of the total heat.

The cooling water becomes heated in all from 15° to 60° C., *i.e.*, through 45°, of which $\frac{85 \times 45}{622} = 6{\cdot}15°$ is accounted for by the period of cooling.

Thus, at the end of the condensation period, when the condensed liquid is still at 100°, the cooling water is at 10° + 6·15° = 16·15° C.

The steam enters at - - - - - - -	100°
The water is finally at - - - - - - -	60°
Difference - - - - - -	40°
The steam is finally at - - - - - -	100°
The water at the same place is at - - -	16·15°
Difference - - - - -	83·85°

40° is the following percentage of 83·85°:—$p = \frac{40 \times 100}{83{\cdot}85} = 47{\cdot}70$ per cent.

The mean temperature difference between steam and water in the first period is, therefore, according to Table 1, $\theta_{mc} = 0{\cdot}7 \times 83{\cdot}85 = 58{\cdot}7°$.

The condensed liquid at the top is at - - -	100°
The cooling water at the top is at -	16·15°
Difference - - - - - -	83·85°
The condensed liquid at the bottom is at - -	15°
The cooling water at the bottom is at - - -	10°
Difference - - - - - -	5°

5° is the following percentage of 83·85°:—$p = \frac{5 \times 100}{83{\cdot}85} = 5{\cdot}96$ per cent.

The mean temperature difference between the condensed liquid and the cooling water during the second period, according to Table 1, is

$$\theta_{mk} = 0{\cdot}339 \times 83{\cdot}85 = 28{\cdot}42°.$$

Table 52 has been calculated in this manner. It shows:—

1. *That the mean temperature difference between vapour and cooling water (first period) decreases with the increase in temperature of the waste water, but that it is very little affected by the extent to which the condensed liquid is cooled. In the latter respect the differences may be neglected in practice.*

TABLE 52.

The temperature differences between vapour and cooling water, θ_{mc}, and between condensed liquid and cooling water, θ_{mk}, for steam at 100°, 60° (611 mm. vacuum), 40° C. (705 mm. vacuum), for alcohol vapour at 80° C. (83·6 per cent. by weight) in closed surface-condensers.

The figures printed vertically are the temperatures of the cooling water at the place where condensation ceases and cooling begins.

Original temperature of cooling water.	Temperature of condensed liquid.	Steam at 100° C. (atmospheric pressure). Latent heat = 537 calories. Final temperature of the cooling water, t_{ke}.													
		20°		30°		40°		50°		60°		70°		80°	
		Mean temperature differences.													
t_{ka}	t_{ue}	θ_{mc}	θ_{mk}	θ_{mc}	θ_{mk}	θ_{mc}	θ_{mk}	θ_{mc}	θ_{mk}	θ_{mc}	θ_{mk}	θ_{mc}	θ_{mk}	θ_{mc}	θ_{mk}
2·5°	5	86·4	26·8	82·2	25·5	75·3	25·1	69	25·7	62·1	24·3	58·4	25·9	45·5	24·5
	7·5	,, 4	32	,, 5·4	31	,, 6·8	30·6	,, 8·4	30·8	,, 9·8	29·3	,, 11	29	,, 14·5	29
	12·5	,,	38	,,	36·8	,,	37·2	,,	36·8	,,	36	,,	36	,,	36
	17·5	,,	44·1	,,	43·4	,,	42·76	,,	42	,,	42·3	,,	41·7	,,	42
5°	7	85·5	25·1	80	24·8	73·8	23·4	67·7	24	60·9	23·45	53·9	23·4	45·7	23·3
	10	,,	31	,,	29·2	,,	30	,,	29·8	,,	29	,,	29	,,	28·9
	15	,, 7	37·2	,, 8	36·7	,, 9·4	36	,, 10·6	35·8	,, 12	34·8	,, 13	34·8	,, 14	34·7
	20	,,	42·8	,,	42·4	,,	42·4	,,	42·6	,,	41·9	,,	41·8	,,	41·7
	25	,,	48·3	,,	47	,,	46·8	,,	46·5	,,	45·2	,,	45·2	,,	45·1
	30	,,	51	,,	49·8	,,	49·5	,,	49	,,	49	,,	49	,,	49
10°	12	84	22·9	77·8	22·6	72	22·3	66	22	58·7	21·8	52·5	21·5	48·4	21
	15	,,	29·2	,,	28·8	,,	28·4	,,	28	,,	27·7	,,	27·4	,,	27·2
	20	,, 11·2	36·4	,, 12·5	36·2	,, 13·8	36	,, 15·3	35·7	,, 16·5	35	,, 17·8	34·3	,, 19·3	33·6
	25	,,	42·2	,,	41·7	,,	41·2	,,	40·8	,,	40·2	,,	39·8	,,	39·2
	30	,,	46·28	,,	45·76	,,	44·7	,,	44	,,	43·42	,,	42·98	,,	42·1
	35	,,	49·84	,,	49·36	,,	48·1	,,	47·4	,,	46·72	,,	46·5	,,	45·8
15°	17	82·7	22·7	76·3	22·4	71	22·4	63·9	21·5	58·8	21·3	51·5	21	41·8	19·8
	20	,,	28·2	,,	27·7	,,	27·7	,,	27	,,	26·8	,,	26·5	,,	25·8
	25	,, 15·6	34·6	,, 16·7	34·8	,, 18	34·8	,, 19	34	,, 20	33·9	,, 12	33·5	,, 23	32·7
	30	,,	39·6	,,	39·6	,,	39·6	,,	38·9	,,	38·8	,,	38·3	,,	38·2
	35	,,	44·7	,,	43·6	,,	43·8	,,	43·7	,,	42·6	,,	42·1	,,	41·
	40	,,	48·1	,,	48	,,	47·8	,,	47	,,	46·6	,,	46	,,	45
20°	22	—	—	74·1	21·4	67·7	21	61·5	20·6	55·5	20·2	48	19·7	40·7	19·3
	25	—	—	,,	27·1	,,	26·6	,,	26·25	,,	25·7	,,	25·8	,,	25
	30	—	—	,,	33·5	,,	32·8	,,	32·25	,,	31·7	,,	31·3	,,	30·7
	35	—	—	,,	39	,,	38·4	,,	37·52	,,	37·1	,,	36·9	,,	36·7

TABLE 52—(*continued*).

<table>
<tr><th rowspan="3">Original temperature of cooling water.</th><th rowspan="3">Temperature of condensed liquid.</th><th colspan="8">Steam at 60° C. (611 mm. vacuum). Latent heat = 564 calories. Final temperature of cooling water, t_{ke}.</th><th colspan="6">Steam at 40° C. (705 mm. vac.). Latent heat = 578 calories. Final temp. of cool'g water, t_{ke}.</th></tr>
<tr><th colspan="2">20°</th><th colspan="2">30°</th><th colspan="2">40°</th><th colspan="2">50°</th><th colspan="2">20°</th><th colspan="2">30°</th><th colspan="2">35°</th></tr>
<tr><th colspan="8">Mean temperature differences.</th><th colspan="6">Mean temperature differences.</th></tr>
<tr><th>t_{ka}</th><th>t_{ce}</th><th>θ_{mc}</th><th>θ_{mk}</th><th>θ_{mc}</th><th>θ_{mk}</th><th>θ_{mc}</th><th>θ_{mk}</th><th>θ_{mc}</th><th>θ_{mk}</th><th>θ_{mc}</th><th>θ_{mk}</th><th>θ_{mc}</th><th>θ_{mk}</th><th>θ_{mc}</th><th>θ_{mk}</th></tr>
</table>

2·5°	5	47·7	17·3	41·5	17·3	34·4	17·5	25·8	17·2	27·5	12·7	20	12	15·9	12·7
	7·5	,,	21·2	,,	51·2	,,	20·7	,,	20·4	,,	16	,,	15·8	,,	16
	12·5	,, 4	26·9	,, 4·6	26·9	,, 4·5	26·1	,, 6	25·8	,, 3·6	20·4	,, 3·7	20	,,	19·9
	17 5	,,	30·8	,,	30·8	,,	30	,,	29·5	,,	24	,,	24	,,	24·3
	22·5	,,	35·3	,,	35·8	,,	35·4	,,	33·8	,,	28	,,	28·5	,,	28·5
5°	7	46·4	16·2	40	15·6	33·3	15·6	25·5	15·3	27	12·2	19·7	12	15·1	11·3
	10	,,	20 8	,,	20 2	,,	20·2	,,	19·9	,,	14·4	,,	14	,,	14
	15	,, 6·6	26·1	,, 7	25·4	,, 7·8	25·4	,, 9·2	25	,, 6·5	19·9	,, 6·2	19	,,	19
	20	,,	31	,,	30·1	,,	30·1	,,	29·6	,,	23·6	,,	23·6	,,	23
	25	,,	34·7	,,	33·8	,,	33·8	,,	33·1	,,	26·5	,,	26·5	,,	26·5
10°	12	44·37	15·7	38·8	15·5	31·7	15·3	24·8	15·2	24	10·9	18	10·9	13·6	9·25
	15	,,	13·7	,,	19·4	,,	19·2	,,	19	,,	13·7	,,	13·7	,,	13·6
	20	,, 10·7	24·7	,, 11·3	24·2	,, 12	24	,, 12·3	23·8	,, 10·3	17·8	,, 10·7	17 8	,,	17·8
	25	,,	28·5	,,	28	,,	27·8	,,	27·55	,,	21·2	,,	21·2	,,	21·2
	30	,,	33	,,	32·5	,,	32	,,	31·6	,,	25	,,	25	,,	24·3
15°	17	42·75	14·4	36·9	14	30·3	13·7	22·8	13·7	22·3	9·87	16·2	9·87	12·5	9·25
	20	,,	18·45	,,	18	,,	17·1	,,	17·6	,,	12·5	,,	12·5	,,	12·5
	25	,, 15	23·8	,, 15·8	23·4	,, 16·2	22·8	,, 16·8	22·8	,, 15·1	16 2	,, 15·3	16·2	,,	16·25
	30	,,	27·9	,,	27·2	,,	26·6	,,	26·6	,,	19·5	,,	19·5	,,	19·5
	35	,,	31	,,	30·2	,,	29·6	,,	29·6	,,	22·5	,,	22·5	,,	22·25
20°	22	—	—	34·9	13·6	28	13·3	20·9	13	—	—	14·4	8·4	10·8	8·4
	25	—	—	,,	16·8	,,	16·4	,,	15·9	—	—	,,	10·8	,,	10·8
	30	—	—	,, 20·5	22	,, 21	21·6	,, 21·2	20·9	—	—	,, 20	14·4	,,	14·4
	35	—	—	,,	25·2	,,	24·4	,,	24	—	—	,,	17·4	,,	17·4
	40	—	—	,,	28·8	,,	28·3	,,	27·4	—	—	,,	—	,,	20

2. That the mean temperature difference between the condensed liquid and the cooling water (second period) is considerably affected by the extent to which the final temperature of the condensed liquid is to approach that of the cooling water, but that it does not depend to any great degree on the temperature of the waste water. In the latter respect the variations may be disregarded, and the mean temperature

TABLE 52—*continued.*

Original temperature of cooling water.	Temperature of condensed liquid.	Alcohol vapour at 80° C., about 90·4 per cent. strength by volume = 86·3 per cent. by weight. Specific heat, $\sigma = 0\cdot8$. Latent heat = 205 calories. Final temperature of the cooling water, t_{ke}.											
		20°		30°		40°		50°		60°		70°	
		Mean temperature differences.											
t_{ka}	t_{we}	θ_{mc}	θ_{mk}	θ_{mc}	θ_{mk}	θ_{mc}	θ_{mk}	θ_{mc}	θ_{mk}	θ_{mc}	θ_{mk}	θ_{mc}	θ_{mk}
2·5°	5	79	21·0	60·4	20·8	53·9	20·5	46·9	20·8	38·2	19·8	29·4	19·3
	7·5	„	25·9	„	25·2	„	24·5	„	23·8	„	23·2	„	23
	12·5	„ 6	32·8	„ 8	31·5	„ 10	30·5	„ 12	29·9	„ 14	29·7	„ 16	29·4
	17·5	„	37	„	36·7	„	35·7	„	35·3	„	34·3	„	33·9
5°	7	67	20·8	58·8	20·3	52	19·7	45	19·1	37·1	18·5	28·5	17·9
	10	„ 8	24·4	„ 10	24·4	„ 12	23·9	„ 14	23·1	„ 16	22·4	„ 18	21·7
	15	„	31·6	„	30·8	„	28·8	„	29	„	28·1	„	27·2
10°	12	64·6	19·7	55·4	19·1	50·5	18·6	43·4	18	36·6	17·4	27·	16·8
	15	„ 12	24·4	„ 14	23·7	„ 16	23	„ 18	22·3	„ 20	21·6	„ 22	20·9
	20	„	30·6	„	29·7	„	29·9	„	27·9	„	27	„	26·1
15°	17	62·7	17·9	55·1	17·36	49·2	16·8	42·3	16·26	35·2	15·68	26·1	15·1
	20	„ 16	23	„ 18	22·3	„ 20	21·6	„ 22	20·9	„ 24	20·1	„ 26	19·4
	25	„	29·44	„	28·5	„	27·6	„	26·7	„	25·76	„	24·85
20°	22	—	-	53·2	16·8	47·6	16·2	41	15·6	32·7	15·1	25	14·5
	25	—	—	„ 22	22	„ 24	21·3	„ 26	20·5	„ 28	19·7	„ 30	19
	30	—	—	„	27·8	„	26·8	„	25·9	„	24·96	„	24

difference for the second period may be taken for all cases as the mean of the temperature differences calculated for waste water temperatures of 20°-80°, without regard to the actual temperature of the waste water in the particular case.

(*b*) *The Coefficients of Transmission of Heat, k_c and k_k.*

The coefficient, k_c, for the passage of heat from steam to non-boiling water (first period) in open copper or brass tubes, is obtained from the empirical expression:

$$k_c = 750 \sqrt[2]{v_a}\ \sqrt[3]{0\cdot007 + v_l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (202)$$

This formula is founded on observations made in actual practice on

large and small condensers of most varied forms; v_d denotes the velocity of the steam when it enters the condenser (initial velocity), v_f, the mean velocity of the cooling water. It appears to be unquestionable that the coefficient of transmission of heat in these cases (condensation of vapours in spaces connected with the atmosphere or with an air-pump) increases with the velocity of the steam and water.

The velocity of the current of steam naturally decreases in the condenser from the beginning to the end, when it is zero. This decrease is in no way uniform, but is first rapid, then slower, following a curve outside our present scope. Since, however, the decrease in velocity must take place in almost all cases in the *same* manner, because the essential conditions, which cause the decrease, are the same in all condensers, it is permissible to assume that the *mean* velocity of the steam, which is the factor to be considered here, is in a simple proportion to the initial velocity.

As already mentioned in Chapter VII., there are many causes besides the velocities which influence the transmission of heat. These influences may be very great and often of such a nature that they cannot be expressed mathematically. The incrustations, which always occur to a greater or less extent, and are *à priori* quite indeterminable, often make any calculation deceptive; but also the position and direction of the surfaces, the width, shape and capacity of the hot space, the air mixed with the vapour, all alter the action to a considerable extent. No equation can be given for k_c, which expresses all these factors.

For coils and tubular coolers, through which the vapours pass equation (202) may be used with some confidence. It is already corrected for an average diminution in efficiency due to the furring of the cooling surface. For extraordinary cases k_c may be taken somewhat larger or smaller. Equation (202) holds good for cooling surfaces of copper and brass; these have walls of tolerably equal thickness, which may therefore be disregarded. For iron surfaces, partly because they generally are more furred than copper surfaces, the value of k_c should be diminished by about 15 per cent., for thick lead surfaces by about 30 per cent.

In Table 53 are collected the values for k_c, calculated by means of equation (202), for initial velocities of steam of 1-65 and velocities of the cooling liquid of 0·001-4 m. These values, k_c, are for the first period—that of condensation.

For the second period, that of cooling, in which *the transfer of heat*

TABLE 53.

The coefficient of the transmission of heat, k_e, between steam at low pressures and water, which does not boil, with copper tubes, for initial velocities of the steam, v_d, of 1-65 m. and velocities of the water, v_r=0·001 4·0 m. (First period).

Velocity of the cooling liquid in m. v_r	Velocity of the steam when it enters the condenser tube, v_d, in m.														
	1	2	4	6	9	12	16	20	25	30	36	42	49	56	65
	Coefficient of transmission, k_e.														
0·001	150	210	300	375	450	525	600	675	750	825	900	975	1050	1125	1200
0·008	187	262	375	448	562	655	750	843	937	1030	1125	1218	1312	1405	1500
0·020	225	315	450	563	675	788	900	1013	1125	1238	1350	1463	1575	1688	1800
0·035	262	367	524	655	786	917	1048	1179	1310	1441	1595	1706	1834	1965	2100
0·056	300	425	600	750	900	1050	1200	1350	1500	1650	1800	1950	2100	2250	2400
0·085	337	475	674	842	1011	1179	1348	1516	1685	1853	2022	2190	2356	2527	2696
0·117	375	528	750	937	1125	1312	1500	1687	1875	2062	2250	2437	2625	2812	3000
0·160	412	580	824	1030	1236	1442	1648	1834	2060	2266	2472	2678	2884	3090	3296
0·210	450	634	900	1110	1350	1575	1800	2025	2250	2475	2700	2925	3150	3375	3640
0·266	487	685	975	1230	1461	1704	1948	2191	2435	2678	2922	3165	8409	3692	3896
0·335	525	742	1050	1325	1575	1837	2100	2362	2727	2987	3150	3412	3675	3937	4200
0·415	562	792	1124	1417	1686	1967	2248	2529	2810	3091	3372	3653	3934	4215	4496
0·505	600	846	1200	1500	1800	2100	2400	2700	3000	3300	3600	3900	4200	4500	4800
0·607	637	897	1274	1592	1912	2230	2548	2866	3185	3503	3822	4140	4459	4777	5096
0·720	675	915	1350	1687	2025	2362	2700	3037	3375	3712	4050	4387	4726	5062	5400
0·850	712	1004	1424	1730	2136	2452	2848	3154	3560	3863	4272	4578	4984	5390	5696
1·00	750	1057	1550	1925	2350	2625	3000	3375	3750	4125	4500	4875	5250	6025	6000
1·50	862	1207	1724	2155	2586	3017	3448	3879	4310	4741	5172	5603	6034	6465	6896
2·00	945	1323	1892	2362	2835	3307	3780	4252	4725	5197	5670	6142	[illegible]	7087	75[illegible]0
2·50	1013	1418	[illegible]026	2532	3039	3545	4052	4558	5065	5571	6078	6584	7091	7597	8104
3·0	1087	1521	2174	2717	3261	3804	4348	4891	5435	5978	6522	7065	7603	8152	8696
3·5	1140	1596	2280	2850	3420	3990	4520	5130	5700	6270	6840	7410	7980	8550	9120
4·0	1200	1800	2100	3000	3600	4200	4800	5400	6000	6600	7200	7800	8400	9000	9600

is between the condensed liquid and the cooling liquid—between two liquids—another coefficient, k_k, holds good.

The coefficient of transmission, k_k, for the transfer of heat between two liquids moving with different velocities, is taken from equation (231) in the following chapter, for copper tubes:

$$k_k = \frac{200}{\frac{1}{1+6\sqrt{v_{r_1}}} + \frac{1}{1+6\sqrt{v_{r_2}}}}.$$

In this expression v_{f1} denotes the velocity of one liquid, v_{f2} of the other.

Table 64 gives, by equation (232), the values of k_k for velocities of the two liquids, v_{f1} and v_{f2}, from 0·001-2 m.

The velocity, v_{f1}, of the cooling liquid (generally water), which is rising and being heated, may be determined in any case after the construction of the apparatus, but is generally calculated previously; it is usually very low. As a rule, in cooling vessels the water rises with a velocity of 1-3 mm. per sec., although there is at times an endeavour to attain a higher velocity. Occasionally 150 or even 200 mm. per sec. is reached.

Apart from the uniform initial velocity, the cooling water acquires, through being heated on the hot surfaces, particular movements, the velocity of which may depend very largely on the temperature difference, the absolute temperature and the shape of the cooling surface. Thus the original velocity alone is not all. The warmer the cooling water is, the more readily it takes up heat (see the example on p. 32).

The velocity, v_{f2}, of the condensed liquid running down in the condenser is not known. It is generally greater than that of the cooling liquid. Certain observations lead to the conclusion that it is rarely more than 1 m. per second; v_{f2} is therefore taken at 0·800. This holds good for cooling surfaces, which are wetted *all over* by the condensed liquid which is to be cooled. It is almost universal in practice to find only a portion of the cooling surface wetted. Therefore, for vertical tubes the calculated surfaces must be approximately doubled. In coil coolers, in which the liquid only runs down on the lower part of the inner wall of the pipe, the upper and larger part remains unused, therefore the calculated cooling surface, H_k, for coils, must be multiplied approximately by 3.

(c) *The Condensing and Cooling Surfaces, H_c and H_k.*

We have now determined the dimensions of the principal factors, θ_{mc}, θ_{mk}, k_c and k_k, upon which depend the size of the condensing surface, H_c, and cooling surface, H_k; we now proceed to calculate the whole surface necessary. It is

$$H_{ck} = H_c + H_k = \frac{C_c}{\theta_{mc}k_c} + \frac{C_k}{\theta_{mk}k_k} \quad . \quad . \quad . \quad . \quad (203)$$

In order to facilitate the estimation of the condensing and cooling surfaces necessary in each separate case, Table 54 is given, from which may be taken the surfaces for condensing and cooling 100 kilos. of water or alcohol vapour per hour.

Table 54 consists of two parts. Part I. gives the surface, H_c, required for condensing 100 kilos. of steam at 100°, 60° and 40° C., and of aqueous alcohol vapour at 80° C. (86·3 per cent. by weight), in one hour, with vapour velocities of 1-64 m. and cooling water velocities of 0·001-1·00 m. per sec. Part II. then gives the surface, H_k, required for cooling the condensed liquid.

In using Table 54 it is therefore necessary first to seek in Part I. the surface necessary for *condensation*, and to add to this the surface required for cooling, obtained from Part II. and multiplied by 2 or 3.

It was assumed in calculating this table that the cooling water enters at 10° C., which is its ordinary temperature. If the water is colder in any particular case, the surfaces may be somewhat smaller, if warmer, they must be increased in proportion to the temperature differences given in Table 54. The figures are for copper heating surfaces. Iron surfaces must be 10-20 per cent. larger, lead surfaces 20-30 per cent. larger. An addition must also be made for exceptionally thick walls.

The first part of Table 54 is based on the assumption that *all* the vapour which enters the condenser is to be condensed. If this is not the case, but only a *part* of the entering vapour is to be liquefied, the other part leaving the condenser as vapour, then the capacity of the cooling surface increases considerably. The increase depends on the velocity with which the vapour leaves. In such cases the *sum* of the initial and final velocities of the vapour is to be taken as the basis of calculation.

The cooling surfaces given for the condensation of steam at 40° C. are probably too low; it would be well in constructing apparatus to make them somewhat larger than is indicated in Table 54—say 15-20 per cent. larger. It appears that highly rarefied steam communicates its heat less rapidly than high pressure steam; this may be on account of the greater distance apart of the molecules or on account of the sluggishness due to this cause. Table 54 assumes that the vapour passes through the tubes and the water flows outside them. If the reverse be the case, the greater velocity of the water is more favourable and the lower velocity of the steam less favourable, but generally

Table 54. Part I.

The cooling surfaces, H_c and H_k, in sq. m., requisite to condense and cool in one hour 100 kilos. of steam at 100° C., 100 kilos. of steam at 60° C., 100 kilos. of steam at 40° C., and 100 kilos. of aqueous alcoholic vapour at 80° C. (86·3 per cent. by weight).

The steam enters at velocities, v_d, from 1-64 m. The cooling water has velocities, v_f, from 0·001-1·00 m.

The initial temperature of the cooling water, $t_{ka} = 10°$ C. The final temperature of the cooling water, $t_{ke} = 20°$-80° C.

The condensed liquid leaves at 2°-25° C. above the initial temperature of the cooling water.

Steam at 100° C. (atmospheric pressure), c = 537.								
Initial velocity of the steam. v_d	Velocity of the cooling water. v_f	Final temperature of the cooling water, t_{ke}.						
		20	30	40	50	60	70	80
		The cooling surface, H_c, in sq. m., required to condense 100 kilos. of steam per hour.						
1·0	0·001	4·29	4·62	5	5·45	6·20	6·90	8·40
	0·009	3·43	3·69	4	4·36	4·96	5·52	6·72
	0·020	2·86	3·08	3·24	3·64	4·14	4·60	5·60
	0·210	1·43	1·54	1·67	1·82	2·07	2·30	2·80
	1·000	0·86	0·93	1·00	1·09	1·24	1·40	1·68
1·5	0·001	3·52	3·78	4·10	4·47	5·10	5·66	7·00
	0·009	2·81	3·00	3·28	3·58	4·08	4·53	5·60
	0·020	2·36	2·52	2·74	2·98	3·40	3·78	5·34
	0·210	1·18	1·26	1·37	1·49	1·70	1·89	2·67
	1·00	0·71	0·76	0·82	0·89	1·02	1·13	1·40
2	0·001	3·01	3·27	3·54	3·83	4·40	4·90	6·00
	0·009	2·41	2·61	2·83	3·06	3·52	3·92	4·80
	0·020	2·02	2·18	2·36	2·56	2·94	3·28	4·00
	0·210	1·01	1·05	1·18	1·28	1·47	1·64	2·00
	1·00	0·61	0·66	0·71	0·77	0·88	0·98	1·20
	0·001	2·15	2·31	2·50	2·73	3·10	3·45	4·20
	0·009	1·72	1·85	2·00	2·18	2·48	2·76	3·36
	0·020	1·44	1·54	1·66	1·82	2·08	2·30	2·80
	0·210	0·72	0·77	0·83	0·91	1·04	1·15	1·40
	1·000	0·43	0·46	0·50	0·55	0·62	0·70	0·84

TABLE 54. PART I.—(*continued*).

Steam at 100° C. (atmospheric pressure), $c = 537$.								
Initial velocity of the steam. v_d	Velocity of the cooling water. v_f	Final temperature of the cooling water, t_{ke}.						
		20	30	40	50	60	70	80
		The cooling surface, H_c, in sq. m., required to condense 100 kilos. of steam per hour.						
9	0·001	1·43	1·54	1·67	1·82	2·07	2·30	2·80
	0·009	1·14	1·25	1·50	1·38	1·66	1·84	2·24
	0·020	0·90	1·02	1·12	1·22	1·38	1·54	1·88
	0·210	0·45	0·51	0·56	0·61	0·69	0·77	0·94
	1·000	0·29	0·31	0·36	0·37	0·42	0·46	0·56
16	0·001	1·08	1·16	1·25	1·36	1·55	1·73	2·10
	0·009	0·86	0·95	1·00	1·09	1·24	1·38	1·68
	0·020	0·58	0·64	0·68	0·74	0·84	0·92	1·12
	0·210	0·29	0·32	0·34	0·37	0·42	0·46	0·56
	1·000	0·22	0·24	0·25	0·27	0·31	0·35	0·42
20	0·001	0·96	1·04	1·12	1·22	1·38	1·54	1·88
	0·009	0·77	0·83	0·89	0·97	1·10	1·23	1·30
	0·020	0·64	0·70	0·75	0·82	0·90	1·02	1·26
	0·210	0·32	0·35	0·38	0·41	0·45	0·51	0·63
	1·000	0·20	0·21	0·23	0·25	0·28	0·31	0·38
25	0·001	0·86	0·93	1·00	1·09	1·24	1·38	1·68
	0·009	0·71	0·75	0·80	0·87	1·00	1·10	1·34
	0·020	0·58	0·62	0·67	0·72	0·64	0·90	1·12
	0·210	0·29	0·31	0·34	0·36	0·32	0·45	0·56
	1·000	0·17	0·19	0·20	0·22	0·25	0·28	0·34

difficult to ascertain. The efficiency of the condensing surfaces may then be taken at about 20 per cent. less than that given in the table, to which extent the surfaces should therefore be increased.

Example.—100 kilos. of steam at 100° C. are to be condensed and the liquid cooled to 15° C. The cooling water is originally at 10° and is to flow away at 60° C. The steam enters with the velocity, $v_d = 30$ m., the water with the velocity, $v_f = 0{\cdot}002$ m.

In order to condense 100 kilos. of steam, (637-100) 100 = 53,700 calories must be withdrawn from it. In order to cool 100 kilos. of water from 100° to 15° (100-15) 100 = 8500 calories must be abstracted.

TABLE 54. PART I.—(*continued*).

Steam at 100° C. (atmospheric pressure), $c = 537$.								
Initial velocity of the steam.	Velocity of the cooling water.	Final temperature of the cooling water, t_{ke}.						
		20	30	40	50	60	70	80
		The cooling surface, H_c, in sq. m., required to condense 100 kilos. of steam per hour.						
v_d	v_f							
30	0·001	0·78	0·84	0·92	1·00	1·15	1·26	1·54
	0·009	0·62	0·67	0·73	0·80	0·92	1·00	1·23
	0·020	0·52	0·56	0·62	0·67	0·76	0·84	1·04
	0·210	0·26	0·28	0·31	0·34	0·38	0·42	0·52
	1·000	0·16	0·17	0·19	0·20	0·23	0·26	0·31
36	0·001	0·72	0·77	0·83	0·91	1·04	1·15	1·40
	0·009	0·57	0·61	0·66	0·73	0·83	0·92	1·12
	0·020	0·48	0·52	0·56	0·62	0·76	0·78	0·95
	0·210	0·24	0·26	0·28	0·31	0·38	0·39	0·47
	1·000	0·15	0·16	0·17	0·19	0·21	0·23	0·28
49	0·001	0·62	0·66	0·72	0·78	0·89	1·00	1·20
	0·009	0·50	0·53	0·58	0·62	0·72	0·80	0·96
	0·020	0·42	0·44	0·48	0·58	0·60	0·68	0·80
	0·210	0·21	0·22	0·24	0·29	0·30	0·34	0·40
	1·000	0·13	0·14	0·15	0·16	0·18	0·20	0·24
64	0·001	0·54	0·58	0·63	0·68	0·78	0·87	1·05
	0·009	0·44	0·47	0·51	0·55	0·62	0·71	0·84
	0·020	0·36	0·38	0·42	0·46	0·52	0·58	0·70
	0·210	0·18	0·19	0·21	0·23	0·26	0·29	0·35
	1·000	0·11	0·12	0·13	0·14	0·16	0·18	0·21

According to Table 52, the temperature differences for the present case are $\theta_{mc} = 58·7°$ and $\theta_{mk} = 27·7°$, and the coefficient of transmission, according to Table 53, is in the first period (condensation) $k_c = 830$, and in the second period (cooling), according to Table 63, $k_k = 212$.

The cooling surface for the (first) period of condensation is therefore

$$H_c = \frac{C}{k^c\theta_{mc}} = \frac{53700}{830 \times 58·7} = 1·13 \text{ sq. m.}$$

The cooling surface for the (second) period of cooling would be

$$H_k = \frac{C}{k_k\theta_{mk}} = \frac{8500}{212 \times 27·7} = 1·44 \text{ sq. m.}$$

if it were all used. The cooler, however, is to be made in the form of a coil; the

TABLE 54. PART I.—(*continued*).

Initial velocity of the steam. v_d	Velocity of the cooling water. v_f	Steam at 60° C. Vacuum = 611 mm. c = 564.				Steam at 40° C. Vacuum = 705 mm. c = 577.		
		Final temperature of the cooling water, t_{ke}.						
		20	30	40	50	20	30	35
		Cooling surface, H_c, in sq. m., required to condense 100 kilos. of steam per hour.						
4	0·001	4·05	4·68	5·50	7·14	6·76	10·20	13·42
	0·009	3·24	3·90	4·20	5·85	5·41	8·16	10·73
	0·020	2·70	2·12	3·68	4·76	4·52	6·80	8·96
	0·210	1·35	1·56	1·84	2·38	2·26	3·40	4·48
	1·000	0·81	0·94	1·10	1·45	1·36	2·04	2·69
9	0·001	2·70	3·13	3·70	4·76	4·51	6·80	8·95
	0·009	2·16	2·50	2·96	3·81	3·61	5·44	7·16
	0·020	1·80	2·10	2·48	3·18	3·02	4·54	5·98
	0·210	0·90	1·05	1·24	1·59	1·51	2·27	2·99
	1·000	0·54	0·63	0·74	0·96	0·91	1·36	1·79
16	0·001	2·03	2·34	2·75	3·57	3·38	5·10	6·70
	0·009	1·62	1·87	2·20	2·86	2·71	4·08	5·16
	0·020	1·36	2·56	1·84	2·38	2·26	3·40	4·46
	0·210	0·68	0·78	0·92	1·19	1·13	1·70	2·23
	1·000	0·41	0·47	0·55	0·72	0·68	1·02	1·34
25	0·001	1·62	1·88	2·22	2·86	2·71	4·08	5·37
	0·009	1·30	1·50	1·77	2·31	2·19	3·26	4·30
	0·020	1·08	1·26	1·48	1·92	1·86	2·72	3·58
	0·210	0·54	0·63	0·74	0·96	0·93	1·36	1·79
	1·000	0·33	0·38	0·44	0·58	0·55	0·82	1·08
36	0·001	1·36	1·57	1·86	2·38	2·26	3·40	4·48
	0·009	1·09	1·26	1·51	1·90	1·81	2·72	3·59
	0·020	0·92	1·06	1·24	1·58	1·52	2·28	2·98
	0·210	0·46	0·53	0·62	0·79	0·76	1·14	1·49
	1·000	0·27	0·32	0·38	0·48	0·46	0·68	0·90

cooling surface must therefore be increased to about $3 \times 1·44 = 4·32$ sq. m., since only one-third is really active. The total surface is therefore

$$H_{ck} = 1·13 + 4·32 = 5·45 \text{ sq. m.}$$

TABLE 54. PART I.—(*continued*).

Aqueous alcohol vapour at 80° C. (80·3 per cent. strength by weight = 90 per cent. by volume).

Initial velocity of the vapour. v_d	Velocity of the cooling water. v_f	c = 252. Final temperature of the cooling water, t_{kw}. Cooling surfaces, H^c, in sq. m., required to condense 100 kilos. of vapour per hour. 20	30	40	50	60	70
1	0·001	2·60	3·03	3·33	3·87	4·59	6·18
	0·009	2·08	2·42	3·66	3·11	3·67	4·95
	0·020	1·74	2·02	2·22	2·58	3·06	4·12
	0·210	0·87	1·01	1·11	1·29	1·53	2·06
	1·000	0·52	0·61	0·66	0·78	0·92	1·24
2	0·001	1·84	2·15	2·36	2·74	3·25	4·38
	0·009	1·47	1·72	1·89	2·19	2·60	3·50
	0·020	1·24	1·44	1·58	1·84	2·18	2·98
	0·210	0·62	0·72	0·79	0·92	1·09	1·49
	1·000	0·37	0·43	0·48	0·55	0·65	0·88
4	0·001	1·30	1·57	1·67	1·94	2·30	3·09
	0·009	1·04	1·26	1·34	1·55	1·84	2·47
	0·020	0·88	1·06	1·12	1·30	1·54	2·06
	0·210	0·44	0·53	0·56	0·65	0·77	1·03
	1·000	0·26	0·32	0·34	0·39	0·46	0·62
6	0·001	1·04	1·21	1·33	1·55	1·84	2·47
	0·009	0·83	0·96	1·06	1·24	1·47	1·97
	0·020	0·70	0·82	0·90	1·06	1·24	1·66
	0·210	0·35	0·41	0·45	0·53	0·62	0·83
	1·000	0·21	0·24	0·27	0·32	0·37	0·50
9	0·001	0·87	1·01	1·11	1·29	1·53	2·06
	0·009	0·71	0·81	0·89	1·02	1·22	1·65
	0·020	0·58	0·68	0·74	0·86	1·04	1·38
	0·210	0·29	0·34	0·37	0·43	0·52	0·69
	1·000	0·18	0·21	0·22	0·26	0·31	0·42

In the practical construction of apparatus the original temperature of the water is frequently unknown, and also several other conditions

Table 54. Part II.

Velocity of the cooling water. v_f	The cooling surface, H_k, for cooling.											Velocity of the cooling water. v_f
	100 kilos. of condensed steam at 100° C. per hour.						100 kilos. of condensed steam at 60° C. (611 mm. vacuum) per hour.					
	Temperature difference between initial temperature of the cooling water and final temperature of the condensed liquid.											
	2°	5°	10°	15°	20°	25°	2°	5°	10°	15°	20°	
	Cooling surface in sq. m.											
0·001	2·00	1·52	1·15	0·92	0·80	0·70	1·60	1·18	0·83	0·63	0·50	0·001
0·009	1·60	1·21	0·92	0·73	0·64	0·56	1·28	0·95	0·66	0·54	0·40	0·009
0·020	1·40	1·06	0·81	0·64	0·56	0·49	1·12	0·83	0·58	0·44	0·35	0·020
0·210	0·86	0·65	0·48	0·40	0·35	0·31	0·69	0·51	0·36	0·27	0·22	0·210
1·000	0·60	0·46	0·34	0 28	0·24	0·21	0·48	0·35	0·25	0·19	0·15	1·000
	100 kilos. of condensed steam at 40° C. (705 mm. vacuum) per hour.						100 kilos. of condensed aqueous alcohol at 80° C. (86·8 per cent. by weight).					
	Cooling surface in sq. m.											
0·001	1·40	0·90	0·56	0·36	0·22	—	1·35	1·07	0·80	—	—	0·001
0·009	1·12	0·72	0·45	0·29	0·18	—	1·08	0·86	0·64	—	—	0·009
0·020	0·98	0·63	0·40	0·25	0·16	—	0·95	0·75	0·56	—	—	0·020
0·210	0·60	0·39	0·24	0·16	0·10	—	0·58	0·46	0·35	—	—	0·210
1·000	0·42	0·27	0·16	0·11	0·06	—	0·41	0·32	0·24	—	—	1·000

The initial temperature of the cooling water is taken at $t_{kw} = 10°$ C.

These cooling surfaces hold good only for surfaces *entirely wetted*. In the case of vertical tubular coolers these surfaces must be at least doubled, in worm coolers they must be at least trebled.

cannot be exactly estimated beforehand; it is therefore necessary to make allowances for these uncertainties. The following assumptions appear to be quite reasonable:—

	Steam.			Aqueous alcohol vapour.
The vapour to be condensed is at - - - -	100°	60°	40°	80°
It enters the cooling coil with the velocity - - $v_d =$	30-50	40-60	45-65	4-5 m.
It enters the tubular cooler with - - - $v_d =$	20-30	20-30	25-35	2-3 m.
The velocity of the water should be as great as possible and at least - - - $v_{fl} =$	0·001	0·001	0·001	0·001 m.
The initial temperature of the water is taken at - $t_{ka} =$	10°	10°	10°	10°
The final temperature of the water is taken at - $t_{ke} =$	70°-80°	40°-50°	30°	60°
The condensed liquid is cooled down to - - - $t_{we} =$	15°	15°	15°	12°

For the sake of convenience in making similar calculations two other tables are given, the first of which, Table 55, contains the weights of steam at 100°, 60°, 40° and 35° C., and of alcohol vapour, ether vapour and air, which pass through pipes of 10-100 mm. diameter in one hour with a velocity of 1 m. per second. At any other velocity, v_d, the weight of vapour passing is v_d times as great.

The second Table, 56, gives the quantity of water which rises in one hour with a velocity of 0·001 m. in vessels of 300-1250 mm. diameter. If the velocity be v_{fl} the quantity of water is v_{fl} times as great. If the quantity of water and the diameter of the vessel are known, Table 56 gives the velocity, v_{fl}.

(d) Estimation of the Dimensions, d and l, of the Cooler Tubes.

As with evaporator tubes (Chapter VIII., Table 13) so also with condenser tubes, in which vapour is to be liquefied, it is necessary to calculate not only their cooling surface, H_c, but also the actual measurements, *i.e.*, to estimate their length and diameter, since too long tubes would be inactive at the end.

TABLE 55.

The weight of steam, in kilos., which passes through tubes of 10-100 mm. in diameter in one hour at the velocity, $v_d' = 1$ m. per second.

Steam. Pressure. Atmos. abs.	Steam. Temperature. °C.	Diameter of the tube in mm. 10	15	20	25	30	35	40	50	60	70	80	90	100
3	134	0·48	1·08	1·92	3·00	4·32	6·00	7·65	11·8	17·2	23·5	30·6	38·9	48·9
2·5	128	0·40	0·91	1·60	2·62	3·66	5·00	6·43	9·78	14·5	18·9	25·7	32·7	40·0
2	121	0·33	0·74	1·31	2·05	2·96	4·00	5·28	7·95	11·8	16·1	20·9	26·6	33·0
1·5	112	0·25	0·56	1·00	1·56	2·24	3·00	4·00	6·03	8·99	12·8	15·9	20·8	25·0
1	100	0·17	0·383	0·685	1·07	1·54	2·10	2·73	4·27	6·16	8·48	10·9	13·9	17·0
0·196	60	0·04	0·083	0·143	0·23	0·33	0·43	0·59	0·93	1·33	1·79	2·26	3·00	3·66
0·1[illegible]1	50	0·023	0·053	0·093	0·15	0·21	0·29	0·38	0·60	0·87	1·14	1·50	1·90	2·34
0·072	40	0·014	0·033	0·058	0·09	0·13	0·18	0·23	0·36	0·5	0·70	0·92	1·17	1·43
0·055	35	0·011	0·015	0·049	0·07	0·10	0·14	0·18	0·28	0·40	0·54	0·72	0·91	1·11
		The weight of the vapour of aqueous alcohol.												
1	80°	0·39	0·88	1·55	2·40	3·50	4·80	6·25	10·0	14·0	19·0	25·0	31·8	39·0
		The weight of ether vapour.												
1	37·5°	0·80	1·70	3·10	5·00	7·00	9·60	12·5	20·0	30·0	41·0	53·0	66·0	82·0
		The weight of air.												
1	15°	0·35	0·78	1·38	2·16	3·11	4·21	5·54	8·65	12·5	16·9	21·1	28·0	34·6

TABLE 56.

The weight of water, W, which rises in one hour at the velocity, $v_f = 0{\cdot}001$ m., through vessels of 300-1250 mm. diameter.

Diameter of vessel-	300	350	400	450	500	550	600	650	700	750
Weight of water, W	252	345	452	572	705	855	1017	1194	1385	1590
Diameter of vessel-	800	850	900	950	1000	1050	1100	1150	1200	1250
Weight of water, W	1800	2042	2289	2520	2820	3117	3420	3738	4068	4417

From the condition, that the quantity of heat given up by the condenser tube to the cooling water in unit time must be equal to

the heat of evaporation (or condensation) of the vapour introduced, we obtain the equation:

$$H_c k_c \theta_{mc}' = \frac{d^2\pi}{4} v_d 3600 c\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad (204)$$

Inserting the values of H_c and k_c, we obtain

$$d\pi l 750 \sqrt{v_d} \sqrt[3]{0{\cdot}007 + v_f}\, \theta_{mc} = \frac{d^2\pi}{4} v_d 3600 c\gamma,$$

from which

$$\frac{l'}{d} = 1{\cdot}2 \frac{c\gamma}{\theta_{mc}} \frac{\sqrt[2]{v_d}}{\sqrt[3]{0{\cdot}007 + v_f}} \quad . \quad . \quad . \quad . \quad . \quad (205)$$

From this equation, the most advantageous proportion of the length to the diameter of the condenser tube may be calculated for each special case.

The great number of possible variations, due to the many variable factors, compels a restricted choice of the cases to be treated in tabular form.

In Table 57 are arranged the ratios of the dimensions of the tube, $\frac{l}{d}$, calculated by means of equation (205), for the condensation of steam at 134°, 121°, 100°, 60° and 40° C., and alcohol vapour at 80° C. (86·3 per cent. by weight = 90·4 per cent. by volume), which enter the tube with velocities, v_d = 4-64 m., for water velocities of v_f = 0·001-3·0 m. and mean temperature differences, θ_m = 10°-70°.

The following is the method of using the table: After fixing the desired entrant velocity of the steam, v_d, the suitable diameter of the tube is obtained, for the quantity of steam to be condensed, from Table 55 by a slight calculation. Table 52 gives also the temperature differences in both periods (condensing and cooling) for the known or assumed initial and final temperatures of the cooling water. Table 57 gives from these the proper ratio of the length of the tube to its diameter.

The size of the resulting surface of *condensation*, H_c, may then be calculated from the dimensions of the tube.

The surfaces, H_k, required for *cooling* may be taken direct from Part II. of Table 54 and multiplied by 2 or 3 before use.

All these assumptions and tables are for copper and brass tubes for those of iron or lead the additions, already frequently mentioned, must be made.

TABLE 57.

The ratio, $\frac{\text{length of pipe}}{\text{diameter of pipe}} = \frac{l}{d}$, of copper condensing pipes (coils) for steam at 134°, 121°, 100°, 60°, 40° C., and aqueous alcohol vapour at 80° C. (86·3 per cent. by weight), when the vapour enters at velocities of v_d = 1-64 m. and the cooling water has velocities of v_f = 0·001-3·0 m., with temperature differences between vapour and cooling water of θ_m = 10°-70° C.

Velocity of cooling water. v_f	Mean temperature difference. θ_m	Steam at 121° C. (2 atmos. abs.) Velocity of steam on entering, v_d, in m.							Velocity of cooling water. v_f	Mean temperature difference. θ_m	Steam at 134° C. (3 atmos. abs.) Velocity of steam on entering, v_d, in m.						
		4	9	16	25	36	49	64			4	9	16	25	36	49	64
m.	° C.	Ratio, $\frac{l}{d}$.							m.	° C.	Ratio, $\frac{l}{d}$.						
0·020	90	60	90	120	150	180	210	240	0·020	90	88	132	174	220	264	308	350
	80	67	102	136	170	204	238	270		80	98	146	198	244	294	342	394
	70	76	114	154	190	228	266	308		70	112	168	224	280	336	392	450
	60	90	136	180	222	270	314	360		60	132	198	264	320	396	462	526
	50	108	162	216	270	324	378	432		50	158	236	316	394	474	580	630
	40	136	202	270	340	406	476	540		40	196	294	394	490	588	686	788
	30	180	270	360	490	540	630	720		30	264	396	526	660	792	924	1052
	20	270	410	540	670	810	938	1080		20	394	590	788	980	1182	1372	1578
0·210	90	30	45	60	75	90	105	120	0·210	90	44	66	87	110	132	154	175
	80	34	51	68	85	102	119	135		80	49	73	98	122	147	171	197
	70	38	57	77	95	114	133	154		70	56	84	112	140	168	196	225
	60	45	68	90	111	135	157	180		60	66	99	132	160	198	231	263
	50	54	81	108	135	162	189	216		50	79	118	158	197	237	275	315
	40	68	101	135	170	203	238	270		40	98	147	197	245	294	343	394
	30	90	125	180	245	270	315	360		30	132	108	263	330	396	462	526
	20	135	205	270	335	405	469	540		20	197	295	394	490	591	686	789
1·00	90	18	27	30	45	54	63	72	1·00	90	26	39	52	65	78	91	105
	80	20	30	40	50	60	70	81		80	29	43	59	72	87	101	118
	70	23	34	46	56	69	80	93		70	34	51	68	85	102	119	135
	60	27	40	54	67	81	94	108		60	39	58	79	97	117	129	158
	50	33	50	65	82	99	115	129		50	47	70	94	117	141	164	189
	40	40	60	81	100	120	140	162		40	59	88	118	177	177	206	236
	30	54	81	108	135	162	189	216		30	79	118	157	195	231	306	315
	20	81	121	162	205	243	283	324		20	118	177	237	295	354	413	473
3·00	90	10	15	21	25	30	35	42	3·00	90	19	28	37	47	57	66	73
	80	12	18	24	30	36	42	48		80	21	31	42	52	63	71	83
	70	14	21	28	35	42	49	55		70	24	36	47	60	72	84	94
	60	16	24	32	40	48	56	64		60	27	40	54	67	81	94	109
	50	19	28	38	47	57	68	76		50	33	50	66	82	99	115	131
	40	24	36	48	60	72	84	95		40	41	61	82	102	123	143	165
	30	32	48	64	80	96	112	127		30	55	82	110	137	165	178	219
	20	47	71	95	117	141	164	190		20	83	125	165	206	249	290	329

TABLE 57—(*continued*).

Velocity of cooling water, v_f	Mean temperature difference, θ_m	Steam at 100° C. Velocity of steam on entering, v_d, in m.							Velocity of cooling water, v_f	Mean temperature difference, θ_m	Steam at 60° C. Velocity of steam on entering v_d, in m.						
		4	9	16	25	36	49	64			4	9	16	25	36	49	64
m.	° C.	Ratio, $\frac{l}{d}$.							m.	° C.	Ratio, $\frac{l}{d}$.						
0·001	70	55·7	82·5	111	139	166	195	220	0·001	50	18	26	35	44	53	62	71
	60	65	97	130	162	195	227	260		40	22	33	44	55	67	78	89
	50	78	117	156	195	234	273	312		30	29	44	59	74	88	103	118
	40	97	146	194	243	282	340	390		20	44	66	88	110	133	145	177
	30	130	195	260	325	390	455	520	0·009	50	14	21	28	36	43	50	57
0·009	70	44·6	67	89	111	133	156	178		40	18	26	35	44	53	62	71
	60	52	78	104	130	156	182	208		30	24	35	47	59	70	83	94
	50	62	93	125	156	187	218	249		20	35	53	71	89	106	124	142
	40	78	117	156	195	234	273	312	0·020	50	12	18	24	30	34	41	47
	30	102	156	208	260	312	364	416		40	15	22	30	37	44	52	59
0·020	70	37	55	74	93	117	130	148		30	20	30	40	50	59	69	79
	60	43	65	86	108	130	151	173		20	30	44	58	74	89	104	118
	50	52	78	104	130	156	182	208	0·210	50	6	9·1	12	15	17	20	24
	40	64	97	130	162	195	227	260		40	7·5	11	15	18	22	26	30
	30	87	130	173	216	259	303	346		30	10	15	20	25	30	35	40
0·210	70	19	28	37	46	55	65	75		20	15	22	30	37	44	52	59
	60	22	33	44	55	66	77	88	1·000	50	3·6	5·3	7·1	9	11	12	14
	50	26	39	52	65	78	91	104		40	4·4	6·7	8·9	11	13·3	15·5	17·7
	40	33	49	65	81	97	114	130		30	6	9	12	15	17	20	24
	30	44	65	86	108	130	152	173		20	8·9	13·2	17·7	22	27	31	35
1·000	70	11	16	22	28	34	40	45									
	60	13	19	26	33	39	46	52									
	50	16	23	31	39	47	55	62									
	40	20	29	39	49	59	69	76									
	30	26	39	52	65	78	91	104									
3·000	70	8	12	16	20	24	28	32									
	60	9	13·5	18	22·5	27	31·5	36									
	50	11	16	21	27	32	38	43									
	40	17·5	20·5	27	34	41	48	55									
	30	18	27	36	45	54	63	72									

In the case of oily substances, or of steam which is bringing oily substances with it, the calculated heating surfaces must be approximately doubled for practical use, because oily matter sticks to the walls and considerably diminishes the conduction of heat.

The figures apply only to pipes of circular section, which are generally used; for pipes of other sections different values must be taken.

TABLE 57—(*continued*).

Velocity of cooling water. v_f	Mean temperature difference. θ_m	Steam at 40° C. Velocity of steam on entering, v_d, in m.						
		4	9	16	25	36	49	64
m.	°C	Ratio, $\frac{l}{d}$.						
0·001	30	12	18	24	30	36	42	48
	20	18	27	36	45	54	63	72
	15	24	36	48	60	72	84	96
	10	36	54	72	90	108	126	144
0·009	30	9	14	19	23	28	33	37
	20	14	21	28	35	42	49	56
	15	19	28	37	46	56	65	74
	10	28	42	56	70	84	98	112
0·020	30	8	12	16	20	24	27	31
	20	12	18	24	30	35	41	47
	15	16	24	32	40	48	56	64
	10	24	35	47	59	71	88	94
0·210	30	4	6	8	10	12	14	16
	20	6	9	12	15	18	21	24
	15	8	12	16	20	24	28	32
	10	12	18	24	30	36	42	48
1·000	30	2·3	3·5	4·6	6	7·0	8·3	9·5
	20	3·5	5·3	7·1	8·9	10·6	12·5	14·0
	15	4·7	7·1	9·5	11·8	14·2	16·5	19·0
	10	7·1	10·6	14·2	17·7	19·3	24·8	28·4

Velocity of cooling water. v_f	Mean temperature difference. θ_m	Aqueous alcohol vapour at 80° C. = 86·3 per cent. by weight = 90 per cent. by volume. Velocity of vapour on entering, v_d, in m.					
		1	2	4	6	9	16
m.	°C.	Ratio, $\frac{l}{d}$.					
0·001	60	30·7	43	61	74	92	122
	50	37	52	74	89	111	148
	40	46	65	92	111	138	184
	30	61	85	122	146	183	244
	20	92	124	184	216	276	368
0·009	60	24·5	34	49	59	73	98
	50	29	40	58	69	87	116
	40	37	52	74	89	111	148
	30	49	60	98	109	147	196
	20	74	104	148	178	222	296
0·020	60	20·5	29	41	50	61	82
	50	24·6	34	49	59	74	98
	40	30·8	43	62	74	92	123
	30	41	58	82	99	123	164
	20	61	85	122	146	183	244
0·210	60	10·2	15	20	25	31	41
	50	12·3	17	25	29	37	49
	40	15·3	21	31	36	46	61
	30	20·4	29	41	49	61	81
	20	30·6	43	61	74	92	122
1·000	60	6·1	8·5	12	15	18	24
	50	7·4	10·4	15	18	22	29
	40	9·2	12·4	18	22	28	37
	30	12·3	17	25	29	37	49
	20	18·4	26	37	44	55	73

Example.—300 kilos. of steam at 100° C. are to be condensed, and the condensed water cooled down to 20° C., by means of water which becomes heated from 10° to 70°.

The velocity at which the steam enters is taken to be about 40 m. and the upward velocity of the cooling water to be $v_f = 0{\cdot}001$ m.

According to Table 55, 300 kilos. of steam pass through a pipe of 65 mm. bore in one hour with a velocity of 42 m. Thus the bore of the tube is fixed at 65 mm.

Table 52 shows that, under the conditions given, the mean temperature difference in condensing, $\theta_{mc} = 52{\cdot}5°$, and in cooling, $\theta_{mk} = 34{\cdot}3°$.

It then follows from Table 57 (by interpolation) that $\frac{l}{d} = 242$, hence the

Table 58.

Examples of the dimensions of condensing and cooling tubes of 10-100 mm. diameter, for steam at 100°, 60°, 40°, and aqueous alcohol vapour at 80° C., for velocities of 40-20 and 2 m. respectively.

Diameter of tube, mm.	10	15	20	25	30	35	40	50	60	70	80	90	100
	Steam at 100°, entering with the velocity, $v_d = 40$ m. Water heated from 10° to 70°; velocity of water, $v_f = 0·001$ m. Condensed liquid at 15°; $\theta_{mc} = 52·5°$, $\theta_{mk} = 27·4°$, $\frac{l}{d} = 234·7$. Copper coils.												
Steam condensed by tube per hour, kilos.	6·80	15·2	27·4	40·3	61 5	84·0	109	171	246	339	438	554	680
For condensation, length	2·35	3·52	4·70	5·87	7·00	8·21	9·38	11·7	14·3	16·4	18·8	21·1	23·5
For condensation, sq. m.	0·07	0·165	0·295	0·46	0·56	1·00	1·17	1·84	2·68	3·79	4·70	5·96	7·87
For cooling, length	10·5	15·0	21·5	24 0	33·0	36·0	40·0	50·0	60·0	71·0	80·0	90·0	99·0
For cooling, sq. m.	0·30	0·69	1·38	1·84	3·14	3·84	4·97	7·80	11·2	15·5	20·0	21·3	30·9
Total length of tube, l	13·0	18·5	26·7	29·8	40·0	44·2	49·5	62·0	74·5	87 4	98·8	103	123
	Steam at 100°, entering with the velocity, $v_d = 20$ m. Water heated from 10° to 70°; velocity of water, $v_f = 0·001$ m. Condensed liquid at 15°; $\theta_{mc} = 52·5°$, $\theta_{mk} = 27·4°$, $\frac{l}{d} = 170$. Vertical cooling tubes.												
Steam condensed by tube per hour, kilos.	3·4	7·6	13·7	20·2	30·8	42·0	54·5	85·5	123	169	219	277	340
For condensation, length	1·70	2·35	3·40	4·05	5·10	5·75	6·80	8·50	10·2	11·9	13·6	15 3	17·0
For condensation, sq. m.	0·052	0·11	0·22	0·31	0·51	0·61	0·85	1·33	1·91	2·00	3·38	4·28	5·20
For cooling, length	4·00	4·80	6·80	8·00	10·0	11·81	13·0	16·3	20·0	23·2	26·4	29·8	32·4
For cooling, sq. m.	0·12	0·23	0·42	0·62	0·93	1·26	1·64	2·58	3·7	5·08	6·58	8·32	10·2
Total length of tube, l	5·70	7·15	10·2	19·1	15·1	17·6	19·8	25·0	30·2	35 5	40·4	45·5	47·4
	Steam at 60°, entering with the velocity, $v_d = 40$ m. Water heated from 10° to 40°; velocity of water, $v_f = 0·001$ m. Condensed liquid at 15°; $\theta_{mc} = 31·7°$, $\theta_{mk} = 19·2°$, $\frac{l}{d} = 95$. Vertical tubes.												
Steam condensed by tube per hour, kilos.	1·48	3·30	5·70	9·20	13·2	17·2	23·6	37·2	52·2	71 6	97·4	120	148
For condensation, length	0·95	1·43	1·90	2·38	2·85	3·33	3·80	4·75	5·70	6·65	7·60	8·55	9·50
For condensation, sq. m.	0·03	0·07	0·12	0·18	0·28	0·37	0·45	0·74	1·06	1·46	1·90	2·39	3·00
For cooling, length	1·10	1·75	2·20	2·80	3·20	4·00	4·40	5·60	6·60	7·70	8·80	10·0	11·1
For cooling, sq. m.	0·034	0·08	0·13	0·22	0·30	0·41	0·55	0·88	1·28	1·68	2·22	2·84	3·46
Total length of tube, l	2·05	3·18	4·10	5·18	6·05	7·33	8·20	10·35	12·3	14·4	66·4	18·6	20·6

TABLE 58—(*continued*).

Diameter of tube, mm.	10	15	20	25	30	35	40	50	60	70	80	90	100
	Steam at 60°, entering with the velocity, $v_d = 20$ m. Water heated from 10° to 40°; velocity of water, $v_f = 0{\cdot}001$ m. Condensed liquid at 15°; $\theta_{mc} = 31{\cdot}7°$, $\theta_{mk} = 19{\cdot}2°$, $\frac{l}{d} = 65$. Vertical tubes.												
Steam condensed by tube per hour, kilos.	0·74	1·66	2·86	4·62	6·60	8·64	11·8	18·6	26·6	35·8	47·2	60·0	73·2
For condensation, length	0·65	0·97	1·30	1·63	1·95	2·27	2·6	3·25	3·90	4·55	5·10	5·85	6·50
For condensation, sq. m.	0·02	0·04	0·08	0·12	0·19	0·25	0·33	0·51	0·73	1·00	1·27	1·63	2·00
For cooling, length	0·55	0·88	1·10	1·40	1·60	2·00	2·20	2·80	3·30	3·90	4·40	5·00	5·50
For cooling, sq. m.	0·02	0·04	0·07	0·11	0·16	0·21	0·28	0·44	0·73	0·84	1·11	1·42	1·78
Total length of tube, l	1·20	1·85	2·30	3·00	3·55	4·27	4·80	6·05	7·20	8·48	9·50	10·8	12·0
	Steam at 40°, entering with the velocity, $v_d = 20$ m. Water heated from 10° to 30°; velocity of water, $v_f = 0{\cdot}001$ m. Condensed liquid at 15°; $\theta_{mc} = 18°$, $\theta_{mk} = 13{\cdot}7°$, $\frac{l}{d} = 45$. Vertical tubes.												
Steam condensed by tube per hour, kilos.	0·28	0·65	1·15	1·80	2·50	3·50	4·60	7·20	10·0	14·0	19·4	23·4	28·6
For condensation, length	0·45	0·68	0·90	1·10	1·35	1·58	1·80	2·25	2·70	3·15	3·60	4·05	4·50
For condensation, sq. m.	0·014	0·03	0·06	0·087	0·13	0·17	0·25	0·35	0·50	0·80	0·90	1·13	1·4
For cooling, length	0·16	0·26	0·34	0·42	0·48	0·60	0·70	0·83	1·00	1·20	1·40	1·60	1·70
For cooling, sq. m.	0·005	0·012	0·021	0·032	0·045	0·063	0·083	0·13	0·18	0·26	0·34	0·42	0·51
Total length of tube, l	0·61	0·94	1·24	1·55	1·83	2·18	2·50	3·05	3·70	4·35	5·00	5·65	6·20
	Aqueous alcohol vapour at 80°, entering with the velocity, $v_d = 2$ m. Water heated from 10° to 60°; velocity of water, $v_f = 0{\cdot}001$ m. Condensed liquid at 12°; $\theta_{mc} = 36{\cdot}6°$, $\theta_{mk} = 17{\cdot}4°$, $\frac{l}{d} = 75$.												
	Vertical tubes.						Coils.						
Vapour condensed by tube per hour, kilos.	0·78	1·76	3·10	4·80	7·00	9·60	12·5	20·0	28·0	38·0	50·0	63·6	78
For condensation, length	0·75	1·13	1·50	1·88	2·25	2·63	3·00	3·75	4·50	5·25	6·00	6·75	7·50
For condensation, sq. m.	0·023	0·052	0·095	0·15	0·22	0·28	0·38	0·58	0·84	1·16	1·50	1·87	2·3
For cooling, length	0·7	1·06	1·40	1·80	2·0	2·50	4·40	5·10	6·25	7·00	8·10	9·90	10·5
For cooling, sq. m.	0·022	0·05	0·084	0·13	0·19	0·25	0·51	0·81	1·17	1·59	2·08	2·75	3·15
Total length of tube, l	1·45	2·2	2·9	3·68	4·25	5·5	7·4	8·9	10·8	12·3	14·1	16·7	18

length of pipe for the condensation is $l = 0.065 \times 242 = 15.73$ m. and the condensing surface $H_c = 3.21$ sq. m.

According to Table 54, the cooling surface must be $H_k = 8 \times 8 \times 1.15 = 10.50$ sq. m., *i.e.*, a pipe of 65 mm. diameter must be 50·8 m. long. The whole condensing and cooling pipe has therefore a length of $15.73 + 50.8 = 66.53$ m. and a surface of $H_{ck} = 3.21 + 10.5 = 13.71$ sq. m.

Since it is impossible to unite all cases, some important ones, chosen from the great number, are alone given in Table 58.

Observations.—Several experiments, calculated out, are now given.

	Water.			Alcohol, 93 per cent. by weight.		Water + Oil.		
Weight of vapour, D, condensed per hour - - kilos.	345	295	3750	139·5	120	315	84	88·2
Oily matter carried in the vapour kilos.	—	—	—	—	—	77	326	31
Temperature of the vapour on entering - - - - -	100°	100°	100°	79°	79°	121°	88°	110°
Temperature of the condensed liquid - - - - - -	34°	25°	100°	5°	79°	26°	22°	22°
Material of the cooling surface -	brass	brass	wrought iron	copper	copper	cast iron	lead	copper
Number and diameter of the tubes	2 × 67	2 × 67	160 × 27	21 × 5	55 × 29	1 × 75	1 × 50	1 × 40
Initial temperature of the cooling water - - - - -	10°	10°	40°	2·5°	8°	6°	10°	13°
Final temperature of the cooling water - - - - - -	75°	65°	96°	20°	61°	48°	42°	38°
Velocity of the cooling water v_f	0·001	0·001	0·032	0·0015	0·002	0·001	0·001	0·001
Actual cooling surface - sq. m.	9·1	9·5	67	6	7	32(a)	14·5(a)	6·3 (a)
Calculation.								
Calories to be abstracted in condensing - - - - -	185262	157341	2130000	32177	68964	170100	45696	47628
Calories to be abstracted in cooling	22770	21976	—	7562	—	13310 2000(b)	5540 8476(b)	6864 860(b)
Temperature of the water at the point of condensation - -	17·1°	16·6°	—	5·6°	—	31·5°	25°	17°
Mean temperature difference in condensing - - - θ_{mc}	48·6°	55·8°	21·6°	67°	42·9°	70°	54·8°	75°
Mean temperature difference in cooling - - - - θ_{mk}	48°	39·8°	—	20·1°	—	39·7°	31·5°	32·2°
Entering velocity of the vapour v_d	22·9	19·5	36	2·73	1·7 0·5	32·8	29	32
Coefficient of transmission in condensing - - - k_c	718·5	663	1425	240	222	855	807	847
Coefficient of transmission in cooling - - - - k_k	$\frac{290}{2}$	$\frac{292}{2}$	—	$\frac{290}{2}$	—	$\frac{292}{4}$	$\frac{290}{4}$	$\frac{290}{3}$
Cold surface for condensing - H_c	5·30	4·26	69	1·96	7·2	3·31	1·00	0·79
Cold surface for cooling - H_k	4·74	5·40	—	3·78	—	12·80	8·88	2·34
Calculated cold surface sq. m.	10·04	9·66	69	5·74	7·2	16·1	9·88	3·16

(a) The exterior surface of the tubes.

(b) The upper figures, 13310, 5540, 6864, are the numbers of calories to be abstracted from the water, the lower figures, 2000, 8476, 860, the calories to be abstracted from the oil.

2. Closed Surface-Condensers with Air Cooling.

In certain rare cases the condensation or cooling is effected by means of air instead of water. The air is then driven over the cooling surfaces by artificial means (fans) or by a natural draught. In both cases it is in the first place necessary to know the quantity of air required to abstract a definite amount of heat, so that the dimensions of the fan and flues may be determined.

Let L be the weight of the air in kilos., $\sigma_l = 0{\cdot}2375$ its specific heat at constant pressure, which is in this case always that of the atmosphere, t_{la} the initial and t_{le} the final temperatures of the air, C the heat, in calories, to be transferred, then

$$L \doteq \frac{C}{\sigma_l(t_{le} - t_{la})} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (206)$$

Thus there are required, in order to take up 100 units of heat from or by the air, if it is to be cooled or heated through

20°	30°	40°	50°	60°	70°	80°	90°	100° C.	
21·05	14·03	10·52	8·42	7·01	6·01	5·25	4·68	4·21	kilos. of air.

The *volume* of the dry air, when the pressure remains constant (which is the case here), depends only on its temperature. 1 cub. m. of dry air at 0° C. and 760 mm. pressure weighs 1·293 kilos., thus under these conditions 1 kilo. of air occupies a space of

$$\frac{1000}{1{\cdot}293} = 773 \text{ litres.}$$

The increase in volume of the air is proportional to the increase in temperature, measured from absolute zero; 1 kilo. of air at the temperature t_{le} thus occupies a space of

$$a_l = \frac{1000(273 + t_{le})}{1{\cdot}293 \times 273} = 773\left(1 + \frac{t_{le}}{273}\right) \text{ litres} \quad . \quad . \quad (207)$$

Example.—At 50° C. and 760 mm. pressure 1 kilo. of air occupies a space of

$$773\left(1 + \frac{50}{273}\right) = 915 \text{ litres.}$$

In Table 59 are given the volumes, a_l, in litres, calculated by means of equation (207), occupied by 1 kilo. of dry air, at the normal barometric height of 760 mm. and various temperatures from − 20° to 400° C. Now, atmospheric air always contains some water vapour —at 15° C. about 0·5-1 per cent. of its weight. The specific heat of

TABLE 59.

The volumes, a_t, of 1 kilo. of dry air at the normal barometric height of 760 mm. and at temperatures from − 20° to 400° C.

Temperature of the air.	1 kilo. of air has the volume, a_t.	Temperature of the air.	1 kilo. of air has the volume, a_t.	Temperature of the air.	1 kilo. of air has the volume, a_t.	Temperature of the air.	1 kilo. of air has the volume, a_t.	Temperature of the air.	1 kilo. of air has the volume, a_t.
° C.	Litres.	° C.	Litres.	° C.	Litres.	° C.	Litres.	° C.	Litres.
− 20	716	60	942	145	1183	235	1438	320	1679
− 15	730	65	956	150	1197	240	1452	325	1693
− 10	745	70	970	155	1211	245	1466	330	1708
− 5	759	75	984	160	1225	250	1480	335	1721
0	773	80	999	165	1249	255	1494	340	1736
1	775	85	1013	170	1254	260	1509	345	1750
5	789	90	1027	175	1268	265	1513	350	1764
10	802	95	1038	180	1282	270	1537	355	1778
15	816	100	1056	185	1296	275	1551	360	1793
20	831	105	1070	190	1319	280	1565	365	1807
25	847	110	1084	200	1330	285	1579	370	1821
30	858	115	1098	205	1344	290	1594	375	1835
35	872	120	1112	210	1367	295	1608	380	1849
40	886	125	1126	215	1381	300	1623	385	1853
45	900	130	1140	220	1396	305	1637	390	1876
50	914	135	1154	225	1410	310	1651	395	1890
55	928	140	1169	230	1424	315	1665	400	1905

When the barometer is at 740 mm. the volume of the air is about 3 per cent. larger, at 780 mm. the volume is about 3 per cent. less.

water vapour is $\sigma_a = 0{\cdot}475$, about double that of air, but the small quantity of vapour in the air causes such a slight increase in the amount of heat required to raise its temperature that we may neglect it in the present case.

The transfer of heat between air in motion and a metal surface (heating surface) may be expressed by the following equation, according to the results of the researches of Joule and Ser and the work of Molier:

$$k_t = 2 + 10\sqrt{v_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (208)$$

in which v_l is the velocity of the air in m. per second. Thus the heating surface, H_l, necessary for the transference of the quantity of heat, C, in the time, z_h (in hours), with the temperature difference, θ_m, is

$$H_l = \frac{C}{z_h\theta_m k_l} = \frac{C}{z_h\theta_m(2 + 10\sqrt{v_l})} \quad . \quad . \quad . \quad . \quad (209)$$

The state of rest, or of motion over the heating surface, of the vapour or water to be cooled is not regarded in the equation (208) which gives the transmission coefficient, k. It is always found, however, that the rapidity of the circulation of vapour or water over heating and cooling surfaces influences very considerably the quantity of heat transferred. There is no doubt this would also be the case with cooling by air, hence we cannot regard the expression (208) as quite correct. Reliable researches on this point are, however, not yet known, and the author has no observations of his own; it is therefore necessary for the present to be content with the above value for k_l. It may be assumed that, in the experiments of which the formula (208) is the result, the velocities of steam and water were not very great, so that with a rapid motion of these substances the transference will be rather greater than calculation indicates.

The temperature difference between air and heating surface is to be taken as the mean. If the entering and leaving temperatures of the water or vapour to be cooled are known, the mean temperature difference, θ_m, is easily found by Table 52, by supposing the cooling air in place of the cooling water.

Example.—The temperature of the vapour to be condensed and cooled is 100° C., the temperature of the condensed liquid is to be 20°; the air enters at 15° and leaves at 60° C. Then the mean difference in temperature, according to Table 52, is:

For the period of condensation . . $\theta_{mc} = 56{\cdot}8°$.
For the period of cooling . . . $\theta_{mk} = 26{\cdot}8°$.

If the temperature difference be obtained in this way and the velocity of the air then fixed, then, in Table 60, calculated by means of equation (209), is found the cooling surface required to transfer 1000 calories in one hour with air velocities of 1-36 m. per second and temperature differences of 5°-100° C.

Finally, the *section* is to be determined across which the air must flow, which depends on the velocity given to the air.

If V_l be the volume of air, in litres, to be sent through the condenser in one hour, q the section of the air channel in sq. dcm., and v the velocity of the air in m. per second, then

$$V_l = qv_l\,3600 \times 10 \quad . \quad . \quad . \quad . \quad . \quad . \quad (210)$$

or

$$q = \frac{V_l}{v_l 36000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (211)$$

An example is calculated in order to make clear the method of estimating the heating surface and section of the air passage.

Example.—100 kilos. of steam at 100° C. are to be condensed in one hour and the condensed water cooled to 20° C. The cooling air is to be heated in the process from 15°-80° C.

In order to convert 100 kilos. of steam at 100° into water at 100° C., 100(637 − 100) = 53,700 units of heat must be withdrawn.

In order to cool the 100 kilos. of condensed water from 100° to 20°, there must be abstracted (100 − 20)100 = 8000 calories. Thus, in all, 53,700 + 8000 = 61,700 calories.

The weight of air required to absorb this heat is, according to equation (206),

$$L = \frac{C}{\sigma_l(t_{le} - t_{la})} = \frac{61{,}700}{0{\cdot}2375(80 - 15)} = 4000 \text{ kilos. of air.}$$

4000 kilos. of air at 15° have (Table 59) a volume of 3,264,000 litres.

4000 kilos. of air have at 80° (Table 59) a volume of 4,000,000 litres.

The mean temperature difference between steam and air is, according to Table 52, $\theta_{mc} = 41{\cdot}8°$.

The mean temperature difference between condensed liquid and air is, according to Table 52, $\theta_{mk} = 25{\cdot}8°$

If we assume the velocity of the air to be 20 m. per second, then the cooling surface required for condensation is, by equation (209),

$$H_l = \frac{C}{z_h\theta_m k_l} = \frac{53{,}700}{1 \times 41{\cdot}8(2 + 10\sqrt{20})} = 28{\cdot}7 \text{ sq. m.,}$$

or, by Table 60, for a difference in temperature of 40° (in round numbers),

$$53{\cdot}7 \times 0{\cdot}545 = 29 \text{ sq. m. (approx.).}$$

For cooling there are required $\dfrac{8000}{25{\cdot}8(2 + 10\sqrt{20})} = 6{\cdot}64$ sq. m. (or, by Table 60, for an approximate difference in temperature of 25°, $\dfrac{0{\cdot}872 \times 8000}{1000} = 6{\cdot}98$ sq. m.). The total cooling surface is thus about 36 sq. m.

The section, across which the air is to pass with a velocity of 20 m., is, by equation (211),

$$q = \frac{V_l}{v_l 3600} = \frac{3{,}264{,}000}{20 \times 36{,}000} = 4{\cdot}53 \text{ sq. dcm.}$$

A tubular heating surface of 36 sq. m., which is to have a section of 4·53 sq. dcm., consists of 147 tubes of 20 mm. bore, each 4000 mm. long.

TABLE 60.

The cooling surface, H_t, in sq. m., required to transfer 1000 calories in one hour, when cooled by air at velocities of v_l = 1-36 m. and at mean differences in temperature of θ_m = 5°-100° C.

Mean temperature difference between air and cooling surface. θ_m	Velocity of the air, v_l, in m. per sec.								
	1	2	3	4	9	16	20	25	36
	Cooling surface, in sq. m., required to transfer 1000 calories per hour.								
5	16·66	12·42	10·46	9·10	6·24	4·76	4·36	3·84	3·220
10	8·33	6·21	5·23	4·55	3·12	2·38	2·18	1·92	1·610
15	5·55	4·14	3·487	3·033	2·080	1·586	1·453	1·280	1·073
20	4·17	3·105	2·615	2·258	1·560	1·190	1·090	0·960	0·805
25	3·33	2·484	2·092	1·820	1·248	0·952	0·872	0·768	0·644
30	2·78	2·07	1·743	1·517	1·040	0·793	0·727	0·640	0·535
40	2·09	1·503	1·308	1·129	0·780	0·595	0·545	0·480	0·403
50	1·67	1·242	1·046	0·910	0·624	0·476	0·436	0·384	0·322
60	1·39	1·035	0·872	0·759	0·520	0·397	0·364	0·320	0·269
70	0·19	0·888	0·748	0·650	0·446	0·340	0·311	0·275	0·229
80	1·05	0·752	0·654	0·565	0·390	0·298	0·273	0·240	0·202
90	0·92	0·690	0·581	0·506	0·347	0·272	0·242	0·214	0·180
100	0·83	0·621	0·523	0·455	0·312	0·238	0·218	0·192	0·161

3. Open Surface-Condensers.

Steam at atmospheric or lower pressures, or other gases or vapours, are condensed in open surface-condensers; it is rarely required also to cool the condensed liquid. In these condensers the vapour to be liquefied flows simultaneously through a number of parallel horizontal tubes, straight or curved, and arranged vertically over one another, or through vertical tubes. The cooling water, in a thin sheet, flows over the uppermost tube, it then flows down over the outside of the tubes and leaves heated at the bottom. The tubes are generally of equal size, but, since in the first case the cooling water is colder when it flows over the upper than the lower tubes, the temperature difference between vapour and water is greater

above than below. The upper tubes therefore condense more vapour and even cool the condensed liquid. The upper tubes have therefore a greater capacity than the lower.

The quantity of heat, C, to be abstracted from the vapour in condensation is known in each case:

$$C = D(c - t_a) \quad \ldots \ldots \ldots \quad (212)$$

The requisite condensing surface, H_c, is obtained from the well-known equation:

$$H_c = \frac{C}{k_c \theta_m} \quad \ldots \ldots \ldots \quad (213)$$

The temperature difference, θ_m, must here be the mean difference calculated for the whole apparatus, as found in the ordinary manner by means of Table 1.

The coefficient of transmission for copper and brass tubes may be taken as

$$k_c = 750 \sqrt[3]{v_a} \sqrt[3]{0{\cdot}007 + v_f} \quad \ldots \ldots \quad (214)$$

For iron tubes it is, at the most, 0·75 times as great.

In this form of condenser there is frequently a very considerable incrustation on the outside of the tubes, the inside is also occasionally coated by slimy or solid deposits. Thus the cooling action often sinks to one-half or to even one-third of the original. This is particularly the case with iron tubes, and must be considered in settling the dimensions.

The initial velocity of the vapour, v_a, may be determined in every case from its weight and volume and the section of the tubes.

The velocity with which the cooling water flows down, v_f, depends on the quantity which is to flow in one hour over 1 m. in length of the apparatus, and increases with that quantity, just as in surface coolers.

With a somewhat economical consumption of water, the velocity, v_f, of flow over the surface of horizontal tubes cannot be taken at more than 0·200 m., then $\sqrt[3]{0{\cdot}007 + v_f} = 0{\cdot}6$.

On vertical tubes v_f may be about 0·400 m., in which case $\sqrt[3]{0{\cdot}007 + v_f} = 0{\cdot}74$.

The ratio between the length and the diameter of the tube, $\frac{l}{d}$, is obtained as in the former similar cases—the quantity of heat transmitted in one hour through the cooling surface must be equal to the

latent heat of the weight of vapour condensed in the tube during one hour. Therefore

$$d\pi l k_c \theta_m = \frac{d^2\pi}{4} v_d 3600 \gamma(c - t_d),$$

or

$$\frac{l}{d} = \frac{v_d 3600 \gamma(c - t_d)}{4 k_c \theta_m}.$$

Inserting the value for k_c from equation (214) we obtain

$$\frac{l}{d} = \frac{\sqrt{v_d}\,1{\cdot}2\gamma(c - t_d)}{\theta_m \sqrt[3]{0{\cdot}007 + v_f}},$$

and, since for horizontal tubes $\sqrt[3]{0{\cdot}007 + v_f} = 0{\cdot}6$ (see above),

$$\frac{l}{d} = \frac{2\sqrt{v_d}\gamma(c - t_d)}{\theta_m} \quad . \quad . \quad . \quad . \quad . \quad . \quad (215)$$

Experimental Observation.—8000 kilos. of steam at a vacuum of 640-650 mm. (53·5° C.) were condensed per hour by 500 vertical iron tubes of 40 mm. bore, 4000 mm. long. The mean temperature of the cooling water was 45°-47°, the cooling surface 250 sq. m.

The amount of heat to be transferred per hour was

$$C = 8000(623 - 53{\cdot}5) = 4{,}556{,}600 \text{ calories.}$$

The volume of steam entering the tubes per second was

$$V_d = \frac{8000 \times 9510}{3600} = 21{,}140 \text{ litres.}$$

The free section of the 500 tubes amounted to

$$q = 0{\cdot}125 \times 500 = 62{\cdot}5 \text{ sq. dcm.,}$$

hence the entrant velocity of the steam was

$$v_d = \frac{21{,}140}{62{\cdot}5 \times 10} = 33{\cdot}9 \text{ m.}$$

The velocity of the cooling water flowing down the vertical tubes was about 0·400 m., consequently the transmission coefficient would have been, for copper,

$$k_c = 750\sqrt{33{\cdot}9}\sqrt[3]{0{\cdot}007 + 0{\cdot}400} = 3232.$$

Since, however, iron tubes were used,

$$k_c = \tfrac{3}{4} \times 3232 = 2424.$$

The temperature difference was $\theta_m = 53{\cdot}5 - 46 = 7{\cdot}5°$.

Consequently the *calculated* cooling surface was

$$H_c = \frac{4{,}556{,}000}{2424 \times 7{\cdot}5} = 250 \text{ sq. m.,}$$

which agrees exactly with the *real* cooling surface of 250 sq. m.

TABLE 61.

The cooling surface, H_c, of copper or brass in open surface-condensers, the consumption of cooling water, W, and the mean temperature difference, θ_m, requisite to condense per hour 100 kilos. of steam at 100°, 60°, 50° and 40° C., by means of cooling water at 15°-50° C.

Initial temperature of the cooling water. t_a.	Entrant velocity of the steam. v_d.	Mean temp. diff., θ_m, cooling water, W, and cooling surface, H_c.	Temperature of the steam, t_d. 100°			60°			50°			40°		
			Final temperature of the cooling water, t_e.											
			80°	90°	98°	40°	50°	58°	30°	40°	48°	20°	30°	38°
15°		θ_m	45	35	21·2	31	23·4	13·5	27	20	11·2	22·5	16·5	9·2
		W	830	733	651	2320	1660	1350	3933	2360	1788	12500	4000	2610
	25	H_c	0·53	0·70	1·13	0·83	1·11	1·93	1·00	1·31	2·34	1·18	1·62	2·96
		$\frac{l}{d}$	73	94	155	54	82	56	18	24	43	14	19	33
	50	H_c	0·38	0·50	0·80	0·58	0·79	1·37	0·71	0·93	1·66	0·83	1·15	2·10
		$\frac{l}{d}$	102	181	217	38	44	78	25	33	60	20	27	46
20°		θ_m	43·2	33·6	20·8	28·8	21·6	12·7	25	18	10·3	—	14·4	7·8
		W	890	786	692	2900	1933	1525	5900	2950	2110	—	6000	3333
	25	H_c	0·55	0·72	1·15	0·90	1·18	2·03	1·05	1·40	2·55	—	1·85	3·42
		$\frac{l}{d}$	76	97	158	26	86	60	19	27	48	—	21	40
	50	H_c	0·39	0·51	0·82	0·64	0·84	1·44	0·74	1·00	1·80	—	1·31	2·42
		$\frac{l}{d}$	106	135	221	36	50	84	27	87	67	—	29	56
25°		θ_m	42	33	19·8	26·6	20	11·4	22·5	16·5	9·2	—	12·3	6 90
		W	982	846	740	3870	2320	1760	11800	3930	2580	—	12500	4616
	25	H_c	0·57	0·73	1·23	1·00	1·28	2·26	1·15	1·60	2·85	—	2·16	3·86
		$\frac{l}{d}$	78	99	165	29	39	66	22	31	54	—	25	44
	50	H_c	0·41	0·56	0·88	0·71	0·91	1·60	0·82	1·10	2·02	—	1·53	2·73
		$\frac{l}{d}$	109	139	231	40	51	92	30	43	75	—	85	61
30°		θ_m	40	31	18·9	24·6	18·8	10·4	—	14·4	7·8	—	—	5
		W	1080	917	800	5800	2900	2075	—	5900	3280	—	—	7500
	25	H_c	0·60	0·79	1·27	1·05	1·41	2·47	—	1·82	3·36	—	—	5·33
		$\frac{l}{d}$	82	105	175	31	41	75	—	33	65	—	—	60
	50	H_c	0·43	0·56	0·89	0·75	1·00	1·74	—	1·29	2·38	—	—	3·77
		$\frac{l}{d}$	114	149	245	48	57	105	—	46	91	—	—	84

TABLE 61—(*continued*).

Initial temperature of the cooling water. t_a.	Entrant velocity of the steam. v_d.	Mean temp. diff., θ_m, cooling water, W, and cooling surface, H_c.	Temperature of the steam, t_d. 100°			60°			50°			40°		
			Final temperature of the cooling water, t_e. 80°	90°	98°	40°	50°	58°	30°	40°	48°	20°	30°	38°
35°		θ_m	38	29·2	18	22·5	16·5	9·2	—	12·3	6·4	—	—	2·8
		W	1200	1000	860	11600	3870	2522	—	11800	4540	—	—	20000
	25	H_c	0·63	0·82	1·33	1·10	1·58	2·81	—	2·13	4·10	—	—	8·00
		$\frac{l}{d}$	87	112	180	35	46	84	—	40	75	—	—	91
	50	H_c	0·45	0·58	0·80	0·78	1·12	2·00	—	1·51	2·90	—	—	5·7
		$\frac{l}{d}$	121	156	252	49	64	117	—	56	105	—	—	127
40°		θ_m	36	27·9	17·4	—	14·5	8	—	—	5	—	—	—
		W	1350	1080	930	—	5640	3130	—	—	9500	—	—	—
	25	H_c	0·67	0·87	1·40	—	1·80	3·10	—	—	5·25	—	—	—
		$\frac{l}{d}$	90	118	190	—	52	94	—	—	97	—	—	—
	50	H_c	0·51	0·66	1·60	—	1·37	2·70	—	—	4·01	—	—	—
		$\frac{l}{d}$	126	165	266	—	88	131	—	—	135	—	—	—
45°		θ_m	34·6	264	16	—	12	6·6	—	—	3·3	—	—	—
		W	1540	1200	1020	—	11280	4340	—	—	57000	—	—	—
	25	H_c	0·71	0·91	1·50	—	2·16	3·95	—	—	8·00	—	—	—
		$\frac{l}{d}$	95	124	200	—	63	114	—	—	147	—	—	—
	50	H_c	0·54	0·71	1·16	—	1·65	3·00	—	—	6·10	—	—	—
		$\frac{l}{d}$	142	173	280	—	88	159	—	—	195	—	—	—
50°		θ_m	32·5	25	15	—	—	—	—	—	—	—	—	—
		W	1800	1350	1125	—	—	—	—	—	—	—	—	—
	25	H_c	0·74	0·95	1·60	—	—	—	—	—	—	—	—	—
		$\frac{l}{d}$	100	135	220	—	—	—	—	—	—	—	—	—
	50	H_c	05·7	0·73	1·23	—	—	—	—	—	—	—	—	—
		$\frac{l}{d}$	140	183	308	—	—	—	—	—	—	—	—	—

Cooling surfaces of iron must be at least 1·33 times as great.

The annexed Table 61 gives for a number of cases the requisite cooling surface (in copper tubes) for the hourly condensation of 100 kilos. of steam at different pressures, which enters the tubes at velocities of 25 or 50 m., and for cooling water at 15°-50° C.

Generally the condensed liquid does not leave the condenser much colder than the steam; if, however, the condensed liquid is intended to be cooled considerably, the cooling surface must be correspondingly increased.

The consumption of cooling water, W, given is the theoretical. In practice, on account of evaporation, it would be 3-5 per cent. less.

CHAPTER XXI.

HEATING LIQUIDS BY MEANS OF STEAM.

A. Steam Heating Coils or Systems of Tubes in the Liquid to be Heated.

1. *The Liquid is not Changed.*

THE heating of liquids by steam has already been mentioned (Chapter VIII.). The steam used for heating liquids (if it is not superheated, a case which is rare and therefore remains untreated here) must condense, and sometimes the condensed water must be cooled. The weight of steam required to heat a given quantity of water through a given range of temperature can always be found. On that account, and because it is convenient to the course of our subject, we proceed to the calculation of the requisite heating surface by first determining *the weight of steam required for heating* and thence the surface requisite for its condensation.

The weight of steam, D, required to heat F kilos. of a liquid of specific heat, σ_f, from t_{fk} to t_{fw}, is

$$D = \frac{F\sigma_f(t_{fw} - t_{fk})}{640 - \frac{t_{fw} + t_{fk}}{2}} \quad . \quad . \quad . \quad . \quad . \quad (216)$$

Example.—In order to heat $F = 100$ kilos. of water from 30°-90° C., there are required $100(90 - 30) = 6000$ calories.

Assuming the condensed water escapes at the mean temperature of the water, $\frac{t_{fw} + t_{fk}}{2} = \frac{90 + 30}{2} = 60°$, then 1 kilo. of steam gives up $640 - 60 = 580$ calories, and $D = \frac{6000}{580} = 10\cdot346$ kilos. of steam are required.

The difference in temperature between the steam and the liquid decreases during the process of heating; it is clear from previous explanations that the mean temperature difference is determined

from the greatest difference at the beginning, θ_a, and the least at the end, θ_e (Chapter I., Table 1).

Example.—If the steam is at 100° C., with the data of the last example, $\theta_a = 100° - 30° = 70°$, $\theta_e = 100° - 90° = 10°$. Consequently

$$\frac{\theta_e}{\theta_a} = \frac{10}{70} = 0{\cdot}143.$$

The mean temperature difference is then, from Table 1,

$$\theta_m = 0{\cdot}442\theta_a = 0{\cdot}442 \times 70 = 30{\cdot}94° \text{ C.}$$

Table 62 gives the number of units of heat required to warm 100 kilos. of water under different conditions, also the consumption of steam and the mean difference in temperature.

If the warming vessel is to be provided with coils or systems of tubes, through which the heating steam passes, its entrant velocity, v_d, can generally be selected (30-40 m. for coils, 10-20 m. for short vertical tubes, would be suitable). From this and the hourly consumption of steam, D, the proper diameter of the coil or tubes can be ascertained by means of Table 55.

The diameter of the tube, the temperature difference and the entrant velocity, all of which are known, *then* give, by means of equation (205) and Table 57, the necessary length of tube, and thence the cooling surface, H_c, if the velocity of the liquid about the tube is known. If this velocity is unknown, the smaller value of k_c from equation (217) should be inserted in the expression:

$$H_c = \frac{C}{k_c\theta_m}.$$

If the liquid is not driven artificially over the heating surface, the rapidity of its motion about this surface increases with the rise in temperature. The real extent of this velocity depends then on the form and dimensions of the surrounding vessel and the arrangement of the heating surface, which naturally is placed at the bottom.

The mean *velocity of the liquid* over the heating surface, in heating without stirrers, may vary in different cases approximately between $v_f = 0{\cdot}02$ and $0{\cdot}300$ m. The smaller figure is for large vessels and liquids at low temperatures, below 60° C.; the larger figure for small vessels and liquids at higher temperatures, 60°-100° C.

The coefficient of transmission should be taken in this case of steam coils, used for heating *without stirrers*, as

$$k_c = 225\sqrt{v_d} \text{ to } 450\sqrt{v_d} \quad . \quad . \quad . \quad . \quad . \quad (217)$$

TABLE 62.

The requisite number of calories, C, weight of steam, D, and mean temperature difference θ_m, between steam and water, for heating 100 kilos. of water from the temperature, t_{fa}, to the higher temperature, t_{fe}.

Initial temperature of the water. t_{fa}.	Steam. Pressure. atmos. abs.	Steam. Temperature. t_d.	Units of heat, C. Weight of steam, D. Mean temp. diff., θ_m.	Final temperature of the heated water, t_{fe} (for $\omega' = 1$). 30	40	50	60	70	80	90	100	
10			$C =$	2000	3000	4000	5000	6000	7000	8000	9000	cals.
			$D =$	3·3	5·5	7·0	9·0	10·5	12·5	14·5	16·7	kilos.
	1	100°	$\theta_m =$	81	75	67	62	54	46	36	—	° C.
	1·5	111°	,,	90	85	79	72	65	60	50	40	,,
	2	121°	,,	100	95	89	83	77	68	62	52	,,
	3	134°	,,	125	110	104	97	90	86	79	73	,,
20			$C =$	1000	2000	3000	4000	5000	6000	7000	8000	cals.
			$D =$	1·7	3·3	5·5	7·2	8·7	11·0	12·7	14·8	kilos.
	1	100°	$\theta_m =$	73	69	60	57	52	43	33	—	° C.
	1·5	111°	,,	85	81	75	69	61	54	46	37	,,
	2	121°	,,	95	90	85	79	73	66	59	50	,,
	3	134°	,,	108	102	95	92	86	79	75	66	,,
30			$C =$	—	1000	2000	3000	4000	5000	6000	7000	cals.
			$D =$	—	1·7	3·5	5·5	7·0	9·1	10·9	13·0	kilos.
	1	100°	$\theta_m =$	—	64	59	55	46	40	30	—	° C.
	1·5	111°	,,	—	75	72	65	58	51	43	35	,,
	2	121°	,,	—	85	81	74	67	61	55	46	,,
	3	134°	,,	—	95	90	85	80	73	67	61	,,
40			$C =$	—	—	1000	2000	3000	4000	5000	6000	cals.
			$D =$	—	—	1·75	3·7	5·3	7·2	9·1	11·1	kilos.
	1	100°	$\theta_m =$	—	—	54	50	43	35	28	—	° C.
	1·5	111°	,,	—	—	64	58	54	45	41	32	,,
	2	121°	,,	—	—	76	70	64	57	52	43	,,
	3	134°	,,	—	—	91	84	79	70	66	58	,,
50			$C =$	—	—	—	1000	2000	3000	4000	5000	cals.
			$D =$	—	—	—	1·8	3·5	5·5	7·2	9·2	kilos.
	1	100°	$\theta_m =$	—	—	—	45	39	32	25	—	° C.
	1·5	111°	,,	—	—	—	54	50	43	36	29	,,
	2	121°	,,	—	—	—	66	59	54	47	40	,,
	3	134°	,,	—	—	—	79	74	68	62	57	,,
60			$C =$	—	—	—	—	1000	2000	3000	4000	cals.
			$D =$	—	—	—	—	1·7	3·7	5·5	7·3	kilos.
	1	100°	$\theta_m =$	—	—	—	—	35	29	22	—	° C.
	1·5	111°	,,	—	—	—	—	45	39	32	25	,,
	2	121°	,,	—	—	—	—	54	50	43	36	,,
	3	134°	,,	—	—	—	—	70	62	57	51	,,

The section of the steam valve may be determined by the aid of Table 14.

When the motion of the liquid is artificially accelerated by *stirrers*, its velocity can in some degree be determined, it will be 1-3 m. A higher velocity is without advantage, for the transmission of heat does not then increase to any great extent, whilst the power required increases considerably. The stirrer should naturally be, as far as possible, constructed so that it always conveys fresh liquid to the heating surface.

The coefficient of transmission for the heating of thin liquids by steam in copper tubes, *with stirrers*, is

$$k_s = 750\sqrt{v_d}\sqrt[3]{0{\cdot}007 + v_f} \quad . \quad . \quad . \quad . \quad . \quad (218)$$

The true velocity of the liquid obtained by means of a stirrer is not easy to estimate, either before or after the construction of the apparatus.

The application of a stirrer is still more necessary in heating and cooling thick sticky masses than with thin and readily mobile liquids. The former cannot be brought into rapid circulation even by very unequal heating. A stirrer is also necessary in the case of those liquids which would be damaged if their particles were heated almost to the temperature of the hot surface.

Example.—5000 litres of water are to be heated in one hour from 20° to 80° C. by steam at 100° by means of a heating pipe.

According to Table 62 there are required for this purpose 50 × 6000 = 300,000 calories and 11 × 50 = 550 kilos. of steam. The temperature difference is 43° C.

The entrant velocity of the steam is taken at 40 m. The diameter of the heating tube must be 90 mm., for, from Table 55, 13·9 × 40 = 556 kilos. of steam pass through a pipe of 90 mm. bore in one hour.

If there is no stirrer in the vessel, the probable velocity of the water about the heating pipe may be assumed to be 0·020 m. Then we obtain the necessary length of pipe from Table 55,

$$l = 194 \times 0{\cdot}090 = 17{\cdot}46 \text{ m.},$$

and the heating surface,

$$H_e = d\pi l = 4{\cdot}92 \text{ sq. m.}$$

The steam valve should be 65 or, better, 80 mm. wide.

If a stirrer is applied in the heating vessel, and it moves the liquid with a velocity of 1 m. over the hot surface, then, with the other conditions the same, according to Table 57, the ratio, $\frac{l}{d} = 66$. Consequently $l = 66 \times 0{\cdot}090 = 5{\cdot}94$ m. and hence the heating surface, $H = 1{\cdot}69$ sq. m. It will be observed that a stirrer considerably decreases the necessary heating surface.

2. *A Continuous Current, in and out, of the Liquid to be heated.*

If the liquid to be heated flows continuously in and out, its velocity, v_f, over the heating surface is known. Also the entrant velocity of the steam into the heating space is known or can be fixed. If all the steam introduced into the heating space is not condensed there, but a portion passes out, then in the equation for k_e the *sum* of its velocities at entering and leaving is to be inserted. This equation is

$$k_e = 750\sqrt{v_d}\sqrt[3]{0{\cdot}007 + v_f}.$$

From the constant difference in temperature at the entry and exit of the liquid, the mean temperature difference, θ_m, is obtained from Table 1.

The quantity of heat to be transferred is

$$C = F\sigma_f(t_{fw} - t_{fk}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (219)$$

and the heating surface

$$H_e = \frac{C}{k_e\theta_m}.$$

The consumption of steam, according to equation (216), is

$$D = \frac{F\sigma_f(t_{fw} - t_{fk})}{640 - \frac{t_{fw} + t_{fk}}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (220)$$

Example.—20,000 litres of water are to be heated per hour from 10°-60° C.; the water flows past the heating surface with the velocity, $v_f = 0{\cdot}20$ m. The steam is at 3 atmos. absolute.

In one hour $C = 20{,}000(60 - 10) = 1{,}000{,}000$ calories are to be transferred, for which $D = \dfrac{20{,}000(60 - 10)}{640 - \left(\dfrac{60 + 10}{2}\right)} = 1627$ kilos. of steam are required.

The steam is at the temperature, $t_d = 134°$ C. (130° is used instead).

The temperature difference at the beginning is $\theta_a = 130° - 10° = 120°$.

The temperature difference at the end is $\theta_e = 130° - 60° = 70°$;

thus the mean temperature difference is

$$\text{(by Table 1, since } \frac{\theta_e}{\theta_a} = \frac{70}{120} = 0{\cdot}583) \ \theta_m = 0{\cdot}77 \times 120 = 92{\cdot}4°.$$

The steam is to be completely condensed and the velocity at which it enters is to be $v^d = 20$ m., therefore

$$k_e = 750\sqrt{20}\sqrt[3]{0{\cdot}007 + 0{\cdot}200} = 1984,$$

consequently the heating surface,

$$H_e = \frac{1{,}000{,}000}{92{\cdot}4 \times 1984} = 5{\cdot}45 \text{ sq. m.}$$

In order to admit 1627 kilos. of steam per hour at a velocity of 20 m., according to Table 55, 7 tubes of 50 mm. bore, and with a heating surface of 5·45 sq. m., are required. Each tube must therefore be $l = 5$ m. long.

B. Steam Vessels with Double Bottoms.

If a liquid is heated, not by steam coils, but in a vessel with a double bottom, then neither the velocity of the liquid nor that at which the steam enters is known. It is necessary to fall back on equation (52) for the heating surface, when there is no stirrer:—

$$H_e = \frac{C}{1400 \text{ to } 1800\theta_m} \quad . \quad . \quad . \quad . \quad . \quad . \quad (221)$$

If the double-bottomed vessel is provided with a suitable *stirrer*, then the expression for estimating the heating surface is

$$H_e = \frac{C}{3500\theta_m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (222)$$

Example.—2000 litres of water are to be heated from 10° to 100° C. in one hour by means of steam at a pressure of 1 atmos. (121° C.) in a double-bottomed vessel.

According to Table 62, 20 × 9000 = 180,000 calories are required, and the temperature difference is 52°. The necessary heating surface, without a stirrer, is therefore

$$H_e = \frac{180{,}000}{1400 \times 52} \text{ to } \frac{180{,}000}{1800 \times 52} = 2{\cdot}48 \text{ to } 1{\cdot}93 \text{ sq. m. (about } 2{\cdot}25 \text{ sq. m.).}$$

If the vessel has a diameter of 1600 mm., then the surface of the double bottom is about 3 sq. m., consequently the 2000 litres will, on the average, be heated in $\frac{60 \times 2{\cdot}25}{3} = 45$ minutes.

If the double vessel is provided with an efficient stirrer, the necessary heating surface is

$$H_e = \frac{C}{3500\theta_m} = \frac{180{,}000}{3500 \times 52} = \text{about 1 sq. m.}$$

The same vessel will then heat the 2000 litres of water in about 20 minutes.

Thick, syrupy, or pasty masses are heated much more slowly.

C. The Liquid to be Heated Flows Through Tubes around which is Steam at Rest.

Steam is hardly ever completely at rest, but we understand in the following pages by steam at rest, steam which moves in a definite direction with a lower velocity than 0·5 m. per second.

TABLE 63.

Copper heating surfaces required to heat per hour 1000 litres of water at 10° or 25° to 50°-90° C., moving through tubes with the velocity 0·01-0·4 m., by means of steam at rest at a temperature of 80°, 90°, 100°, or 120° C.

Velocity of the liquid. v_f.	Initial temperature of the liquid. t_{fa}.	Mean temp. diff., θ_m, and heating surface, H_e in sq. m.	Temperature of the hot vapour (alcohol or water), t_d. 80°			90°			100°			120°		
			Final temperature of the liquid to be heated, t_f. 50°	60°	75°	50°	70°	85°	60°	80°	90°	60°	80°	90°
0·010	10	θ_m =	47·6	40	24·5	58	43·5	27	62	46·5	36	83	69	62
		H_e =	4·3	6·4	13·6	3·6	7·0	14·3	5·0	7·7	11·5	3·1	5·2	6·8
	25	θ_m =	41	34·6	21	51	37·7	23	55·5	41	32	76	64	56
		H_e =	3·1	5·2	12·2	2·4	5·9	13·0	3·4	6·9	10·4	2·4	4·4	6·0
0·050	10	θ_m =	47·6	40	24·5	58	43·5	27	62	46·5	36	83	69	62
		H_e =	3·0	4·3	9·2	2·4	5·0	9·6	3·2	5·2	8·0	2·1	3·5	4·6
	25	θ_m =	41	34·6	21	51	37·7	23	55·5	41	32	76	64	56
		H_e =	2·1	3·5	8·0	1·7	4·1	8·8	2·3	4·7	7·2	1·6	3·6	4·0
0·100	10	θ_m =	47·6	40	24·5	58	43·5	27	62	46·5	36	83	69	62
		H_e =	2·4	3·5	7·4	2·0	3·9	8·0	2·6	4·2	6·3	1·7	2·9	3·7
	25	θ_m =	41	34·6	21	51	37·7	23	55·5	41	32	76	64	56
		H_e =	1·7	2·9	6·7	1·4	3·4	7·2	1·8	3·8	5·7	1·3	2·4	3·3
0·200	10	θ_m =	47·6	40	24·5	58	43·5	27	62	46·5	36	83	69	62
		H_e =	2·0	2·8	6·0	1·6	3·1	6·3	2·1	3·4	5·1	1·4	2·3	3·0
	25	θ_m =	41	34·6	21	51	37·7	23	55·5	41	32	76	64	56
		H_e =	1·4	2·3	5·4	1·1	2·7	5·7	1·3	3·0	4·6	1·1	2·0	2·6
0·300	10	θ_m =	47·6	40	24·5	58	43·5	27	62	46·5	36	83	69	62
		H_e =	1·7	2·5	5·3	1·4	2·7	5·5	1·9	3·0	4·5	1·2	2·0	2·7
	25	θ_m =	41	34·6	21	51	37·7	23	55·5	41	32	76	64	56
		H_e =	1·2	2·0	4·7	1·0	2·4	5·0	1·3	2·7	4·1	0·9	1·7	2·3
0·400	10	θ_m =	47·6	40	24·5	58	43·5	27	62	46·5	36	83	69	62
		H_e =	1·6	2·3	4·8	1·3	2·5	5·0	1·7	2·7	4·2	1·1	1·8	2·4
	25	θ_m =	41	34·6	21	51	37·7	23	55·5	41	32	76	64	56
		H_e =	1·1	1·8	4·1	0·90	2·2	4·5	1·2	2·4	3·7	0·83	1·6	2·1

If the liquid to be heated is passed with the velocity, v_f, through tubes, whilst the steam moves round the tubes with its slight velocity, then the transmission coefficient for copper tubes and thin liquids may be taken as

$$k_e = 750\sqrt[3]{0{\cdot}007 + v_f} \quad . \quad . \quad . \quad . \quad . \quad . \quad (223)$$

so that the requisite heating surface is

$$H_e = \frac{C}{\theta_m 750 \sqrt[3]{0 \cdot 007 + v_f}} \quad . \quad . \quad . \quad . \quad . \quad (224)$$

For thick liquids k_e is about 10-15 per cent. lower, H_e consequently about as much greater.

For iron tubes k_e is about 15 per cent. lower.

The temperature difference is obtained in the ordinary manner, by Table 1, from the temperature of the steam, which is generally constant, and the initial and final temperatures of the liquid.

If the liquid is sent simultaneously through a considerable number of (vertical) tubes, round which the steam passes, if only at velocities of 0·5-1 m. per second, the efficiency of the heating surface is greater, and may easily be in this case 1·5 times as great as with steam at rest.

The next, Table 63, gives the temperature differences and requisite heating surfaces for a number of cases. The figures given for steam at 80° and 90° C. apply also to aqueous alcohol vapour of 86 and 58 per cent. strength by weight respectively.

Experimental Example.—5890 kilos. of wort were heated in one hour from 31° to 49° C. by aqueous alcohol vapour at rest (velocity about 0·3 m.) at a temperature of 79·1° C. The wort was passed with a velocity of 0·205 m. through a copper pipe, with a bore of 100 mm. and the heating surface, $H_e = 6 \cdot 9$ sq. m.

The specific heat of the liquor being taken as $\sigma_f = 1$, there were to be transferred in one hour

$$C = 5890(49 - 31) = 106{,}020 \text{ calories.}$$

The temperature difference at the beginning was $\theta_a = 79 \cdot 1° - 31° = 48 \cdot 1°$.

The temperature difference at the end was $\theta_e = 79 \cdot 1° - 49° = 30 \cdot 1°$.

Then $\frac{\theta_e}{\theta_a} = \frac{30 \cdot 1}{48 \cdot 1} = 0 \cdot 625$, accordingly, by Table 1, the mean temperature difference is

$$\theta_m = 0 \cdot 8 \times 48 \cdot 1 = 38 \cdot 48°.$$

The coefficient of transmission is

$$k_e = 705 \sqrt{0 \cdot 007 + 0 \cdot 205} = 447 \cdot 75.$$

The *calculated* heating surface is therefore

$$H_e = \frac{106{,}020}{38 \cdot 48 \times 447 \cdot 75} = 6 \cdot 15 \text{ sq. m.}$$

On account of the thickness of the liquid, 10 per cent. is to be added, which gives $6 \cdot 15 + 0 \cdot 615 = 6 \cdot 8$ sq. m., which agrees well with the actual heating surface.

CHAPTER XXII.

THE COOLING OF LIQUIDS.

There are various different methods for cooling liquids, in most of which the liquid is cooled by the consequent heating of the means of cooling. Thus the consideration of the cooling of liquids may also serve for the operation of heating, for which what is about to be said may also bo useful.

Liquids may be artificially cooled by the following methods:—

A. By the direct introduction of ice.

B. By the direct addition of cold to hot liquids.

C. By the evaporation of a portion of the liquid without the application of heat.

D. By flowing over metal surfaces which are in contact with a colder liquid (surface or closed coolers).

E. By flowing free over surfaces which are in contact with the colder liquid on the other side, by which means the surrounding air takes up a portion of the heat (open coolers).

F. By contact with metal surfaces which are traversed by cold air.

G. By spreading out and dividing the liquid in the open, and subjecting it to the action of air in natural or artificial motion (as in cooling water).

These methods of cooling will be dealt with in turn.

A. The Direct Introduction of Ice.

This method of cooling is only employed when it is desired to produce very low temperatures. The ice employed is generally only a few degrees below 0° C., its latent heat is 79 calories. Having

regard to its specific heat ($\sigma_e = 0{\cdot}504$) for the 2°-3° through which it must be heated before melting, it may be assumed that each kilo. of ice in melting to water at 0° C. takes up 80 units of heat. If t_{fa} and t_{fe} be the temperatures of the liquid before and after cooling, and σ_f its specific heat, then the amount of heat to be withdrawn is

$$C = F\sigma_f(t_{fa} - t_{fe}) \quad \ldots \ldots \ldots \quad (225)$$

The weight of ice to be used is

$$E = \frac{F\sigma_f(t_{fa} - t_{fe})}{80 + t_{fe}} \quad \ldots \ldots \ldots \quad (226)$$

In order to cool 100 kilos. of water from

	10°	9°	8°	7°	6°	5°	4° C.
To 5° C. } there are required	5·9	4·8	3·6	2·4	1·2	—	—
To 2° C. } E kilos. of ice	9·8	8·6	7·4	6·1	4·9	3·7	2·44

B. The Direct Addition of Cold to Hot Liquid.

If F kilos. of a cold liquid at the temperature, t_f, be added to F_w kilos. of a warmer liquid, of the same specific heat, at the temperature, t_{fw}, the temperature of the mixture is

$$t_m = \frac{F_w t_{fw} + F\, t_{fk}}{F_w + F_k} \quad \ldots \ldots \ldots \quad (227)$$

Example.—$F_w = 100$ kilos. of water at $t_{fw} = 80°$, and $F_k = 200$ kilos. of water at $t_f = 20°$, give

$F_w + F_k = 300$ kilos. of water at the temperature

$$t_m = \frac{100 \times 80 + 200 \times 20}{100 + 200} = 40°.$$

C. Cooling Liquids by Evaporation.

Liquids are best cooled in this manner by bringing them into a vacuum. If a space be provided over a hot aqueous liquid, in which a lower pressure is maintained than corresponds to steam at the temperature of the liquid, the latter is cooled down to that temperature, the steam at which corresponds to the pressure over the liquid, the heat of the liquid given out in falling from the original temperature to the lower being utilised in the formation of steam. The temperatures of steam (and also of liquid) corresponding to every degree of vacuum are to be obtained from Table 9.

If the weight of liquid, F_w, at the original temperature, t_{fw}, is cooled *in vacuo* to t_{fk}, then the weight of steam evolved is

$$D = \frac{F_w(t_{fw} - t_{fk})}{640 - \frac{t_{fw} + t_{fk}}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (228)$$

whence we obtain the following small table:—

Vacuum.	Temperature of the cooled liquid, t_{fk}.	100 kilos. of aqueous liquid at the original temperature, t_{fw} =				
		100°	90°	80°	70°	60°
mm.	°C.	Evolve the following weights of steam, D, in being cooled to the temperatures, t_{fk}, given in the second column.				
234	90	1·82	—	—	—	—
405	80	3·67	1·82	—	—	—
526	70	5·25	3·50	1·75	—	—
611	60	7·00	5·25	3·50	1·75	—
668	50	8·50	6·80	5·10	3·40	1·70
705	40	10·00	8·33	6·66	5·00	3·33

D. Cooling a Hot Liquid by means of a Colder Liquid.

The cooling of a hot liquid by another colder liquid, or, what is the same thing, the heating of a cold liquid by a hot one, may be effected in two different ways, *viz.*:—

1. *By sending the two liquids continuously in opposite directions (counter-currents) with the highest possible velocity over the common wall of separation.*

In this method the warm liquid falls through straight or bent tubes (coils) or channels, whilst the cold liquid rises in the surrounding vessel or in a surrounding tube concentric with the first, or rises, whilst being warmed, in a channel surrounding the first.

If we put σ_w for the specific heat of the warm liquid, σ_k for that of the cold, t_{wa} and t_{we} for the temperature of the warm, t_{ka} and t_{ke} for the temperatures of the cold liquid, then the quantity of heat to be transferred is

$$C = F_w\sigma_w(t_{wa} - t_{we}) = F_k\sigma_k(t_{ke} - t_{ka}) \quad . \quad . \quad . \quad (229)$$

TABLE 64.

The transmission coefficient, k_k, between two liquids, the one taking or brass diaphragm with the

$v_{/1}$ = / $v_{/2}$ =	0·001	0·002	0·004	0·006	0·008	0·01	0·02	0·04
0·001	119	122	128	130	132	136	144	155
0·002	122	128	132	136	140	142	150	160
0·004	128	132	138	140	144	148	157	170
0·006	130	136	140	145	150	153	162	173
0·008	132	140	144	150	154	156	168	176
0·01	136	142	148	153	156	160	170	185
0·02	144	150	157	162	169	170	185	200
0·04	155	160	170	175	176	185	200	210
0·06	160	168	177	183	188	194	210	234
0·08	165	172	183	188	196	200	218	242
0·10	169	176	186	194	200	206	225	250
0·20	180	188	200	208	214	224	246	274
0·40	190	200	214	224	232	240	266	302
0·60	196	206	222	232	240	250	280	316
0·80	200	212	226	238	246	256	285	328
1·00	204	214	230	240	252	259	294	336
1·25	206	218	234	247	256	266	298	344
1·50	208	222	238	250	260	270	302	350
2·0	210	225	240	253	264	274	308	358

From this equation is also obtained the necessary weight of hot liquid, F_w for heating the weight of cold liquid, F_k.

If θ_m be the mean temperature difference and k_k the coefficient of transmission, then the surface required for the cooling is obtained from the known equation:—

$$H = \frac{C}{k_k\theta_m} = \frac{F_w\sigma_w(t_{wa} - t_{we})}{k_k\theta_m} \quad . \quad . \quad . \quad . \quad (230)$$

The coefficient of transmission of heat, k_k, between two moving liquids at different temperatures is found from an equation calculated by Molier from Joule's researches (Zeits. d. V. d. Ing., 1897, Nos. 6 and 7) on copper and brass separating walls. The equation, which

TABLE 64.

heat from the other, which flow in opposite directions over a copper different velocities, v_{f1} and v_{f2}.

0·06	0·08	0·10	0·2	0·4	0·6	0·8	1·0	1·25	1·50	2·0
160	165	169	180	190	196	200	204	206	208	210
168	172	176	188	200	206	212	214	218	222	225
176	183	186	200	214	222	226	230	234	238	240
183	188	194	208	224	232	238	240	247	250	253
188	196	200	216	232	240	246	252	256	260	264
194	200	206	224	240	250	256	259	266	270	274
210	218	225	246	266	280	285	294	298	302	308
234	242	250	274	302	316	328	336	344	350	358
250	256	267	296	324	344	356	362	377	380	392
256	270	276	312	344	362	376	392	400	408	420
267	276	289	328	362	384	400	408	425	440	443
296	312	328	370	416	454	464	486	500	512	531
324	344	362	416	476	530	540	570	588	606	636
344	362	384	454	530	570	606	624	660	680	709
356	376	400	464	540	606	644	666	700	724	782
362	392	408	486	570	624	666	700	735	762	810
377	400	425	500	588	660	700	735	768	800	850
380	408	440	512	606	680	724	762	800	833	888
392	420	443	531	636	709	782	810	850	888	947

neglects the thickness of the diaphragm (of little influence because of the thinness and high conductivity of the metal), is

$$k = \frac{300}{\frac{1}{1+6\sqrt{v_{f1}}} + \frac{1}{1+6\sqrt{v_{f2}}}} \quad . \; . \; . \; . \quad (231)$$

which v_{f1} and v_{f2} are the velocities of the two liquids.

In order to allow for the furring of the pipes, which is never wanting in practice, we shall take, in estimating the coefficient of transmission, k_k, *for practical purposes*, the expression

$$k_k = \frac{200}{\frac{1}{1+6\sqrt{v_{f1}}} + \frac{1}{1+6\sqrt{v_{f2}}}} \quad . \; . \; . \; . \quad (232)$$

The coefficients, k_k, calculated from this equation for velocities of 0·01-2 m. are collected in Table 64, from which most actual cases may be taken.

The *mean temperature difference*, θ_m, is obtained by means of Table 1 from the ratio

$$\frac{t_{wa} - t_{ke}}{t_{we} - t_{ka}} = \frac{\theta_e}{\theta_a}$$

The mean difference in temperature for certain special conditions may be taken from the later Table 68, in which it is given for open surface-coolers.

When the cooling surface is formed of tubes of circular section it can be calculated from the dimensions of the tube, $H = d\pi l$, and the weight of liquid, F_w, passing through per hour, may be expressed as the product of the section of the tube, the velocity and the specific gravity :—

$$F_w = \frac{d^2\pi}{4} v_f 3600 s_w 1000 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (233)$$

The quantity of heat passing through the cooling surface in one hour must be equal to that lost in this period by the liquid :—

$$d\pi l k_k \theta_m = \frac{d^2\pi}{4} v_f \, . \, 3600 \, s_w \, . \, 1000 \, . \, \sigma_w (t_{wa} - t_{we}) \quad . \quad . \quad (234)$$

Hence follows the length of the cooling pipe :—

$$l = \frac{d}{k_k \theta_m} 900{,}000 \, v_f \, . \, s_w \, . \, \sigma_w (t_{wa} - t_{we}) \quad . \quad . \quad (235)$$

in which, for water, σ and $s = 1$.

The desired velocity of flow and diameter of pipe, required to cool a definite weight of liquid through a definite range of temperature, cannot be arbitrarily chosen, and from them the length of the pipe calculated, because in most cases impossibly long pipes would be the result. The diameter of the pipe, the velocity and quantity of liquid depend one on the other. It requires some practice to select proper proportions.

In order to facilitate the selection, two tables are here given.

1. Table 65, which gives the necessary lengths of tube for the required inner surface of 0·5-7 sq. m. in tubes of 10-70 mm. diameter.

2. Table 66, which shows :—

(*a*) The volume of liquid, V_f, which flows per hour through pipes of 10-30 mm. diameter with velocities from 0·02-0·4 m. (*b*) The

TABLE 65.

The length of a cooling pipe of 10-70 mm. diameter, when its internal surface is 0·25-7 sq. m.

Bore of pipe.	In order that a heating or cooling pipe may have an internal cooling surface, H_k, in sq. m., of														
	0·25	0·5	1	1·5	2	2·5	3	3·5	4	4·5	5	5·5	6	6·5	7
mm.	it must have the following lengths, l, in m., with the diameters given in the side column.														
10	8·00	16·1	32·2	48·3	64·5	80·5	96·6	—	—	—	—	—	—	—	—
15	5·30	10·6	21·2	31·8	42·4	53·0	74·2	84·8	84·9	95·4	106·0	—	—	—	—
20	4·00	8·0	15·9	23·9	31·8	39·8	47·7	55·7	63·6	71·6	79·5	87·5	95·4	103·4	—
25	3·20	6·4	12·7	19·1	25·4	31·8	38·1	44·5	50·8	57·2	63·5	69·9	79·2	85·6	91·9
30	2·65	5·3	10·6	15·9	21·2	26·5	31·8	37·1	42·4	47·7	53·0	58·3	63·6	68·9	74·2
35	2·30	4·6	9·1	18·7	18·2	22·8	27·3	31·9	36·4	41·0	45·5	50 1	54·6	59·2	63·7
40	2·00	4·0	8·0	12·0	16·0	20·0	24·0	28·0	32·0	36·0	40·0	44·0	48·0	52·0	56·0
45	1·80	3·6	7·1	10·7	14·2	17·8	21·3	24·9	28·4	32·0	35·5	39·1	42·6	46·2	49·7
50	1·58	3·15	6·3	10·0	12·6	15·9	18·9	22·6	25·2	28·9	31·8	35·5	37·8	41·5	44·1
55	1·45	2·9	5·8	8·7	11·6	14·5	17·4	20·3	23·2	26·1	29·0	31·9	34·8	37·7	40·6
60	1·35	2·7	5·3	8·0	10·3	13·3	15·6	18·3	20·1	23·0	26·5	29·2	31·2	33·9	36·2
65	1·25	2·5	4·9	7·4	9·8	12·3	14·7	17·2	19·6	22·1	24·5	27·0	29·4	31·9	34·3
70	1·15	2·3	4·6	6·9	9·2	11·4	13·8	16·1	18·4	20·7	22·7	25·0	27·6	29·9	32·2

lengths of tube, l (and thence the cooling surface), required to cool the volumes of liquid, V_f, given in column 3 (in this case water: $\sigma = 1$, $s = 1$) from the initial temperature, t_{wa}, to the final temperature, t_{we}, by means of cooling water at the different initial and final temperatures, t_{ka} and t_{ke}, and of different velocities, $v_f = 0·02$-$0·4$ m.

This Table 66 is calculated by means of equation (235). The very great number of the possible variations of all cases has permitted only a restricted selection of variables. The table shows that, if the pipe is not to be too long, the velocity of the liquid to be cooled may only be low. Therefore, in the case of a large quantity of liquid, many narrow pipes, arranged parallel to one another, must be used in place of one long pipe.

If it is expected that the cooling surface will be very clean, the number of tubes found from Table 66, or their length, may be diminished by about 25 per cent.

TABLE 66.

(*a*) The volume of liquid, V_f, in litres, which passes through tubes of 10-30 mm. diameter in one hour, with velocities of v_f = 0·02, 0·05, 0·1, 0·2 and 0·4 m. per second.

(*b*) The necessary length of pipe, l, in m., by which, with continuous working, the above volumes of water, V_f, may be cooled from the initial temperature, t_{wa}, to t_{we}, by means of cooling water with the temperatures, t_{ka}, to t_{ke}.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Bore of pipe. d	Velocity of the liquid to be cooled. v_{f1}	(*a*) Litres of liquid passing through the pipe per hour. V_f	Velocity of the cooling liquid. v_{f2}	Initial temperature of the warm liquid, t_{wa}.																	
				100°				80°					60°					50°			
				Final temperature of the warm liquid, t_{we}.																	
				3	3	15	25	10	20	20	25	30	3	15	20	25	30	15	20	20	30
				Final temperature of the cooling water. t_{ke}.																	
				60	80	60	60	60	60	60	50	60	50	40	40	50	50	30	40	30	30
				Initial temperature of the cooling water, t_{ka}.																	
				2	2	10	10	5	10	15	10	15	2	10	15	10	15	10	10	15	15
				(*b*) Requisite length of cooling pipe in m.																	
10	0·02	5·6	0·001	11·35	18·9	6·2	3·56	8	7·0	5	4·62	3·6	18·5	5	3·3	3·6	3	3·8	3·6	3·3	1·3
			0·10	8·2	12	4	2·2	5·2	4·5	3·2	2·8	2·3	12	3·7	2·1	2·3	2	2·4	2·3	2·1	0·82
			0·00	5·6	9·4	3·1	1·8	4	3·5	2·5	2·3	1·8	9·3	2·5	1·7	1·8	1·5	1·9	1·8	1·6	0·7

	0·05	14·1	0·001	25·8	43	14	8·1	18·1	16	16	10·5	8·1	42	11·5	7·5	8	6·8	8·6	8·2	7·5	3
			0·10	16·5	26·5	9	5·2	12	10·2	10·2	7·2	5·2	27	7·5	4·8	5·2	4·4	5·6	5·3	4·8	2
			1·00	12	19	6·3	3·6	8·	7·2	7·2	5	3·4	19	5·3	3·5	3·4	3·1	3·9	3·7	3·5	1·4
	0·10	28·2	0·001	49	81·5	26·6	15·6	34·5	30	22	20	15·3	30	22	14	15	13	17	16	14·5	6
			0·10	31·5	52	16·5	10·2	22·2	20	14	13	10	52	14	9	10	8·2	10·8	10·2	9·8	4
			1·00	22	37	12	7	16	14	10	9	7	36	10	6·3	6·8	6	7·7	7·2	6·5	2·7
	0·20	56·4	0·001	—	—	52	30	—	—	43	39	29·2	—	42	28	29	25·5	34	30	28	13
			0·10	—	—	33	20	49	45	27·6	26	13·5	—	27	18	18·4	16	22	19	20	8·[illegible]
			1·00	—	—	17	10·3	26	23	15	14	10	—	15	10	10	9	12	11	10	4·6
	0·40	112·8	0·001	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	21·6
			0·10	—	—	—	43	—	—	—	—	39	—	—	37	38	33	44	39	42	13·9
			1·00	—	—	38	23	54	48	32	29	21	—	32	21	21	19	25	28	21	7
15	0·02	12·7	0·001	17	28·5	10·3	5·3	12	10·5	7·5	6·9	5·3	28	7·5	5	5·4	4·5	5·6	5·4	5	2
			0·10	11	18	7	3·9	7·7	7	4·8	4·5	3·5	18	4·8	3·7	3·5	2·9	3·6	3·5	3·7	1·3
			1·00	8·5	14·3	5·2	2·7	6	5·3	3·3	3·5	2·7	14	3·8	2·5	2·7	2·3	2·8	2·7	2·5	1
	0·05	31·7	0·001	38·7	64·5	21	12·2	27	24	17	15·8	12·2	58	17·5	12	12	10·2	13	12·2	12	4·5
			0·10	26	42	13·5	7·7	17	15·5	11	10·1	7·8	39	11·3	7·8	7·8	7·1	3·2	7·8	7·8	2·9
			1·00	13	29	9·5	5·6	12·5	11	7	7·2	5·5	29	8	5·4	5·4	4·5	6	5·4	5·4	6
	0·10	63·5	0·001	77·7	—	40	23·4	52	45	33	30·2	23	—	33	21	22·5	19·5	26	24	22	9·1
			0·10	49	—	25	15·4	33	29	21	20	15·2	—	21	13·5	14·2	13	16·6	16	14	5·8
			1·00	36	—	18	11	24	20·5	15	14	11·5	56	15	9·5	10·5	9	12	11	10	4·2
	0·20	127	0·001	—	—	78	45	—	—	—	59	44	—	61	41	—	38·2	—	45	41	18
			0·10	—	—	49	29	—	52	37	38	28	—	38·6	26	27	25	30	29	26	11·6
			1·00	—	—	27	16	43	29	21	21	16	—	21	15	15	13	18	16	14	5
20	0·02	22·6	0·001	22·7	37·8	12·4	7·1	16	14	10	9·3	7·1	37	10	6·6	7·1	6	7·2	7·1	6·6	2·7
			0·10	14·4	24·4	7·9	4·5	10·2	9	7·1	6	4·5	24	7·1	4·1	4·5	3·8	4·6	4·5	4·1	1·8
			1·00	11·4	1·9	6·2	3·6	·8	7	5	4·7	3·6	18·5	5	3·3	3·6	3	3·6	3·6	3·3	1·4

TABLE 66—(*continued*).

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Bore of pipe.	Velocity of the liquid to be cooled.	(a) Litres of liquid passing through the pipe per hour.	Velocity of the cooling liquid.	Initial temperature of the warm liquid, t_{wa}.																	
				100°				80°					60°					50°			
				Final temperature of the warm liquid, t_{we}.																	
				3	3	15	25	10	20	20	25	30	3	15	20	25	30	15	20	20	30
				Final temperature of the cooling water, t_{ke}.																	
				60	80	60	60	60	60	60	50	60	50	40	40	50	50	30	40	30	30
				Initial temperature of the cooling water, t_{ka}.																	
				2	2	10	10	5	10	15	10	15	2	10	15	10	15	10	10	10	15
d	v_{f1}	V_f	v_{f2}	(b) Requisite length of cooling pipe in m.																	
20	0·05	56·5	0·001	52	86	28	16·3	36·2	32	23	21	16·2	—	23	15	24	13·6	17	37	15	6
			0·10	33	55	18	10·5	23	10	15·4	13·5	10·3	48	15·4	10	15	9	11	11	9·8	3·8
			1·00	24	38	13	7·3	17	15	10·5	9·5	7·4	37	10·5	7	11	6	8	8	7	2·7
	0·10	113	0·001	97	163	53	31·2	69	60	44	40·2	30	—	44	28	30	26	34	22	29	12
			0·10	62	104	33·5	20	44	39	28	26·7	19·6	—	28	18	20	16·6	22	20·8	18·5	7·8
			1·00	29	76	24	14·5	20	27	20	18	14·5	—	20	13	14	12	16	15	23	5·4
	0·20	226	0·001	—	—	—	60	—	—	—	—	58	—	—	—	—	51	—	—	—	24
			0·10	—	—	—	40	—	—	—	—	37	—	—	—	—	32	44	41	37	15·6
			1·00	—	—	36	21	30	50	30	27	20	—	30	19	21	18	24	23	19	9

25	0·02	35·2	0·001	28·3	47	16·5	8·9	20	21	12·5	11·5	8·9	46	12·5	8·3	9	7·5	9·4	9	8·3	3·8
			0·10	18·3	30	10·6	5·6	14	13·5	17·9	7·5	5·6	29·2	7·9	5·8	5·7	4·8	6·1	5·7	5·3	2·1
			1·00	14·2	24	8·3	4·5	10	10·5	6·3	5·8	4·4	28	6·3	4·2	4·5	3·8	4·7	4·5	4·2	1·7
	0·05	88	0·001	64	187	35	21	54·5	45	29	26·3	20	—	34·5	19	20	17	22	20	19	7·5
			0·10	41	68	33·5	13·5	34·5	28	18·5	16·7	13	—	22·3	12	13	11	14	13	12	4·8
			1·00	29	49	16·5	9·5	25	18	13·5	12	9	47	16	8·7	9	8	10	9	7·7	8·5
	0·10	176	0·001	122	204	67	39	86·5	—	55	50·3	38	—	—	35	37·5	33	43	40	39	15
			0·10	78	134	43	24·4	55	—	35	32	24	—	42	22·4	64·4	21	27·4	26	23·4	9·8
			1·00	55	92	30	12	39	33	25	23	17	—	30	16	17	15	19·5	18	16·2	6·8
	0·20	352	0·001	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	30
			0·10	—	—	—	43	—	—	—	—	43	—	—	40	43	38	—	—	42	20
			1·00	—	—	—	14	—	—	42	39	29	—	—	27	29	96	33	30	27	10
30	0·01	25·4	0·001	18·2	30·3	10	8·7	12·2	7·7	10·8	7·8	5·7	24·2	9	8·6	5·6	5·1	6·4	6	5·4	2·3
			0·10	12	20	6·7	5·8	8	5·2	7·2	5·	3·7	16	6	5·4	3·7	3·4	4·3	4	3·6	1·5
			1·00	9	16·5	5·5	4·7	6·7	4	6	4·2	3·1	13	5	4·7	3·1	2·8	3·5	3·3	3	1·23
	0·02	50·8	0·001	24·3	39·5	13·3	11·6	15·9	10·2	14·4	10·5	7·6	32	12	11·5	7·5	6·8	8·6	8	7·2	3·1
			0·10	22	37	12	10·7	15	9	11	9	7	29	13·6	10·6	6·6	6·3	7·6	7·3	6·6	2·8
			1·00	16·2	27	9	7·8	10·8	7·1	9·6	7	5·1	21	8	7·5	5	4·5	5·7	5·4	4·8	2·0
	0·05	127	0·001	80	132	44	35	5·2	33	47	34	24	—	39	37	24	22	27	26	23	10
			0·10	47	78	26	22·6	31·7	20	28	20	13·8	62	23·4	22	14·5	13·2	16	15·6	14	6
			1·00	36	60	20	17	24	15	21	15·6	11	48	18	17	11	10	12·8	12	10·8	4·1
	0·10	254	0·001	—	—	—	—	—	—	—	—	45	—	—	—	68	45	51	48	43	19
			0·10	—	—	47	41	57	35	51	36	26	—	42	40	36	23·5	30	28·2	25	11
			1·00	—	—	33	28	40	25	25	26	18·5	—	29	27	18	16·2	21	19·8	17	7·5
	0·20	508	0·001	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
			0·10	—	—	—	—	—	64	—	—	45	—	—	—	44	40	51	48	43	18
			1·00	—	—	66	57	—	37	50	80	50	—	60	56	36	33	52	39	35	15

Iron tubes must be about 20 per cent. greater in number. In cooling thick liquids the same increase is necessary.

If the specific gravity and specific heat of the liquid to be cooled are not equal to unity, but are s and σ respectively, the number of tubes is to be multiplied by $s\sigma$.

Example.—2000 litres of water are to be cooled per hour from 80° to 30° C. by means of cooling water which becomes heated from 15° to 60° C. The velocity of the warm water is 0·02 m., that of the cold water 0·01 m., the cooling pipe is to have a diameter of 20 mm.

According to equation (229) the amount of heat to be transferred is

$$C = F_w \sigma_w s_w (t_{wa} - t_{we}) = 2000 \times 1 \times 1\,(80 - 30)$$
$$= 100{,}000 \text{ calories.}$$

The volume of cooling water is

$$F_k = \frac{C}{t_{ke} - t_{ka}} = \frac{100{,}000}{60 - 15} = 2222 \text{ litres.}$$

Through a tube of 20 mm. diameter there flow in one hour at $V_f = 0{\cdot}02$ m. per second, according to Table 66, 22·6 litres. There must therefore be $\frac{2000}{22{\cdot}6} =$ 89 tubes.

The length of each tube is obtained from equation (235):

$$l = \frac{d}{k_k \theta_m} 900{,}000 v_f (t_{wa} - t_{we}),$$

in which, by equation (232) and Table 64, $k_k = 170$.

Now $\frac{30 - 15}{80 - 60} = \frac{15}{20} = 0{\cdot}75$, therefore, by Table 1,

$$\theta_m = 0{\cdot}872 \times 20 = 17{\cdot}44°,$$

thus $l = \frac{0{\cdot}02}{170 \times 17{\cdot}44} 900{,}000 \times 0{\cdot}02(80 - 30) = 6{\cdot}07$ m.

The cooling surface is therefore $H = 89\,dl = 35{\cdot}8$ sq. m.

If 2000 litres of alcohol (86·8 per cent. by weight), for which $\sigma_w = 0{\cdot}7$ and $s_w = 0{\cdot}8$, are to be cooled under the same conditions of temperature as above, then

$$C = 100{,}000 \times 0{\cdot}7 \times 0{\cdot}8 = 56{,}000 \text{ calories;}$$

therefore $$F_k = \frac{56{,}000}{60 - 15} = 1244 \text{ litres.}$$

The number of tube is, as above, 89.

The length of each tube, $l = 6{\cdot}07 \times 0{\cdot}7 \times 0{\cdot}8 = 8{\cdot}4$ m.

The cooling surface, H_k, is about 19 sq. m.

Experiment.—Hentschel's wort cooler. A hollow spiral (conveyor) of 350 mm. diameter turns in an open trough of about 360 mm. diameter at 40-45 revolutions per minute, and carries the wort from end to end. The cooling water flows in the hollow spiral in the opposite direction to the wort in the trough. ..

2800 litres of warm wort were in this way cooled by means of 14 sq. m. of cooling surface from 58·8° to 16·25° C. in 45 minutes by 2400 litres of cooling water, which was heated from 10° to 40° C.

$$\text{Now, } \theta_a = 58{\cdot}8 - 40 = 18{\cdot}8^\circ$$
$$\theta_e = 16{\cdot}25 - 10 = 6{\cdot}25^\circ$$
$$\text{thus } \frac{\theta_e}{\theta_a} = \frac{6{\cdot}25}{18{\cdot}8} = 0{\cdot}3.$$

Therefore, by Table 1 the mean temperature difference is

$$\theta_m = 0{\cdot}583 \times 18{\cdot}8 = 10{\cdot}96^\circ.$$

It was observed, in regard to the wort, that

$$k_k = \frac{4 \times 2800(58{\cdot}8 - 16{\cdot}25)}{3 \times 14 \times 10{\cdot}96} = \text{about } 1035,$$

or in regard to the water:—

$$k_k = \frac{4 \times 2400(40 - 10)}{3 \times 14 \times 10{\cdot}96} = \text{about } 621.$$

The velocity of the wort over the cooling surface is

$$v_{f1} = \frac{0{\cdot}350 \,.\, \pi \,.\, 45}{2 \times 60} = 0{\cdot}41 \text{ m. per second.}$$

The velocity of the water is equally great, but there is to be added to it the velocity in the hollow spiral, which is, if the section of the spiral be 0·15 sq. dcm.:

$$v_{f2} = \frac{2400 \times 4}{60 \times 60\,80 \times 15 \times 10} = \text{about } 0{\cdot}6 \text{ m. per second.}$$

Thus the water is carried with a velocity of 0·41 + 0·60 = 1·01 m. over the diaphragm between water and wort.

The coefficient of transmission for the water, *calculated* by equation (232), is

$$kk = \frac{200}{\dfrac{1}{1 + 6\sqrt{0{\cdot}41}} + \dfrac{1}{1 + 6\sqrt{1{\cdot}01}}} = 572 \text{ (approx.)}.$$

This result agrees with the *observed* coefficient $kk = 626$ with sufficient accuracy, since the metal surface is always kept clean by the wash of the liquid, and the coefficient thus somewhat increased.

The transmission coefficient for the wort *appears* to be considerably higher, because it is in contact with the air and is thus cooled by evaporation to a considerable extent, which is the advantage of this method of cooling.

In refrigerating machines the exchange of heat generally takes place at a low temperature; for this reason, and because the liquids used are not always as mobile as water, the coefficient of transmission appears to be somewhat lower. H. Lorenz (Zeits. f. d. gesammte Kälteindustrie, 1897, Heft 9) found, for liquid carbonic acid which was cooled in an iron pipe from 34·58° to 21·61° C. by means of water which became heated from 9·9° to 21·61° C., $k_k = 105$. In another

case, when the liquid carbonic acid was cooled from 19·45° to 11·8° C., and the cooling water warmed from 9·9° to 11·08°, k_k was 125 (when the real mean temperature difference was used in the calculation).

2. *The second method (discontinuous or periodic)* consists in bringing the whole quantity of liquid to be cooled at once into a vessel and allowing the cooling fluid (usually water) to flow round the external walls of the vessel, or through pipes or plates, at rest or in motion, until the liquid is sufficiently cooled. The operation is shortened if the liquid to be cooled is moved artificially at a fair speed over the cooling surface or the cooling surface is moved through the liquid, since the very small differences of temperature existing at the same time in the liquid cause only a slow circulation. The amount of heat to be extracted from the weight of liquid, F_w, which is cooled from t_{wa} to t_{we}, and thus to be taken up by the cooling agent is

$$C = F_w \sigma_w (t_{wa} - t_{we}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (236)$$

The cooling surface required for the transfer of this amount of heat is

$$H_k = \frac{C}{k_k \theta_m} = \frac{C}{\dfrac{200}{\dfrac{1}{1+6\sqrt{v_{f1}}} + \dfrac{1}{1+6\sqrt{v_{f2}}}}\theta_m} \quad . \quad . \quad (237)$$

If we assume that a uniform temperature prevails throughout the warm liquid at any instant, so that all portions take a regular part in the cooling, then the mean temperature difference between the liquid and the cooling medium diminishes continuously, the latter being heated from its constant initial temperature to a final temperature which decreases during the progress of the operation.

The mean *temperature difference* at the beginning, θ_{ma}, is obtained from the greatest and least temperature differences between the warm liquid and the cooling medium at the beginning, θ_{a1} and θ_{e1}. The mean temperature difference at the end, θ_{me}, is obtained from the greatest and least temperature differences at the end, θ_{a2} and θ_{e2}.

The true mean temperature difference, θ_m, for the whole operation, is obtained from the two mean temperature differences at the beginning and the end, θ_{ma} and θ_{me}.

By means of Table 1, $\dfrac{\theta_{e1}}{\theta_{a1}}$ gives the mean temperature difference of the beginning: $\theta_{ma} = a\theta_{a1}$; similarly, $\dfrac{\theta_{e2}}{\theta_{a2}}$ gives the mean tempera-

ture difference at the end: $\theta_{me} = \beta\theta_{a2}$. Finally, $\frac{\theta_{me}}{\theta_{ma}}$ gives the true mean temperature difference:

$$\theta_m = \gamma\theta_{ma} = \gamma\alpha\theta_{a1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (238)$$

When the true mean temperature difference, θ_m, is found, and also the mean temperature, t_m, of the warm liquid calculated in the well-known simple manner, then by subtraction the mean escape temperature of the cooling water is found: $t_{ke} = t_m - \theta_m$; from this the mean increase in temperature is obtained: $t_{em} = t_{ke} - t_{ka}$, and thence the weight of cooling water requisite to extract the quantity of heat, C:—

$$W = \frac{C}{t_{em}} = \frac{C}{t_{ke} - t_{ka}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (239)$$

If we now arrange that the ratios $\frac{\theta_{e1}}{\theta_{a1}}$ and $\frac{\theta_{e2}}{\theta_{a2}}$ are equal, *i.e.*, that $\alpha = \beta$, the calculation and explanation are simplified. We shall therefore now assume that the ratio of the temperature differences at the beginning is equal to the ratio of the temperature differences at the end—a very good and natural condition.

In order to estimate the necessary cooling surfaces we still require to know the *velocities of the liquid and the cooling water*, v_{f1} and v_{f2}. The former may be taken at about 0·02 m. if there is no stirrer and the cooling surfaces are favourably arranged.

If the cooling vessel be provided with a stirrer it may be arranged so as to give the mass a velocity of 1 m. or rather more, but not more than 3 m.

The velocity of the cooling water, when it flows through pipes, may be determined by means of Table 66. It will generally be very low.

Example.—2000 litres of water are to be cooled in 1 hour from 80° to 20° C. by water at 10° C. which is to be heated at first to 60°.

The quantity of heat to be transferred is

$$C = 2000(80 - 20) = 120{,}000 \text{ calories.}$$

The mean temperature difference at the beginning, is, by Table I,

$$\left(\text{since } \frac{\theta_{e1}}{\theta_{a1}} = \frac{80 - 60}{80 - 10} = \frac{20}{70} = 0{\cdot}286\right)$$

$$\theta_{ma} = 0{\cdot}575\theta_{a1} = 0{\cdot}575 \times 70 = 40{\cdot}25°.$$

At the end,

$$\left(\text{since } \frac{\theta_{e2}}{\theta_{a2}} \text{ is to be equal to } \frac{\theta_{e1}}{\theta_{a1}}\right)$$

$$\theta_{me} = 0.575\theta_{a2} = 0.575(20 - 10) = 5.75^\circ.$$

The true mean temperature difference is therefore

$$\left(\text{since } \frac{\theta_{me}}{\theta_{ma}} = \frac{5.75}{40.25} = 0.143\right)$$

$$\theta_m = 0.575 \times 0.441 \times 70 = 17.7^\circ.$$

The mean temperature of the liquid is

$$\left(\text{since } \frac{t_{we}}{t_{wa}} = \frac{20}{80} = 0.25\right)$$

$$t_m = 0.544 \times 80 = 43.52^\circ.$$

Consequently the mean temperature at which the cooling water leaves is

$$t_{ke} = 43.52 - 17.7 = 25.82^\circ.$$

$$\text{Now } t_{em} = 25.82 - 10 = 15.82^\circ,$$
$$\text{and } C = 2000(80 - 20) = 120{,}000,$$
$$\text{therefore } W = 7580 \text{ litres.}$$

If the water flows through the pipe with a velocity of 0·1 m., and if the stirrer gives the liquid to be cooled a velocity of 1 m. over the cooling surface, then, by Table 64, $kk = 408$.

The requisite cooling surface is therefore

$$H_k = \frac{C}{k_k\theta_m} = \frac{120{,}000}{408 \times 17.7} = 16.7 \text{ sq. m.}$$

Since the velocity in the pipe is to be 0·1 m., the cooling surface may consist of:—

1 tube	of 160 mm.	diameter,	33·4 m.	long.
4 tubes	of 80	,,	16·7	,,
8	,, 57	,,	11·7	,,
18	,, 40	,,	8·4	,,

The desired data for a few cases are collected in Table 67.

Experiment.—In the mash-tun of a distillery, with 8·4 sq. m. of cooling surface in the shape of brass tubes of 45 mm. bore and 48 mm. external diameter, 3000 litres of wort were cooled in 105 minutes from 62·5° to 16·25° C., by means of 9632 litres of cooling water (91·73 litres per minute) at 10·62° C., which was heated to 50° at the commencement, to 13·4° at the end.

The average velocity of the water in the cooling pipe was 0·877 m., that of the wort over the cooling surface about 0·85 m. per second. (Tun 2300 mm. in diameter, stirrer gives 30 revolutions per minute, hence its mean velocity is 1·7 m. The motion of the liquid moved by the stirrer was assumed to be half as great.) The wort lost 3000(62·5 − 16·25) = 138,750 calories. The water gained

TABLE 67.

Discontinuous (periodic) cooling. Mean temperature difference, θ_m, mean temperature of outflow of cooling water, t_{ke}, the requisite quantity of cooling water, W, and cooling surface, H_k, for velocities of the liquid of 1 m., of the cooling water of 0·1 m., in order to cool 100 kilos. of water in one hour.

Original temperature of cooling water.	Liquid to be cooled. From	Liquid to be cooled. to	Cooling water, temp. of outflow. Beginning.	Cooling water, temp. of outflow. End.	Mean temperature difference.	Mean temperature of cooling water outflow.	Cooling water required for 100 kilos. of liquid.	Cooling surface for $v_{f1} = 1$, $v_{f2} = 0·1$.	Original temperature of cooling water.	Liquid to be cooled. From	Liquid to be cooled. to	Cooling water, temp. of outflow. Beginning.	Cooling water, temp. of outflow. End.	Mean temperature difference.	Mean temperature of cooling water outflow.	Cooling water required for 100 kilos of liquid.	Cooling surface for $v_{f1} = 1$, $v_{f} = 0·1$.
t_{ka}	t_{wa}	t_{we}			θ_m	t_{ke}	W	H_k	t_{ka}	t_{wa}	t_{we}			θ_m	t_{ke}	W	H_k
° C.	° C.	° C.	° C.	° C.	° C.	° C.	kilos.	sq. m.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	kilos.	sq. m.
10	100	80	80	64 5	41	48·6	52	0·12	10	70	30	60	26·7	16·8	30·8	192	0·60
”	”	80	60	49	54·9	34·7	81	0·09	”	”	80	50	23·3	22·2	25·4	260	0·44
”	”	60	80	49	35	43·6	119	0·28	”	”	20	60	18·3	12·6	27·3	290	1·00
”	”	60	60	38	46·8	31·8	183	0·21	”	”	20	50	16·7	16·8	23·1	382	0·73
”	”	40	80	33·3	28·8	36·9	223	0·50	15	70	50	60	43·3	21·1	38·4	70	0·23
”	”	40	60	26·6	38	27·7	339	0·40	”	”	50	50	37·3	29	30 5	98	0·17
”	”	20	80	17·8	18	32	363	1·09	”	”	30	60	27·3	14·5	33·1	173	0·68
”	”	20	60	15·6	24·5	25·5	516	0·80	”	”	30	50	24·5	20	27·6	228	0·49
15	100	80	80	64·7	39·5	50·1	57	0·122	”	”	20	60	19	10·3	29·6	255	1·20
”	”	80	60	49·4	52	37·6	88·5	0·095	”	”	20	50	18	14	25·9	315	0·87
”	”	60	80	49·4	32·3	46 3	128	0·30	10	60	40	50	34	19·6	29·6	102	0·25
”	”	60	60	38·8	43·3	35·3	200	0·23	”	”	40	40	28	25·9	23·3	150	0·18
”	”	40	80	34	25·5	40·2	238	0·58	”	”	20	50	18	12·5	24·7	272	0·80
”	”	40	60	28·3	34	31·7	360	0·48	”	”	20	40	16	16·5	20·7	374	0·49
”	”	20	80	18·8	14·4	35·6	390	0·36	15	60	40	50	34·4	17·5	31·7	120	0·28
”	”	20	60	17·6	19·5	30·5	516	1 00	”	”	40	40	28 9	23	26·2	178	0·22
10	80	60	60	45·7	33·3	32·3	90	0·15	”	”	20	50	18·9	9	28·2	303	1·10
”	”	60	40	31·4	45	20·6	195	0 11	”	”	20	40	17·2	12·4	24·8	408	0·80
”	”	40	60	31·4	26·3	31·6	281	0·37	10	50	50	40	25	15·7	23·6	147	0·31
”	”	40	40	23	35	22·9	311	0·28	”	”	30	30	20	20·9	18·4	238	0·24
”	”	20	60	17·4	17·5	26	375	0·83	”	”	20	40	17·5	11·8	21·0	273	0·63
”	”	20	40	19·4	23·3	20·2	590	0·63	”	”	20	30	15	15·7	19·1	330	0·48
15	80	60	60	46	37	28·6	147	0·14	15	50	30	40	25	13·6	25·7	190	0·36
”	”	60	40	32	42·9	22·7	220	0·12	”	”	30	30	21·4	17·9	21·4	315	0·28
”	”	40	60	32·3	24·7	33·2	220	0·40	”	”	20	40	18·6	8·9	23·9	339	0·83
”	”	40	40	24·6	38	20	817	0·27	”	”	20	30	17·1	12·1	20·7	526	0·61
”	”	20	60	18·4	13·7	29·8	405	1·08	10	40	20	30	16·7	11	17·9	253	0·45
”	”	20	40	17	18·9	24·6	625	0·80	”	”	20	20	13·3	15	13·9	513	0·33
10	70	50	60	48·4	22·6	36·9	74·3	0·22	15	40	20	30	18	8·3	20·6	355	0·60
”	”	50	50	26.7	30	29·5	103	0·16	”	”	20	20	16	11·2	17·7	741	0·44

9632 × 12·1 = 116,547 calories. The difference, 138,750 − 116,547 = 22,203 calories, was lost by radiation and evaporation.

The mean temperature difference was $\theta_m = 12{\cdot}03°$, hence the *observed* coefficient of transmission is:

$$k_k = \frac{C}{H\theta_m z_h} = \frac{116{,}547}{8{\cdot}4 \times 12{\cdot}1 \times \frac{105}{60}} = 665 \text{ calories.}$$

The *calculated* coefficient of transmission is:

$$k_k = \frac{200}{\frac{1}{1+6\sqrt{v_{fl}}} \times \frac{1}{1+6\sqrt{v_{fl}}}}$$

$$= \frac{200}{\frac{1}{1+6\sqrt{0{\cdot}877}} + \frac{1}{1+6\sqrt{0{\cdot}85}}} = 656 \text{ calories.}$$

The agreement is sufficiently good.

The following table gives the course of the experiment

After minutes.	Temperature of wort.	Temperature of waste water.	Temperature differences.				Rise in temperature of water.	
			At outlet.	At inlet.	Observed mean.	Total mean.	Observed.	Mean.
	t_{we}	t_{ke}	θ_e	θ_a		θ_m		
0	62·5	30	12·5	51·9	28	5 × 27·5	39·4	5 × 35·2
5	56·25	41·25	15	45·65	27	6 × 25·8	30·65	6 × 28·15
11	50	36·25	13·75	39·4	24·6	6 × 22·6	25·65	6 × 23 15
17	43·75	31·25	12·5	33·15	21·1	8 × 19·6	20·65	8 × 18·77
25	37·5	27·5	10	26·9	17·4	8 × 15·5	16·9	8 × 14·4
33	31·25	22·5	8·75	20·65	13·58	25 × 11·25	11·9	25 × 10·9
58	25	20	5	14·4	9·21	6 × 8·15	9·9	6 × 8·9
64	22·5	18·5	.4	11·9	7·1	10 × 6·95	7·9	10 × 6·77
74	20	16·25	8·75	9·4	6·18	16 × 5·5	5·65	16 × 4·73
90	17·5	14·4	8·1	6·9	4·9	15 × 4·5	3·8	15 × 3·3
105	16·25	13·4	2·85	5·65	4·1		2·8	
						$\frac{1263}{105} = 12{\cdot}03°$		$\frac{1267}{105} = 12{\cdot}1°$

E. Open Surface-coolers.

Many hot liquids are cooled by allowing them to flow down, exposed to the atmosphere, over metallic surfaces, on the other side of which passes cold water. This form of apparatus is here called the open surface-cooler. Its cooling surfaces consist of straight or

bent tubes arranged one above the other; the section of a tube is circular, oval or approximately triangular. More rarely plane surfaces, vertical or inclined, or vertical tubes, are used.

The liquid flows down over the cooling surface with various velocities, which increase with the smoothness of the surface, the height of flow, and with the quantity of liquid which flows in unit time over unit length of the apparatus, *i.e.*, with the thickness of the flowing layer. The velocity decreases with the inclination of the surfaces to the horizon and with the consistency, thickness or viscosity of the liquid.

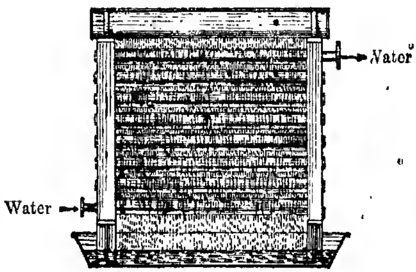

FIG. 20.

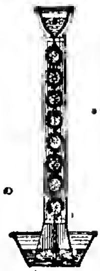

FIG. 21.

Over smooth plane vertical surfaces, the height of which is	1	2	3	4 m.,
the mean velocity at which water flows down is about	0·5-0·7	0·6-0·9	0·8-1·1	0·9-1·3 m.

The quantity of liquid, which flows down in one hour over 1 m. length of the cooling surface, may be greater in larger apparatus than in smaller. With an apparatus which can cool in one hour

	100	300	500	800	1000	2000	3000 (or more) litres,
there may flow over a length of 1 m. in one hour	125	300	390	420	550	700	800 litres.

The cooling water enters below and leaves above; it is desirable that it should pass through the cooling tubes with a tolerable velocity, which may be about 0·5 mm. in small apparatus, 1·0 m. or more in a large apparatus.

TABLE 68.

The copper or brass cooling surface, H_k, in sq. m., and the cooling water, W, in litres, for open surface coolers, required to cool $F_w = 100$ kilos. of aqueous liquid in one hour from $t_{wa} = 100°$-$30°$ C. down to $t_{we} = 30°$-$3°$ C., by means of cooling water at $t_{ka} = 2°$-$15°$ C.

Original temperature of the liquid to be cooled. t_{wa}	Temperature of the outflow of cooling water. t_{ka}		Original temperature of the cooling water, t_{ka}. 2°	5°			10°			15°		
			Temperature of the cooled liquid, t_{we}. 3°	6°	10°	20°	11°	15°	25°	16°	20°	30°
00°	90°	$\theta_m =$	3·91	3·91	7·24	12·40	3·91	7·24	12·40	3·91	7·24	12·40
		$H_k =$	2·50	2·42	1·26	0·646	2·26	1·18	0·604	2·16	1·11	0·56
		$W =$	111	111	107	94·2	112	107	94	112	106	94
	80°	$\theta_m =$	6·34	6·34	10·88	17·44	6·34	10·88	17·44	6·34	10·88	17·44
		$H_k =$	1·55	1·48	0·83	0·460	1·40	0·78	0·43	1·33	0·74	0·40
		$W =$	115	125	120	107	128	122	108	130	123	108
	60°	$\theta_m =$	10·56	10·56	16·96	25·60	10·56	16·96	25·60	10·56	16·96	25·60
		$H_k =$	0·92	0·90	0·53	0·31	0·84	0·50	0·29	0·8	0·48	0·27
		$W =$	168	171	164	146	178	170	150	187	179	155
80°	70°	$\theta_m =$	3·91	3·91	7·24	12·40	3·91	7·24	12·40	3·91	7·24	12·40
		$H_k =$	1·98	1·82	0·97	0·49	1·62	0·89	0·45	1·61	0·83	0·45
		$W =$	114	114	108	93	115	109	92	116	110	90
	60°	$\theta_m =$	6·34	6·34	10·88	17·44	6·34	10·88	17·44	6·34	10·88	17·44
		$H_k =$	1·22	1·21	0·65	0·36	1·09	0·60	0·34	1·01	0·56	0·34
		$W =$	133	129	121	104	140	130	110	144	133	110
	40°	$\theta_m =$	10·56	10·56	16·96	25·60	10·56	16·96	25·60	10·56	16·96	25·60
		$H_k =$	0·73	0·70	0·41	0·35	0·69	0·38	0·22	0·60	0·36	0·20
		$W =$	200	212	200	171	230	217	184	260	240	200
60°	50°	$\theta_m =$	3·91	391	7·24	12·40	3·91	7·24	12·40	3·91	7·24	12·40
		$H_k =$	1·46	1·40	0·70	0·33	1·73	0·63	0·28	1·15	0·56	0·25
		$W =$	119	120	110	90	123	112	88	126	114	89
	40°	$\theta_m =$	6·34	6·34	10·88	17·44	6·34	10·88	17·44	6·34	10·88	17·44
		$H_k =$	0·90	0·84	0·46	0·20	0·80	0·42	0·20	0·72	0·37	0·20
		$W =$	150	150	143	90	163	150	117	180	160	120
50°	40°	$\theta_m =$	3·91	3·91	7·24	12·40	3·91	7·94	12·40	3·91	7·24	12·40
		$H_k =$	1·24	1·15	0·56	0·24	0·99	0·48	0·22	0·80	0·42	0·17
		$W =$	124	124	114	89	130	117	83	136	120	80
	30°	$\theta_m =$	6·34	6·34	10·88	17·44	6·34	10·88	17·44	6·34	10·88	17·44
		$H_k =$	0·74	0·71	0·37	0·20	0·61	0·32	0·17	0·55	0·28	0·12
		$W =$	170	178	160	120	195	175	125	226	200	133

TABLE 68—(*continued*).

Original temperature of the liquid to be cooled. t_{wa}	Temperature of the outflow of cooling water. t_{ka}		Original temperature of the cooling water, t_{ke}.									
			2°	5°			10°			15°		
			Temperature of the cooled liquid, t_{we}.									
			3°	6°	10°	20°	11°	15°	25°	16°	20°	30°
40°	30°	θ_m =	3·91	3·91	7·24	12·40	3·91	7·24	12·40	3·91	7·24	12·40
		H_k =	0·90	0·80	0·42	0·16	0·75	0·35	0·12	0·65	0·28	0·09
		W =	132	136	120	80	145	125	75	160	133	66
	20°	θ_m =	6·34	6·34	10·88	17·44	6·34	10·88	17·44	6·34	10·88	17·44
		H_k =	0·61	0·45	0·28	0·12	0·45	0·35	0·09	0·40	0·19	0·06
		W =	200	227	200	133	290	250	150	480	400	200
30°	25°	θ_m =	2·5	2·5	5·0	9·0	2·5	5	9	2·5	5	—
		H_k =	1·09	0·97	0·40	0·12	0·77	0·30	0·06	0·57	0·2	—
		W =	118	120	140	50	180	100	33	140	100	—
	20°	θ_m =	3·91	3·91	7·24	12·40	3·91	7·24	12·40	3·91	7·24	—
		H_k =	0·70	0·64	0·28	0·09	0·49	0·21	0·05	0·25	0·15	—
		W =	150	160	133	67	190	150	50	280	280	—

The cooling action of this apparatus is generally very good, because the thin layer of liquid greatly favours the transfer of heat, and because the velocity of both liquids—the cooling and the cooled—may be greater here than in closed coolers, since the air itself takes up heat and by evaporation accelerates the cooling, and, finally, because the surfaces are easily accessible and can therefore always be kept clean and active. A small amount of the heat is also lost by radiation.

As a rule, open coolers are placed inside the works, and occasionally air is blown over the surfaces in order to increase the cooling action. The surrounding air rises very slowly over the liquid, with small coolers and not very warm liquids, at a velocity of 0·2-0·3 m.; with higher apparatus and warmer liquids, at about 1 m. per second. The air is heated approximately in proportion to the temperature of the liquid to be cooled, and, in proportion to the degree of heating and its original amount of moisture, it takes up water, as will be described in treating of cooling water. The liquid loses by evaporation 1-3 per cent. of its weight, according to circumstances.

There are no reliable experimental figures as to the heating of the air and its evaporative effect in this form of cooler; it is therefore necessary to calculate the quantities of heat taken up by the air and by the cooling water *separately* in open surface-coolers. It would appear that the heat given up to the air is approximately proportional to the mean temperature difference between water and air.

The hotter is the liquid to be cooled when it reaches the cooler, the better the apparatus works, since then a tolerable quantity of heat is taken up by evaporation. It is of considerable importance to the cooling capacity that the liquid should flow down quietly over the whole surface, without splashing. It will be assumed that in this case the coefficient of transmission, $k_k = 1000$. The amount of moisture in the surrounding air also affects the cooling action.

Experiments.—1. An open surface-cooler with a cooling surface of 18·4 sq. m. cooled 1600 litres of beer per hour from 70° to 13° C. by means of cooling water at 10°, which left the apparatus at 33° C. This gives $k_k = 800$.

2. A similar apparatus with a surface of 13·5 sq. m. cooled 3500 litres of beer per hour from 70° to 18° C. by means of cooling water at 18° C., which flowed away at about 40° C. This gives $k_k = 1010$.

3. A similar apparatus with a surface of 20 sq. m. (16 tubes of 55 mm. external diameter and 4200 mm. long = 11·5 sq. m., fed by water at 8·75-25°, *plus* 12 tubes of the same size = 8·66 sq. m., fed by ice-water at 1°-7·5° C.) cooled 6000 litres of beer per hour from 43·7°-6° C. The temperature of the beer at the outlet of the ice-water was 14·1° C. This gives for the 11·5 sq. m. $k_k = 1000$, for the 8·66 sq. m. $k_k = 670$.

As a result of those and other similar experiments not given here, we assume that it is permissible, in estimating the necessary cooling-surface of open coolers, to take

$$k_k = 1000 \quad \ldots \ldots \quad (240)$$

and thence the surface required to abstract C calories is

$$H_k = \frac{C}{1000\delta_{m z_k}} \quad \ldots \ldots \quad (241)$$

This expression is applicable to copper and brass cooling tubes, cooled by water, and to thin warm liquids.

If the original temperature of the liquid is low, say under 15° C., we may only take

$$k_k = 700 \quad \ldots \ldots \quad (242)$$

If the cooling surface is of iron, then for warm liquids $k_k = 800$.

If the liquid to be cooled is somewhat thicker than water, H_k must be increased by about 20 per cent.

Table 68, which is clear without further explanation, has been compiled in this manner.

Example.—In one hour $F_k = 1000$ kilos. of an aqueous liquid at $t_{wa} = 80°$ C. are to be cooled to $t_{we} = 17°$. The cooling water is at 15°, and is to flow away at 60° C.

Now, $s_h = 1$, $C = F(t_{wa} - t_{we}) = 1000(80 - 17) = 63{,}000$ calories.

The greatest temperature difference is: $\theta_a = 80° - 60° = 20°$.

The least temperature difference is: $\theta_e = 17° - 15° = 2°$.

Since $\frac{\theta_e}{\theta_a} = \frac{2}{20} = 0{\cdot}1$, it follows, from Table 1, that

$$\theta_m = 0{\cdot}391 \times 20 = 7{\cdot}88°.$$

Thus the necessary cooling surface is

$$H_k = \frac{C}{k_k \theta_m s_h} = \frac{63{,}000}{1000 \times 7{\cdot}88 \times 1} = 8 \text{ sq. m.}$$

The requisite weight of cooling water is given by

$$C = W(t_{ke} - t_{ka}) = W(60 - 15),$$

$$\text{or } W = 1400 \text{ litres.}$$

F. Cooling by Contact with Metallic Surfaces which are Traversed by Cold Air.

This method has been sufficiently treated in Chapter XX., B. 2, page 283.

G. Cooling Water by Air.

In cooling large quantities of water, the method is generally used of exposing the water with the greatest possible surface to air at rest or in motion. The water is allowed to stand in shallow tanks with a great surface, to flow through a long shallow channel, to flow down in sheets over terraces or over vertical or inclined plane walls; it also falls in the form of jets and drops down cooling towers or is finely divided and sprayed by roses, to sink down as dust.

The cooling air either moves with its natural velocity, or is artificially driven, over the water. In these arrangements it is endeavoured to bring the greatest volume of air in direct contact with water in the finest possible state of division.

The cold air has a *twofold* cooling action on the warm water; in the first place it acts directly by abstracting heat and itself becoming hotter. If the atmospheric air, at its first contact with the water, has the temperature t_{la} and leaves it at t_{le}, then L kilos. of air take from the water in being heated:

$$C_e = L0{\cdot}2375(t_{le} - t_{la}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (243)$$

In the second place the air cools the water by causing a portion of it to evaporate. The atmospheric air, which is practically never saturated with moisture, readily takes up more, especially when it is warmed, as by the water in this case.

In regard to the quantity of water which can be taken up by air, and other questions of interest here, more detail will be found in the author's work, *Drying by Means of Steam and Air* (Scott, Greenwood & Co., London), from which the numerical values required below are taken.

If 1 kilo. of air before contact with the water contains d_a kilo. of vapour, and on leaving the water, d_e kilo., this 1 kilo. of air has taken up during the contact $(d_e - d_a)$ kilo. of water vapour. If the mean temperature of the water was t_{wm}, the number of calories withdrawn from the water for the evaporation of the water taken up by 1 kilo. of air was

$$C_v = L(d_e - d_a)(640 - t_{wm}) \quad . \quad . \quad . \quad . \quad . \quad (244)$$

Thus, in all, L kilos. of air take from the water

$$C_k = C_t + C_v = L[0{\cdot}2375(t_{le} - t_{la}) + (d_e - d_a)(640 - t_{wm})] \quad (245)$$

calories.

If W kilos. of water at the temperature t_{wa} are to be cooled to the temperature t_{we}, then there are to be withdrawn for that purpose $W(t_{wa} - t_{we})$ calories; the *principal equation* is therefore

$$C_k = C_t + C_v = W(t_{wa} - t_{we})$$

$$= L[0{\cdot}2375(t_{le} - t_{la}) + (d_e - d_a)(640 - t_{wm})] \quad . \quad (246)$$

The temperature of the external air, t_{la}, is very variable, and so also is the quantity of moisture in it; the temperature of, and moisture in, the air when it leaves are variable, and the temperature of the cooling water is different in each case. In order to obtain a view of the prevailing conditions and actions in the many different and varying cases, Table 69 has been calculated for temperatures of the outer air of $t_{la} = -20°$ to $+30°$ C. and of the emergent air of $t_{le} = 5°$ to $40°$ C.

For Table 69, the amount of heat required for the evaporation of 1 kilo. of water was taken at 600 calories, which is perhaps somewhat low. It is also assumed that the atmospheric air is completely saturated at the prevailing temperature, but that it leaves the cooler at temperatures from 5° to 40° C. *only three-fourths saturated.* The

values of d_a and d_e, which give the amount of water in 1 kilo. of air, are taken from Tables I. and III. of the above-mentioned work.

Table 69 gives, in the first lines, the number of units of heat taken up from the water by 1 kilo. of air in becoming heated $[0{\cdot}2375(t_{le} - t_{la})]$, and, in the lines 2, the number of calories abstracted by *the same* kilo. of air through partial evaporation of the water $[(d_e - d_a)(600 - t_{wm})]$. The sum of these two lines would then show how many calories are withdrawn *in all* by 1 kilo. of air.

The lines 3 give the *ratio* of the absorption of heat through *heating* to that through *evaporation*.

The fourth lines give the weight of air, L, required to abstract 1000 calories from the water.

Example.—If the air reaches the water at 0° C. and leaves it at 20° C., the ratio of the heat withdrawn by heating the air to that by evaporation is, by section 5, line 3, 0·527 : 0·473.

If a total of 1000 calories is to be abstracted, then the air must take for heating itself $C_e = 1000 \times 0{\cdot}527 = 527$ calories, and by evaporation $C_v = 1000 \times 0{\cdot}473 = 473$ calories.

Now, by equation (243),

$$C_e = L0{\cdot}2375(t_{le} - t_{la}) = L0{\cdot}2375(20 - 0) = 527 \text{ calories.}$$

and thence the necessary weight of air (Table 69, section 5, line 1) is

$$L = \frac{527}{4{\cdot}75} = 111 \text{ kilos. (approx.).}$$

[To confirm. These 111 kilos., if the air is quite saturated at 0° and only three-fourths saturated at 20° C., can in fact take up for *evaporation* $C_v = 1000 \times 0{\cdot}473 = 473$ calories, for, by Table 1 (see *Drying by Means of Steam and Air*), the amount of water which can be absorbed by 1 kilo. of air under these conditions is $d_e - d_a = 0{\cdot}01103 - 0{\cdot}00387 = 0{\cdot}00716$ kilo., therefore 111 kilos. absorb $111(d_e - d_a) = 0{\cdot}79476$ kilo. of water, for which (on our assumption) $C_v = 0{\cdot}79476 \times 600 = 476{\cdot}8$ calories are required.]

The fifth lines contain the *volume*, v_l, of the weight of air, L, at the external temperature, t_{la}. This volume of air is obtained by dividing the weight of air, L, by the weight of 1 cub. m. of dry air at the proper temperature (obtained from Table 1, column 8, of *Drying by Means of Steam and Air*).

In the above example, 111 kilos. of air at 0° C. occupy a space of $\frac{111}{1{\cdot}293} = 86$ cub. m.

The sixth lines then give the weight of vapour which is evaporated from the water by the calculated weight of air, L, which weight may thus be regarded as loss in the cooling apparatus. This is for a total

Table 69.

The heat taken up by 1 kilo. of air in becoming heated, C_e, and by evaporation, C_v. The fraction of the total absorption of heat due to heating, $\frac{C_e}{C_e + C_v}$, and to evaporation, $\frac{C_v}{C_e + C_v}$. The requisite weight of air, L, and volume, V_{la}, and also the evaporation of water for the abstraction of 1000 calories. For temperatures of the completely saturated external air of – 20° to + 30° C. and temperatures of the outlet of the three-fourths saturated air from 5° to 40° C.

Number of line.	Temp. of the atmos. air. t_{la}		Temperature of the air outlet, t_{le}. 5°	10°	15°	20°	25°	30°	35°	40°
1	– 20	$(t_{le} - t_{la})$ 0·2375 =	5·94	7·12	8·30	9·50	10·68	11·78	12·9	14·22
2	For 1 kilo. of air:	$(d_e - d_a)$ $(640 - t_w)$ =	2·04	3·006	4·38	6·16	8·4	11·86	15·78	20·68
3		By heating - -	0·744	0·704	0·659	0·607	0·5[illegible]	[illegible]·490	0·449	0·407
		By evaporation - -	0·256	0·296	0·346	0·393	[illegible]	[illegible]10	0·551	0·593
4	For 1000 cals.	Weight of air, L =	125	100	80	64	[illegible]	[illegible]	35	29
5		Volume of air, V_{la} =	90	70	57·6	46	42	[illegible]	25·2	21
6		Water evap't'd, kilos.	0·422	0·501	0·584	0·656	[illegible]747	0·828	0·953	0·995
1	– 15	$(t_{le} - t_{la})$ 0·2375 =	4·75	5·94	7·125	8·30	9·50	10·68	11·78	12·9
2		$(d_e - d_a)$ $640 - t_w)$ =	1·80	2·772	4·08	5·93	8·16	11·62	15·48	20·34
3		By heating - -	0·725	0·682	0·635	0·583	0·539	0·479	0·432	0·389
		By evaporation -	0·275	0·318	0·365	0·417	0·461	0·521	0·568	0·611
4		Weight of air, L =	153	115	90	70	57	45	37	30
5		Volume of air, V_{la} =	112	84	65·7	51·2	41·7	33	27	22
6		Water evap't'd, kilos.	0·457	0·521	0·622	0·692	0·780	0·[illegible]70	0·966	1·019
1	– 10	$(t_{le} - t_{la})$ 0·2375 =	3·57	4·75	5·94	7·125	8·30	9·54	10·68	11·78
2		$(d_e - d_a)$ $(640 - t_w)$ =	1·44	2·43	3·80	4·98	7·84	11·27	15·18	19·98
3		By heating - -	0·700	0·661	0·610	0·572	0·514	0·458	0·413	0·370
		By evaporation -	0·300	0·339	0·390	0·428	0·486	0·542	0·587	0·630
4		Weight of air, L =	200	139	103	80	62	48	39	31
5		Volume of air, V_{la} =	149·5	104	76·9	59·8	46·3	35·9	29·1	23·1
6		Water evap't'd, kilos.	0·484	0·562	0·653	0·745	0·780	0·903	0·985	1·034
1	– 5	$(t_{le} - t_{la})$ 0·2375 =	2·375	3·57	4·75	5·94	7·125	8·30	9·50	10·68
2		$(d_e - d_a)$ $(640 - t_w)$ =	0·96	1·95	3·21	4·51	7·85	10·78	14·65	19·33
3		By heating - -	0·713	0·647	0·590	0·568	[illegible]·492	0·435	0·385	0·356
		By evaporation - -	0·187	0·353	0·410	0·432	[illegible]·508	0·565	0·615	0·644
4		Weight of air, L =	300	180	124	96	70	53	40	34
5		Volume of air, V_{la} =	228	136	94·3	73	53	40·3	30·4	25·8
6		Water evap't'd, kilos.	0·480	0·581	0·671	0·815	0·813	0·951	0·978	1·106

TABLE 69—(*continued*).

Number of line.	Temp. of the atmos. air. t_{la}		Temperature of the air outlet, t_{le}.							
			5°	10°	15°	20°	25°	30°	35°	40°
1	0	$(t_{le} - t_{la})$ 0·2375 =	1·187	2·37	3·57	4·75	5·94	7·18	8·30	9·50
2		$(d_e - d_a)(640 - t_w)$ =	0·162	1·14	2·52	4·26	6·55	9·96	18·87	18·79
3		By heating	0·880	0·675	0·586	0·527	0·475	0·418	0·374	0·336
		By evaporation	0·120	0·325	0·414	0·473	0·525	0·582	0·626	·0664
4		Weight of air, L =	746	284	165	111	81	60	45	85·5
5		Volume of air, V_{la} =	581	221	128·5	86·5	73	46·7	85	27·6
6		Water evap't'd, kilos.	0·202	0 540	0·680	0·794	0·786	0·998	1·040	1·108
1	5	$(t_{le} - t_{la})$ 0·2375 =	—	1·187	2·37	3·57	4·75	5·94	7·125	8·30
2		$(d_e - d_a)(640 - t_w)$ =	—	0·160	1·53	3·30	5·58	8·94	12·90	17·70
3		By heating	—	0·885	0·608	0·518	0·458	0·400	0·356	0·319
		By evaporation	—	0·115	0·392	0·482	0·541	0·600	[illegible]	0·681
4		Weight of air, L =	—	750	252	145	99	67	50	38
5		Volume of air, V_{la} =	—	600	201	116	80	54	40	80·5
6		Water evap'd, kilos.	—	0·180	0·637	0·797	0·745	0·998	1·073	1·123
1	10	$(t_{le} - t_{la})$ 0·2375 =	—	—	1·187	2·37	3·57	4·75	5·94	7·13
2		$(d_e - d_a)(640 - t_w)$ =	—	—	0·21	1·97	4·25	7·68	11·52	16·44
3		By heating	—	—	0·854	0·546	0·457	0·3[illegible]2	0·840	0·325
		By evaporation	—	—	0·146	0·454	0·543	0·618	0·660	0·675
4		Weight of air, L =	—	—	720	23[illegible]	129	80	57	44·4
5		Volume of air, V_{la} =	—	—	583	186·5	104·5	65	46·2	36
6		Water evap't'd, kilos.	—	—	0·259	0·759	0·916	1·024	1·100	1·216
1	15	$(t_{le} - t_{la})$ 0·2375 =	—	—	—	1·18	2·37	3·57	4·75	5·94
2		$(d_e - d_a)(640 - t_w)$ =	—	—	—	0·12	2·4	6·72	9·72	14·58
3		By heating	—	—	—	0·902	0·495	0·347	0·328	0·290
		By evaporation	—	—	—	0·098	0·505	0·653	0·672	0·710
4		Weight of air, L =	—	—	—	765	208	97	69	49
5		Volume of air, V_{la} =	—	—	—	635	172·6	80·5	57·3	40·6
6		Water evap't'd, kilos.	—	—	—	0·153	[illegible]·832	0·990	1·118	1·191
1	20	$(t_{le} - t_{la})$ 0·2375 =	—	—	—	—	1·187	2·37	3·57	4·75
2		$(d_e - d_a)(640 - t_w)$ =	—	—	—	—	—	3·42	7·32	12·18
3		By heating	—	—	—	—	—	0·409	0·327	0·281
		By evaporation	—	—	—	—	—	0·591	0·673	0·719
4		Weight of air, L =	—	—	—	—	—	172	90	59
5		Volume of air, V_{la} =	—	—	—	—	—	146	76·5	50
6		Water evap't'd. kilos.	—	—	—	—	—	0·980	1·098	1·192
1	25	$(t_{le} - t_{la})$ 0·2375 =	—	—	—	—	—	1·18	2·375	3·57
2		$(d_e - d_a)(640 - t_w)$ =	—	—	—	—	—	0·18	4·08	8·98
3		By heating	—	—	—	—	—	0·869	0·369	0·284
		By evaporation	—	—	—	—	—	0·131	0·631	0·716
4		Weight of air, L =	—	—	—	—	—	730	156	80
5		Volume of air, V_{la} =	—	—	—	—	—	631	135	69·2
6		Water evap't'd, kilos.	—	—	—	—	—	0·219	1·061	[illegible]·192

TABLE 69—(*continued*).

Number of line.	Temp. of the atmos. air. t_{la}		Temperature of the air outlet, t_{le}.							
			5°	10°	15°	20°	25°	30°	35°	40°
1	30	$(t_{le} - t_{la})\ 0{\cdot}2375 =$	—	—	—	—	—	—	1·181	2·37
2		$(d - d_a)\ (640 - t_w) =$	—	—	—	—	—	—	—	4·56
3		By heating . . .	—	—	—	—	—	—	—	0·342
		By evaporation .	—	—	—	—	—	—	—	0·658
4		Weight of air, $L' =$	—	—	—	—	—	—	—	145
5		Volume of air, $V_{la} =$	—	—	—	—	—	—	—	130
6		Water evap't'd, kilos.	—	—	—	—	—	—	—	1·098

abstraction of heat of 1000 calories and on the assumption that the external air is completely, and the emergent air three-fourths, saturated with water vapour.

It often happens that the external air is not completely and the emergent air is more than three-fourths saturated. In that case 1 kilo. of water absorbs more moisture than is assumed in the table. Consequently less air is used for cooling the water and, on the other hand, more water is evaporated. In many cases $\frac{1}{40}$ to $\frac{1}{30}$ of the water to be cooled is removed by the air.

In using Table 69, it is first necessary to calculate how many calories must be withdrawn in one hour from the water to be cooled; the table then gives the weight and volume of the air and the evaporation of water per 1000 calories.

The surface of the water, which must be in contact with the air in order to produce the desired cooling, is still to be calculated.

If C_e be the heat to be taken from the water to warm the air, not by evaporation, O the surface of the water in sq. m., z_h the time of cooling in hours, θ_m the mean difference in temperature between water and air, k_t the coefficient of transmission, v_t the velocity in m. per sec. with which the air passes over the water, then, by the usual principles,

$$C_e = z_h O k_t \theta_m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (247)$$

and the surface requisite for the cooling by means of air is

$$O = \frac{C_e}{z_h k_t \theta_m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (248)$$

The transmission coefficient for towers, in which drops are abundantly formed, is

$$k_t = 2 + 18 \sqrt{v_t},$$

for plane surfaces over which the water flows,

$$k_t = 2 + 12 \sqrt{v_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (249)$$

for water quite at rest a smaller coefficient must be taken,

$$k_t = 2 + 10 \sqrt{v_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (250)$$

The velocity of the air, v_t, in the atmosphere is very variable; it may be as high as 40 m., but even when there is no wind it is generally about 1·5-2 m., which figures must be employed in calculation. In cooling apparatus made after the fashion of a chimney, in which the air rises in consequence of being heated, it moves with a velocity of about 3 m. When the air is blown by fans through the chimney, the velocity may be arbitrarily fixed at 6-12 m. The large volumes of air required are rarely moved by artificial means on account of the cost.

The fresh air from fans is naturally made to enter *below* in order to obtain counter-currents of air and water.

The *mean difference in temperature*, θ_m, is to be determined by means of Chapter I., Table 1.

It may be seen from the third lines of Table 69 that the heat to be abstracted by warming the air, in proportion to the whole amount to be given up, is least when the air is heated by the water to about 15° C., on the hypothesis that the atmospheric air enters the apparatus completely saturated and leaves it three-fourths saturated.

If the external air is cold, the emergent air will also be cool, and the temperature difference between air and water will then be large. On the other hand, if the external air is warm, it leaves still warmer, and the mean temperature difference is then much less. As Table 69 shows, in the former case the air takes up more heat by being warmed, in the latter case more by the formation of vapour.

The consumption of air is the least when it enters very cold and leaves very warm. The necessary water-surface is the least when unlimited quantities of air flow over it. If, in a definite case, the air is always to receive the same increase in temperature, then, whilst the temperatures of the water remain the same, a lower temperature of the air necessitates more air and a smaller surface for the water.

Air which is originally cold naturally is warmed through a greater range of temperature than air originally warm; thus the consumption of air is approximately constant, but the former takes up more heat from the same surface. *Ceteris paribus*, cold air cools better than warm air.

Example.—In $z_h = 1$ hour, 10,000 kilos. of water are to be cooled from 40° to 22° C., for which $C_k = 10{,}000(40 - 22) = 180{,}000$ calories are to be abstracted. The air moves with a velocity of 2 m.—(1) it is originally at 0°, and is warmed to 25° C.; (2) it is at 20°, and is warmed to 35° C. The temperature-differences between air and water are:—

1. Air warmed from 0° to 25°—

   at the top, $\theta_a = 40° - 25° = 15°$; at the bottom $\theta_e = 22° - 0° = 22°$.

The mean difference is, by Table 1 (since $\frac{15}{22} = 0{\cdot}682$),

$$\theta_m = 0{\cdot}44 \times 22 = 9{\cdot}68°.$$

2. Air warmed from 20° to 35°—

   at the top, $\theta_a = 40° - 35° = 5°$, at the bottom, $\theta_e = 22° - 20° = 2°$.

The mean difference, by Table 1 (since $\frac{2}{5} = 0{\cdot}4$) is

$$\theta_m = 0{\cdot}658 \times 5 = 3{\cdot}39°.$$

In the first case, from Table 69, 0·475 of the total amount of heat is to be withdrawn by heating the air, $C_\epsilon = 180{,}000 \times 0{\cdot}475 = 85{,}500$ calories. In the second case, $C_\epsilon = 180{,}000 \times 0{\cdot}327 = 58{,}860$ calories.

Thus, when *cold* air enters, the water-surface necessary in a cooling tower is

$$O = \frac{85{,}000}{(2 + 18\sqrt{2})9{\cdot}68} = 300 \text{ sq. m. (approx.)},$$

and when *warm* air enters

$$O = \frac{58{,}860}{(2 + 18\sqrt{2})3{\cdot}39} = 730 \text{ sq. m. (approx.)}.$$

The requisite weight of air is in the first case

$$L = \frac{85{,}500}{0{\cdot}2375(25 - 0)} = 14{,}400 \text{ kilos. } (= 11{,}250 \text{ cub. m.}),$$

in the second case

$$L = \frac{58{,}860}{0{\cdot}2375(35 - 20)} = 16{,}900 \text{ kilos. } (= 14{,}360 \text{ cub. m.}).$$

The *surface* which the *water* presents to the air must change as frequently and rapidly as possible. For heat penetrates slowly into a mass of water at rest (Chapter XX., 8, Table 46), rapidly warming the external layers to a slight depth, but then entering the interior very slowly, and the laws which govern this action also apply, if the expression be permitted, to the penetration of cold into the mass of water. The figures given in Table 50 hold good also for the *decrease* in temperature of jets of water which fall from step to step in a current of cold air.

The best cooling apparatus will thus always be in the form of a staging with the greatest possible number of low steps, over which the

air passes rapidly, either sideways or drawn upwards by a chimney. Mechanical acceleration of the motion of the air will be advantageous in but a few rare cases.

1000 litres of water, which fall through 5 m. in the finest state of division, form a surface of about 4-6 sq. m., which is however insufficient to cool the water. The remaining surface required must be provided in another way, as by surfaces over which the water flows, which must be of ample dimensions since they are generally not wetted throughout.

We now give a few examples, collected in Table 70, of open stagings (cooling towers) through which air circulates freely. In quite open stagings without a chimney the temperature difference is greater, which is an advantage, but then the motion of the air is somewhat slower than with a chimney.

Observed Examples.—By means of a cooling tower, with many steps and a natural access of air, 3 × 12 = 36 sq. m. in ground area, 4800 mm. high, and with 322·5 sq. m. of wooden surface over which the water flowed, 22,800 litres of water were cooled in one hour from 50° to 20° C., when the air entered at 2·5° C. and left at the different stages at 8·5°, 14·5°, 20·5° C. From the water were to be abstracted

$$Ct = 22{,}800(50 - 20) = 684{,}000 \text{ calories.}$$

1 kilo. of saturated air at	2·5°	contains	0·0046	kilo. of water.
1 " " "	8·5°	"	0·0069	" "
1 " " "	14·5°	"	0·0107	" "
1 " " "	20·5°	"	0·0153	" "

The mean of the last three numbers is 0·01096 kilo.

If the air which leaves the staging is only saturated to the extent of 80 per cent., then 1 kilo. contains 0·01096 × 0·8 = 0·008768 kilo. of water.

1 kilo. of air thus taken up by evaporation 0·008768 − 0·0046 = 0·00416 kilo. of vapour, which corresponds to 2·496 calories.

The air is heated on the average from 2·5° to 12·5°, *i.e.*, through 10° C., consequently 1 kilo. taken up *by being heated* 10 × 0·2375 = 2·375 calories.

Thus 1 kilo. of air takes up a total of 2·496 + 2·375 = 4·871 calories.

Of the total quantity of heat to be abstracted from the water, the air takes

$$\text{by evaporation, } \frac{2{\cdot}496 \times 684{,}000}{4{\cdot}871} = 380{,}438 \text{ calories;}$$

$$\text{by heating, } \frac{2{\cdot}375 \times 684{,}000}{4{\cdot}871} = 293{,}562 \text{ calories.}$$

The surface of the apparatus over which water flowed was	322·5 sq. m.
The wetted surface underneath was estimated at - -	60·0 "
The surface of the falling drops was about 6 sq. m. per 1000 litres, *i.e.*, = 6 × 22·8 = - - - - - -	136·0 "
Total - O =	518·5 "

TABLE 70.

Examples of the direct cooling by air

1000 kilos. of water per hour are to be cooled — from t_{wa}	40	40	40	40	40
to t_{we}	20	20	15	10	10
The air enters the cooler at - - - t_{la}	25	10	10	10	-10
And leaves it at - - - - - t_{le}	35	25	30	20	5
The temp. difference is at the top θ_e °C.	5	15	10	20	35
The temp. diff. is at the bottom - θ_a °C.	5	10	5	10	20
The ratio of the temperature differences $\frac{\theta_e}{\theta_a}$	$\frac{5}{5}$	$\frac{10}{15}$	$\frac{5}{10}$	$\frac{10}{20}$	$\frac{30}{35}$
Hence the mean temp. diff. by Table 1 θ_m	5	12·3	7·24	14·48	19·9
Total calories to be withdrawn from the water - - - C_k	20000	20000	25000	30000	30000
Of above to warm the air - - - C_e	7380	9140	9550	15810	21000
Of above to evaporate the water - - C_v	12620	10860	15450	14190	9000
The water loses by evaporation - kilos.	21·1	18·1	25·75	24	15
Necessary surface of the water, in sq. m. O	50	26	45	37·5	36
Necessary weight of air at entry, in kilos. L	3108	2570	2000	3330	5900
Necessary volume of air at entry, in cub. m. V_l	2716	2085	1625	2440	4400

TABLE 70.

of water in a fine state of division.

50	50	50	50	50	50	50	50	60	60	60
30	25	20	15	20	30	35	25	25	40	30
25	10	0	- 10	5	10	20	10	10	10	15
35	25	20	15	20	25	35	20	30	25	25
15	25	30	35	30	25	15	30	30	35	15
5	15	20	25	15	20	15	15	15	30	35
$\frac{5}{15}$	$\frac{15}{25}$	$\frac{20}{30}$	$\frac{25}{35}$	$\frac{15}{30}$	$\frac{20}{25}$	$\frac{15}{15}$	$\frac{15}{30}$	$\frac{15}{30}$	$\frac{30}{35}$	$\frac{15}{35}$
9	19·65	24·6	29·75	21·7	21·8	15	21·7	21·7	32·2	24·1
20000	25000	30000	35000	30000	29000	15000	25000	35000	20000	30000
7380	11425	15810	21350	15540	13253	4905	12950	13370	9140	12750
12620	13575	14190	13650	14480	15747	10095	12050	21620	10860	17250
21	22·6	22	22·8	24·1	26·2	16·8	20·1	36	18·1	28·7
24	19	21	23	23	19·5	11	19·5	20	11	17
3108	3208	3330	3600	4370	4300	1380	5450	2810	2600	5350
2716	2620	2440	2700	3470	3500	1190	1420	2280	2100	4460

The mean temperature-difference was 27°, hence the coefficient of transmission

$$k_l = \frac{C}{O\theta_m} = \frac{293,562}{518 \cdot 5 \times 27} = 21 \cdot 1.$$

The weight of air required for cooling is

$$L = \frac{293,562}{2 \cdot 375} = 123,600 \text{ kilos.}$$

The volume $V_l = \dfrac{123,600}{1 \cdot 27} = 100,000$ cub. m. (approximately), *i.e.*, 28 cub. m. per sec. If the air meets the apparatus obliquely, the velocity would be about 1·2 m., and the *calculated* coefficient would be

$$k = 2 + 18\sqrt{1 \cdot 2} = 22.$$

2. A chimney cooler with 18 plates, 1500 by 4800 mm., having a total wetted surface of 259 sq. m., cooled 18,500 litres of water per hour from 39° to 22° C. by means of 44,000 cub. m. of air, blown in by a fan (1100 mm. diameter, 300 revolutions) at 12·5° C. and leaving at 18·8° C. at the top. The air was saturated originally to the extent of 67 per cent.

From the water are to be taken

$$C^k = 18,500(39 - 22) = 314,500 \text{ calories.}$$

1 kilo.	of air at	12·5°	contains	0·00926	kilo. of water when	completely saturated.
1	,,	,,	12·5°	,,	0·0062042	,,	,,	67·5 per cent. ,,
1	,,	,,	18·8°	,,	0·0140	,,	,,	completely ,,

Thus, 1 kilo. of air takes up by *evaporation*,

0·014 − 0·0062042 = 0·0078 kilo. of water, which requires	4·68 calories.
1 kilo. of air absorbs in being heated from 12·5° to 18·8°, 6·3 × 0·2375 =	1·496 ,,
Total	6·176 ,,

Accordingly the air takes up

$$\text{by evaporation,} \frac{4 \cdot 68 \times 314,500}{6 \cdot 176} = 238,307 \text{ calories;}$$

$$\text{by heating,} \quad \frac{1 \cdot 496 \times 314,500}{6 \cdot 176} = 76 \cdot 193 \text{ calories.}$$

The velocity of the air was 3·8 m. per sec., the temperature-difference 14° C., consequently the *observed* coefficient of transmission

$$k_l = \frac{C}{H\theta_m} = \frac{76,193}{259 \times 14} = 23 \cdot 8.$$

The *calculated* coefficient of transmission is

$$k = 2 + 12\sqrt{3 \cdot 6} = 24.$$

H. Cooling Air by Water.

Atmospheric air always contains more or less moisture in the form of vapour. The maximum amount of vapour in 1 cub. m. of air is equal to the weight of 1 cub. m. of saturated vapour at the temperature of the air. If air which contains much moisture is considerably cooled, it generally reaches a condition in which it can contain only a smaller weight of vapour, and consequently the excess of vapour must separate, *i.e.*, be condensed.[1]

Thus, if a certain volume of air is to be artificially cooled in a certain time, it is necessary to take from it as much heat as is required.

1. To cool the dry air itself.
2. To condense the vapour which must be separated.

Let L = weight of air to be cooled,

σ_l = its specific heat = 0·2375,

t_{la} = its temperature before cooling (at the beginning),

t_{le} = " after " (at the end),

d_a = the weight of vapour in 1 kilo. of air before cooling,

d_e = " " " " after "

c = the total heat of 1 kilo. of vapour.

Then in order to cool the air from t_{la} to t_{le} it is necessary to abstract the following amount of heat:—

$$C = L\sigma_l(t_{la} - t_{le}) + L(d_a - d_e)(c - t_{le}).$$

In atmospheric air there is rarely more than 95 per cent. of the maximum quantity of vapour possible, generally there is considerably less. Even when moist air is strongly cooled, so that it deposits water, it does not remain saturated with vapour.

If we assume that the atmospheric air is saturated to the extent of 80 per cent., and also that its degree of saturation is 80 per cent. after cooling through a certain range of temperature, then the above equation gives, for cooling 100 cub. m. of air, the quantities of heat which are arranged in the table on the next page.

[1] See *Drying by Means of Steam and Air* for amount of vapour in air at different temperatures.

Temperature to which the air is to be cooled, t_{le}.	Weight of vapour in 1 cub. m. of the cooled air d_c.		Original temperature of the air, t_{la}.				
			30°	25°	20°	15°	10°
			Weight of 1 cub. m. of this air, in kilos., when saturated with moisture to the extent of 80 per cent.				
			1·1412	1·1630	1·1881	1·2154	1·2408
			Weight of the moisture, d_a, in kilos. in cub. m. of this air.				
			0·0244	0·01849	0·0141	0·01041	0·0076
			Number of calories necessary to cool 100 cub. m. of this air.				
°C.	kilo.						
25°	0·01849	Cals. for cooling the air	133	—	—	—	—
		" " condensing vapour	373	—	—	—	—
		Total	506	—	—	—	—
20°	0·0141	Cals. for cooling the air	265	136	—	—	—
		" " condensing vapour	644	275	—	—	—
		Total	909	411	—	—	—
15°	0·01041	Cals. for cooling the air	398	272	145	—	—
		" " condensing vapour	875	505	221	—	—
		Total	1273	777	366	—	—
10°	0·0076	Cals. for cooling the air	530	407	279	143	—
		" " condensing vapour	1060	686	385	177	—
		Total	1590	1093	667	320	—
5°	0·0056	Cals. for cooling the air	663	544	418	286	146
		" " condensing vapour	1198	821	507	308	130
		Total	1861	1365	925	594	276

The necessary quantity of *cooling water* depends on its initial and final temperatures, t_a and t_e, it is

$$W = \frac{C}{t_e - t_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (251)$$

The *cooling surface*, for the cooling of definite quantities of air, is obtained from the ordinary equation :

$$H_k = \frac{C_k}{k_l \theta_m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (252)$$

TABLE 71.

The temperature difference, θ_m, consumption of cooling water, W, and the necessary surface, H_k, of water in rapid motion, in order to cool hourly 100 cub. m. of air, which flows with the velocity, $v_l = 1$ m, from 30°-10° C. down to 20°-5° C.

Temp. of the cooled air. t_{le}	Initial temp. of the cooling water. t_a	Mean temp. diff. - θ_m; Consumption of cooling water - - - W; Cooling surface - - H_k; For $v_l = 1$ and metal walls.	Initial temp. of the air, t_{la}: 30°	30°	25°	20°	20°	15°	15°	10°
		Final temp. of the cooling water, t_e:	20°	15°	15°	15°	12°	12°	10°	5°
20°	15°	θ_m	7·24	—	—	—	—	—	—	—
		W	185	—	—	—	—	—	—	—
		H_k	6·35	—	—	—	—	—	—	—
	10°	θ_m	10	12·8	10	—	—	—	—	—
		W	92·5	185	82·2	—	—	—	—	—
		H_k	4·61	3·74	2·06	—	—	—	—	—
15°	10°	θ_m	7·24	8·4	7·24	5	6·4	—	—	—
		W	127	255	156	73·2	189	—	—	—
		H_k	8·80	7·6	5·40	3·60	2·74	—	—	—
10°	5°	θ_m	7·24	8·4	7·24	5	6·4	3·9	5	—
		W	107	150	109	66·7	95	45	32	—
		H_k	11·0	9·5	7·60	6·69	5·18	4·10	3·20	—
	2°	θ_m	8·97	11·3	8·97	6·4	8	5·2	6·4	—
		W	89	123	91	51·1	66·7	32	40	—
		H_k	8·90	7·1	6·10	5·18	7·15	3·07	2·50	—
5°	2°	θ_m	5·83	7·5	6·1	3·9	3·8	3	3·9	3·9
		W	104	143	105	71·2	92·5	60	75	92
		H_k	16·0	12·6	11·2	11·9	17·0	10·0	8·00	3·20

If the velocity of the air is greater than 1 m. per sec., *viz.*,

1 | 2 | 3 | 4 | 5 | 6 m.

the surfaces of direct contact with the rapidly moving cooling water, H_k, required to cool 100 cub. m. per hour, are obtained by multiplying the figures in the above Table by 1 | 0·73 | 0·60 | 0·53 | 0·48 | 0·44

If the air flows past a cooled metallic surface, its necessary superficies is obtained by multiplying the above surfaces, H_k, by

1·66 | 1·06 | 1·04 | 0·90 | 0·82 | 0·75

The *coefficient of transmission of heat*, k_t, in this equation may be assumed to be:

1. When the cooling surfaces are metallic walls,

$$k_t = 2 + 10\sqrt{v_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (253)$$

2. When the cooling surface consists of moving and rapidly changing surfaces of water, jets or drops,

$$k_t = 2 + 18\sqrt{v_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (254)$$

The *mean temperature difference* is obtained from the initial and final differences in temperature between air and cooling water, and must be calculated in the usual manner for each case by means of Chapter I., Table 1.

CHAPTER XXIII.

THE VOLUMES TO BE EXHAUSTED FROM CONDENSERS BY THE AIR-PUMPS.

A. General.

In this chapter we proceed to determine the volume of gas and vapour which the air-pump must exhaust from any condenser, whence the dimensions of the pump are obtained.

The air and incondensible gases which obtain admittance to the condenser are derived from:

1. The liqnid to be evaporated.
2. The injected cooling water.
3. Leaks in the apparatus and pipes, which are rarely entirely absent.

The volume of air, introduced into the condenser by each of these sources *separately*, is seldom to be ascertained in any particular case. It is therefore necessary to be content with an approximate estimate of the total quantity of air introduced in all three ways and afterwards to be removed. It is usual to express this total quantity of air as a fraction of the injected water. Although there are certain connections between the quantity of the cooling water and that of the air to be exhausted. yet the latter is certainly not directly proportional to the quantity of cooling water. If we however assume snch a proportionality, as is the custom, it is done because only in this manner is a basis for our considerations to be found. It will of course be permissible to modify or specialise for particular conditions the assumptions here made.

In view of the large volumes of gas which cold water can contain (97 volumes per cent. of carbonic acid at 17° C., 15,200 per cent. of

sulphurous acid at 14° C., 326 per cent. of sulphuretted hydrogen at 14·6°, 73,700 per cent. of ammonia at 14·14) it is necessary to assume that the injected water used for condensation may frequently contain considerable quantities of gases.

On the other hand, it is usual to assume (after Bunsen, *Gasometrische Methoden*, 1857) that rain-water and most spring waters contain about 2·5 volumes per cent. of *atmospheric air*. Springs are known the water of which contains 12 volumes of gas per cent.

The liquids to be evaporated also contain very variable, and often considerable, quantities of gases, especially ammonia. In this case also 2·5 per cent. may be taken as the average.

Finally, the leakages in the apparatus and pipes are to be considered. We assume that the quantity of air entering through faulty joints, cracked glasses and defective metallic connections, is equal to 10 volumes per cent. of the cooling water employed.

Thus the air introduced into the condenser is 2·5 + 2·5 + 10 = 15 volumes per cent. of the cooling water. For safety, and in order to allow for the possible presence of other gases than air in the cooling water, this number will be still further increased. We shall assume that incondensible gases to the extent of about 20 volumes per cent. of the cooling water are carried into the condenser, *i.e.*, that for every 1000 litres of cooling water 200 litres of air (and other gases) enter the condenser.

Now 1 cub. m. of air under atmospheric pressure at 0° C. weighs 1·294 kilo. and at 15° C. 1·2266 kilo., thus 200 litres of air weigh about 0·25 kilo.; therefore we shall take as the basis of the following calculation the assumption that, for every 1000 litres of cooling water, 0·25 kilo. of air is introduced into the condenser and must be pumped out.

From equation (176), $W = \dfrac{D(c - t_e)}{t_e - t_a}$, and Table 41, we know the quantity of cooling water required in each case; therefore we can at once find, on the basis of the above somewhat arbitrary but sufficient assumption, the weight of air to be exhausted from the condenser.

The so-called wet and dry air-pumps must now be considered separately.

B. The Volume of Air to be exhausted from Wet Jet-Condensers.

By a "wet" air-pump is understood a pump which, together with the air, takes in the whole of the water from the condenser and forces it away.

The air to be removed from the condenser is invariably mixed with vapour at the same temperatnre as the air. The common temperature of the air and vapour depends on that of the water with which they were last in contact. In wet condensers the mixture of air and vapour remains together with the quite warm water to be drawn off (formed from the injected water and the condensed steam), and goes with it into the pump. It has therefore almost the same temperature as the water. In counter-current condensers the air is last in contact with cold injected water, which has just entered, and thus is cold when it reaches the air-pump.

A wet condenser can be so arranged that the air-pump exhausts the warm water from the bottom, and the air, which is then cold, because it was last in contact with the injected water, at the top. The cold air, however, then enters the pump along with the warm water, and is rapidly heated by it and the vapours rising from it, since its weight is small in proportion to that of the water. The final condition between air and vapour is thus also in this case quite similar to the ordinary condition in which air and water are taken off together, although not quite the same. The vapour, which is mixed with the air, has always the temperature of the waste water in wet condensers, consequently the pressure it exerts is the greater the warmer the water which flows away. The pressure of the air (and thus its weight per cub. m.), which, together with the pressure of the vapour, gives the total pressure, is the greater the colder the water exhausted by the pump.

The volume of the air depends on its pressure (which is only a portion of the total pressure in the condenser) and its temperature; it may be calculated as was done in Chapter XX., 9, and in Table 47.

Let W = the weight of injected water.

L = the weight of air in the water. On our assumption

$$L = W\frac{0{\cdot}25}{1000} \text{ kilos.} \qquad (255)$$

V_m = the volume of air in cub. m., which is to be exhausted from the wet condenser, V_u from the dry condenser, and V_w from the surface condenser.

a_t = the volume of 1 kilo. of air in cub. m.

γ_t = the weight of 1 cub. m. of air in kilos.

p = the pressure of the atmosphere in kilos. per sq. m. = 10,336 kilos.

t_e = the temperature of the waste water.

a = the coefficient of expansion of air = 0·003665.

b = the pressure of the air in the condenser in mm. of mercury.

T = the absolute temperature, $T = \frac{1}{a} + t_a = 273 + t_a$.

By the laws of Mariotte and Gay Lussac $\frac{\gamma \cdot p_t}{T} = R$, a constant, which for air is 29·27.

Thus 1 kilo. of air has the volume

$$a = \frac{273 + t_e}{p} 29{\cdot}27 \quad \ldots \ldots \quad (256)$$

and L kilos. of air have the volume

$$V_m = \frac{L(273 + t_e)}{p} 29{\cdot}27 \quad \ldots \ldots \quad (257)$$

For a pressure, which is $\frac{b}{760}$ of the atmospheric when measured in mm. of mercury, the volume of the L kilos. of air is

$$V_m = \frac{L(273 + t_e)}{p} 29{\cdot}27 \frac{760}{b} \quad \ldots \quad (258)$$

or, inserting the numerical values,

$$V_m = \frac{W0{\cdot}25(273 + t_e)29{\cdot}27 \times 760}{1000pb} = 0{\cdot}5385 \frac{W(273 + t_e)}{b} \quad (259)$$

In the case of every evaporator the weight of steam passed into the condenser, which is equal to the weight of water to be evaporated, is given. The weight of the injected water, W, then follows by means of equation (176) and Table 41, if its initial and final temperatures are known. Both these temperatures may be given under certain circumstances, but under others they must be assumed after examining the case. From the weight of the injected water there follows, on our hypothesis, the weight of the air introduced into the condenser.

The vacuum, or, what is the same thing, the absolute pressure in the condenser, can generally be fixed as desired. It will naturally be endeavoured to reach the highest possible vacuum, *i.e.*, the lowest possible pressure.

The volume of air to be exhausted is obtained at once, from its known weight and the vacuum decided upon, by equation (209) and Table 47.

Example.—Water at $t_a = 10°$ C. is at disposal to condense 100 kilos. of steam; it is to flow away at $t_e = 40°$ C. The vacuum is to be 680 mm., *i.e.*, the absolute pressure is to be 760 – 680 = 80 mm. By Chapter XX., Table 41, the injected water is then $W = 1960$ kilos.; the tension of the vapour is 54·9 mm. at 40° C., and since the total pressure is 80 mm., the pressure of the air, $b = 80 - 54{\cdot}9 = 25{\cdot}1$ mm. All the necessary figures for calculating out the equations are now given.

The weight of the air $L = \frac{1960 \times 0{\cdot}25}{1000} = 0{\cdot}484$ kilo.

The volume of 1 kilo. of air at 40° C. and 25·1 mm. pressure is, by Table 47, $a_t = 27{,}020$ litres. Consequently the volume of 0·484 kilo. of air is (for 100 kilos. of steam)

$$V_m = La_t = 0{\cdot}484 \times 27{,}020 = 13{,}070 \text{ litres.}$$

The wet air-pump has therefore to remove, in the condensation of 100 kilos. of steam, 1960 kilos. of water + 100 kilos. from steam and 13,070 litres of air, in all 15,130 litres.

In Table 72 are given the quantities of injected water and the volumes of air, which must be exhausted by wet air-pumps, for vacua of 600-740 mm., for initial temperatures of the cooling water of $t_a = 5°$-35° C., and final temperatures of $t_e = 10°$-50° C.

If the injected water and the liquid to be evaporated contain more or less air and gases, and the apparatus is more or less air-tight than we have assumed, the volume of air given in Table 72 must be increased or diminished in proportion to the altered circumstances. The figures in the table are determined for actual use, and for most cases are to be regarded as abundant. But if the water employed contains, *e.g.*, not 20 per cent. (by volume), but 15 per cent. of gases, the volume of air to be exhausted is $\frac{15}{20}$ of that given in Table 72.

Table 72 not only gives the actual quantities of water and air to be exhausted, it also shows that for any determined vacuum and any temperature of the injected water there is a definite most favourable temperature for the waste water, at which the volume of air to be exhausted is least. The reason for this is, that the higher the temperature of the waste water the less water is required, and consequently the less air is introduced into the condenser; but the warmer the waste

TABLE 72.

The cooling water required, and the volume of air to be exhausted, in litres, for the evaporation of 100 kilos. of water at vacua of 600-740 mm., with the cooling water at initial temperatures of t_a = 5°-30° C., and at final temperatures of t_e = 10°-50° C., for wet *jet-condensers.*

Vacuum.	Absolute pressure.	Steam. Temperature.	Steam. Total heat.	Cooling water. Initial temperature.	Cooling water. Final temperature.	Cooling water. Weight, W.	Air. Pressure.	Air. Weight.	Air. Volume.
mm.	mm.	° C.	c.	a.	t_e.	kilos.	mm.	kilos.	Litres.
600	160	61·5	625	5	10	12300	150·8	3·075	12484
"	"	"	"	"	15	6100	147·3	1·525	6451
"	"	"	"	"	20	4033	142·61	1·008	4496
"	"	"	"	"	25	3000	136·45	0·750	3541
"	"	"	"	"	30	2380	128·45	0·595	3032
"	"	"	"	"	35	1967	118·17	0·492	2775
"	"	"	"	"	40	1671	105·1	0·418	2690*
"	"	"	"	"	45	1450	88·61	0·363	3035
"	"	"	"	"	50	1278	68·02	0·320	3284
"	"	"	"	10	15	12200	147·3	3·050	12902
"	"	"	"	"	20	6050	142·61	1·512	6744
"	"	"	"	"	25	4000	136·45	1·000	4721
"	"	"	"	"	30	2975	128·45	0·744	3789
"	"	"	"	"	35	2360	118·17	0·590	3328
"	"	"	"	"	40	1950	105·1	0·488	3137*
"	"	"	"	"	45	1686	88·61	0·422	3524
"	"	"	"	"	50	1438	68·02	0·360	3696
"	"	"	"	15	20	12100	142·61	3·033	13527
"	"	"	"	"	25	6000	136·45	1·500	7081
"	"	"	"	"	30	3966	128·45	0·992	5051
"	"	"	"	"	35	2950	118·17	0·738	4162
"	"	"	"	"	40	2340	105·1	0·585	3844
"	"	"	"	"	45	1933	88·61	0·483	3743*
"	"	"	"	"	50	1643	68·02	0·411	4952
"	"	"	"	20	25	12000	136·45	3·000	14163
"	"	"	"	"	30	5950	128·45	1·488	7587
"	"	"	"	"	35	3933	118·17	0·983	5543
"	"	"	"	"	40	2925	105·1	0·732	4706

* Indicates the most favourable condition.

TABLE 72—(*continued*).

Vacuum.	Absolute pressure.	Steam.		Cooling water.			Air.		
		Temperature.	Total heat.	Initial temperature.	Final temperature.	Weight, *W*.	Pressure.	Weight.	Volume.
mm.	mm.	° C.	*c*.	t_a.	t_e.	kilos.	mm.	kilos.	Litres.
600	160	61·5	625	20	45	2320	88·61	0·580	4495*
,,	,,	,,	,,	,,	50	1917	68·02	0·479	4924
,,	,,	,,	,,	25	30	11900	128·45	2·975	15155
,,	,,	,,	,,	,,	35	5900	118·17	1·475	8319
,,	,,	,,	,,	,,	40	3900	105·1	0·975	6274
,,	,,	,,	,,	,,	45	2900	88·61	0·725	6061
,,	,,	,,	,,	,,	50	2300	68·02	0·575	5911
,,	,,	,,	,,	30	35	11800	118·17	2·950	16638
,,	,,	,,	,,	,,	40	5850	105·1	1·463	9414
,,	,,	,,	,,	,,	45	3866	88·61	0·967	8080
,,	,,	,,	,,	,,	50	2875	68·02	0·719	7389*
,,	,,	,,	,,	35	40	11700	105·1	2·925	18892
,,	,,	,,	,,	,,	45	5800	88·61	1·450	12122*
620	140	58·5	624	5	10	12280	130·8	3·070	14346
,,	,,	,,	,,	,,	15	6090	127·3	1·522	7314
,,	,,	,,	,,	,,	20	4026	122·61	1·006	5191
,,	,,	,,	,,	,,	25	29950	116·45	0·749	4143
,,	,,	,,	,,	,,	30	2376	108·45	0·594	3588
,,	,,	,,	,,	,,	35	1963	98·17	0·491	3331
,,	,,	,,	,,	,,	40	1669	85·1	0·417	3312*
,,	,,	,,	,,	,,	45	1448	68·61	0·362	3594
,,	,,	,,	,,	,,	50	1276	48·02	0·319	4645
,,	,,	,,	,,	10	15	12180	127·3	3·045	14634
,,	,,	,,	,,	,,	20	6040	122·61	1·510	7792
,,	,,	,,	,,	,,	25	3993	116·45	0·998	5520
,,	,,	,,	,,	,,	30	2970	108·45	0·743	4485
,,	,,	,,	,,	,,	35	2356	98·17	0·389	3996
,,	,,	,,	,,	,,	40	1947	85·1	0·487	3868*
,,	,,	,,	,,	,,	45	1683	68·61	0·421	4180
,,	,,	,,	,,	,,	50	1435	48·02	0·359	5227
,,	,,	,,	,,	15	20	12080	122·61	3·020	15568
,,	,,	,,	,,	,,	25	5990	116·45	1·498	8291
,,	,,	,,	,,	,,	30	3960	108·45	0·990	5980
,,	,,	,,	,,	,,	35	2945	98·17	0·736	5053
,,	,,	,,	,,	,,	40	2336	85·1	0·584	4638*
,,	,,	,,	,,	,,	45	1930	68·61	0·483	4834

TABLE 72—(*continued*).

Vacuum.	Absolute pressure.	Steam.		Cooling water.			Air.		
		Temperature.	Total heat.	Initial temperature.	Final temperature.	Weight, *W*.	Pressure.	Weight.	Volume.
mm.	mm.	°C.	*c*.	t^a.	t^e.	kilos.	mm.	kilos.	Litres.
620	140	58·5	624	15	50	1640	48·02	0·410	5970
,,	,,	,,	,,	20	25	11980	116·45	2·995	16565
,,	,,	,,	,,	,,	30	5940	108·45	1·485	8969
,,	,,	,,	,,	,,	35	3927	98·17	0·982	6662
,,	,,	,,	,,	,,	40	2920	85·1	0·730	5798*
,,	,,	,,	,,	,,	45	2316	68·61	0·579	5802
,,	,,	,,	,,	,,	50	1913	48·02	0·478	6960
,,	,,	,,	,,	25	30	11880	108·45	2·970	17939
,,	,,	,,	,,	,,	35	5890	98·17	1·473	9991
,,	,,	,,	,,	,,	40	3893	85·1	0·973	7727
,,	,,	,,	,,	,,	45	2895	68·61	0·724	7168*
,,	,,	,,	,,	,,	50	2296	48·02	0·574	8357
,,	,,	,,	,,	30	35	11780	98·17	2·945	19982
,,	,,	,,	,,	,,	40	5840	85·1	1·460	11595
,,	,,	,,	,,	,,	45	3860	68·61	0·965	9581*
,,	,,	,,	,,	,,	50	2870	48·02	0·718	10447
,,	,,	,,	,,	35	40	11680	85·1	2·920	23191
,,	,,	,,	,,	,,	45	5790	68·61	1·448	14377*
640	120	55	623	5	10	12260	110·8	3·062	16908
,,	,,	,,	,,	,,	15	6080	107·3	1·520	8811
,,	,,	,,	,,	,,	20	4020	102·61	1·005	6205
,,	,,	,,	,,	,,	25	2990	96·45	0·748	5014
,,	,,	,,	,,	,,	30	2372	88·45	0·593	4390
,,	,,	,,	,,	,,	35	1960	78·17	0·490	4171*
,,	,,	,,	,,	,,	40	1666	65·1	0·417	4280
,,	,,	,,	,,	,,	45	1445	48·61	0·361	5103
,,	,,	,,	,,	,,	50	1273	28·02	0·318	7956
,,	,,	,,	,,	10	15	12160	107·3	3·040	17632
,,	,,	,,	,,	,,	20	6030	102·61	1·508	9310
,,	,,	,,	,,	,,	25	3991	96·45	0·998	6675
,,	,,	,,	,,	,,	30	2965	88·45	0·741	5488
,,	,,	,,	,,	,,	35	2352	78·17	0·588	5005
,,	,,	,,	,,	,,	40	1943	65·1	0·486	5061
,,	,,	,,	,,	,,	45	1680	48·61	0·420	5937
,,	,,	,,	,,	,,	50	1433	28·02	0·358	8957
,,	,,	,,	,,	15	20	12060	102·61	3·015	18618

TABLE 72—(*continued*).

Vacuum.	Absolute pressure.	Steam. Temperature.	Steam. Total heat.	Cooling water. Initial temperature.	Cooling water. Final temperature.	Cooling water. Weight, W.	Air. Pressure.	Air. Weight.	Air. Volume.
mm.	mm.	°C.	c.	t_a.	t_e.	kilos.	mm.	kilos.	Litres.
640	120	55	623	15	25	5980	96·45	1·495	9990
″	″	″	″	″	30	3953	88·45	0·988	7316
″	″	″	″	″	35	2940	78·17	0·735	6262
″	″	″	″	″	40	2332	65·1	0·583	6085*
″	″	″	″	″	45	1927	48·61	0·482	8599
″	″	″	″	″	50	1637	28·02	0·409	10233
″	″	″	″	20	25	11960	90·45	2·990	21979
″	″	″	″	″	30	5930	88·45	1·482	10971
″	″	″	″	″	35	3920	78·17	0·980	7342*
″	″	″	″	″	40	2915	65·1	0·729	7592
″	″	″	″	″	45	2312	48·61	0·578	8167
″	″	″	″	″	50	1910	28·02	0·478	11959
″	″	″	″	25	30	11860	88·45	2·965	21950
″	″	″	″	″	35	5880	78·17	1·470	12513
″	″	″	″	″	40	3857	65·1	0·972	10122*
″	″	″	″	″	45	2890	48·61	0·723	10213
″	″	″	″	″	50	2292	28·02	0·573	14336
″	″	″	″	30	35	11760	78·17	2·940	25025
″	″	″	″	″	40	5830	65·1	1·458	15184
″	″	″	″	″	45	3854	48·61	0·964	13620*
″	″	″	″	″	50	2865	28·02	0·716	17914
″	″	″	″	35	40	11660	65·1	2·915	30357
″	″	″	″	″	45	5780	48·61	1·445	20427*
660	100	52	622	5	10	12240	90·8	3·060	20869
″	″	″	″	″	15	6070	87·3	1·518	10823
″	″	″	″	″	20	4013	82·61	1·003	7692
″	″	″	″	″	25	2985	76·45	0·746	6284
″	″	″	″	″	30	2368	68·45	0·592	5673
″	″	″	″	″	35	1957	58·17	0·489	5599*
″	″	″	″	″	40	1663	45·1	0·416	6232
″	″	″	″	″	45	1443	28·61	0·361	8718
″	″	″	″	″	50	1271	8·02	0·318	28458
″	″	″	″	10	15	12140	87·3	3·035	21640
″	″	″	″	″	20	6020	82·61	1·505	11543
″	″	″	″	″	25	3980	76·45	0·995	8382
″	″	″	″	″	30	2960	68·45	0·740	7091

TABLE 72—(*continued*).

Vacuum.	Absolute pressure.	Steam. Temperature.	Steam. Total heat.	Cooling water. Initial temperature.	Cooling water. Final temperature.	Cooling water. Weight, *W*.	Air. Pressure.	Air. Weight.	Air. Volume.
mm.	mm.	°C.	c.	t_a.	t_e.	Kilos.	mm.	kilos.	Litres.
660	100	52	622	10	35	2348	58·17	0·587	6721*
"	"	"	"	"	40	1940	45·1	0·485	7265
"	"	"	"	"	45	1677	28·61	0·419	10118
"	"	"	"	"	50	1430	8·02	0·358	31791
"	"	"	"	"	20	12040	82·61	3·010	22966
"	"	"	"	"	25	5970	76·45	1·493	12578
"	"	"	"	"	30	3946	68·45	0·987	9462
"	"	"	"	"	35	2935	58·17	0·734	8403*
"	"	"	"	"	40	2328	45·1	0·582	8718
"	"	"	"	"	45	1923	28·61	0·481	11611
"	"	"	"	"	50	1634	8·02	0·409	36555
"	"	"	"	20	25	11940	76·45	2·985	25164
"	"	"	"	"	30	5920	68·45	1·480	14181
"	"	"	"	"	35	3913	58·17	0·978	11098
"	"	"	"	"	40	2910	45·1	0·728	11020*
"	"	"	"	"	45	2308	28·61	0·577	13715
"	"	"	"	"	50	1907	8·02	0·477	42687
"	"	"	"	25	30	11840	68·45	2·960	28364
"	"	"	"	"	35	5870	58·17	1·468	16803
"	"	"	"	"	40	3880	45·1	0·970	14331*
"	"	"	"	"	45	2885	28·61	0·721	17219
"	"	"	"	"	50	2288	8·02	0·572	51188
"	"	"	"	30	35	11740	58·17	2·935	33306
"	"	"	"	"	40	5820	45·1	1·455	21796*
"	"	"	"	"	45	3847	28·61	0·962	23232
"	"	"	"	"	50	2860	8·02	0·715	63965
"	"	"	"	35	40	11640	45·1	2·910	43592
"	"	"	"	"	45	5770	28·61	1·443	34836*
680	80	48	621	5	10	12220	70·8	3·073	24759
"	"	"	"	"	15	6060	67·3	1·515	14053
"	"	"	"	"	20	4006	62·61	1·001	10150
"	"	"	"	"	25	2980	56·45	0·745	8508
"	"	"	"	"	30	2364	48·45	0·591	6961*
"	"	"	"	"	35	1453	38·17	0·488	8535
"	"	"	"	"	40	1660	28·1	0·415	11176
"	"	"	"	"	45	1440	8·61	0·360	29635

TABLE 72—(*continued*).

Vacuum. mm.	Absolute pressure. mm.	Steam. Temperature. ° C.	Steam. Total heat. c.	Cooling water. Initial temperature. t_a.	Cooling water. Final temperature. t_b.	Cooling water. Weight, W. kilos.	Air. Pressure. mm.	Air. Weight. kilos.	Air. Volume. Litres.
680	80	48	621	5	50	1269	—	—	—
,,	,,	,,	,,	10	15	12120	67·3	3·030	28106
,,	,,	,,	,,	,,	20	6010	62·61	1·502	15230
,,	,,	,,	,,	,,	25	3970	56·45	0·993	11334
,,	,,	,,	,,	,,	30	2955	48·45	0·739	9352*
,,	,,	,,	,,	,,	35	2344	38·17	0·586	10249
,,	,,	,,	,,	,,	40	1937	25·1	0·484	13070
,,	,,	,,	,,	,,	45	1674	8·61	0·419	44492
,,	,,	,,	,,	15	20	12020	62·61	3·005	30501
,,	,,	,,	,,	,,	25	5960	56·45	1·490	17016
,,	,,	,,	,,	,,	30	3940	48·45	0·985	13337
,,	,,	,,	,,	,,	35	2930	38·17	0·732	12600*
,,	,,	,,	,,	,,	40	2324	25·1	0·581	15646
,,	,,	,,	,,	,,	45	1920	8·61	0·480	39513
,,	,,	,,	,,	20	25	11920	56·45	2·980	34034
,,	,,	,,	,,	,,	30	5910	48·45	1·478	19909
,,	,,	,,	,,	,,	35	3903	38·17	0·976	17070*
,,	,,	,,	,,	,,	40	2905	25·1	0·726	19602
,,	,,	,,	,,	,,	45	2304	8·61	0·576	47992
,,	,,	,,	,,	25	30	11820	48·45	2·960	39804
,,	,,	,,	,,	,,	35	5860	38·17	1·465	25623*
,,	,,	,,	,,	,,	40	3877	25·1	0·969	26102
,,	,,	,,	,,	,,	45	2880	8·61	0·720	59270
,,	,,	,,	,,	30	35	11720	38·17	2·930	51246
,,	,,	,,	,,	,,	40	5810	25·1	1·453	39116*
,,	,,	,,	,,	,,	45	3840	8·61	0·996	79027
,,	,,	,,	,,	35	40	11620	25·1	2·905	78234*
,,	,,	,,	,,	,,	45	5760	8·61	1·440	118541
700	60	44	619	5	10	12180	50·8	3·045	36723
,,	,,	,,	,,	,,	15	6040	47·3	1·510	17818
,,	,,	,,	,,	,,	20	3993	42·61	0·998	14870
,,	,,	,,	,,	,,	25	2970	36·45	0·743	13166*
,,	,,	,,	,,	,,	30	2356	28·45	0·589	13641
,,	,,	,,	,,	,,	35	1947	18·17	0·487	17946
,,	,,	,,	,,	,,	40	1654	5·1	0·414	51936
,,	,,	,,	,,	10	15	12080	47·3	3·020	37616

TABLE 72—(*continued*).

Vacuum.	Absolute pressure.	Steam. Temperature.	Steam. Total heat.	Cooling water. Initial temperature.	Cooling water. Final temperature.	Cooling water. Weight, W.	Air. Pressure.	Air. Weight.	Air. Volume.
mm.	mm.	°C.	c.	t_a.	t_e.	kilos.	mm.	kilos.	Litres.
700	60	44	619	10	20	5990	42·61	1·498	22320
"	"	"	"	"	25	3960	36·45	0·990	17543
"	"	"	"	"	30	2945	28·45	0·736	17046*
"	"	"	"	"	35	2336	18·17	0·584	21520
"	"	"	"	"	40	1930	5·1	0·483	60520
"	"	"	"	15	20	11980	42·61	2·995	44495
"	"	"	"	"	25	5940	36·45	1·485	26314
"	"	"	"	"	30	3927	28·45	0·982	22743*
"	"	"	"	"	35	2920	18·17	0·730	27500
"	"	"	"	"	40	2316	5·1	0·579	77169
"	"	"	"	20	25	11880	36·45	2·970	52628
"	"	"	"	"	30	5890	28·45	1·473	34115*
"	"	"	"	"	35	3893	18·17	0·976	35965
"	"	"	"	"	40	2895	5·1	0·724	90826
"	"	"	"	25	30	11780	28·45	2·945	68204
"	"	"	"	"	35	5840	18·17	1·460	53801*
"	"	"	"	"	40	3860	5·1	0·965	121059
"	"	"	"	30	35	11680	18·17	2·920	107602*
"	"	"	"	"	40	5790	5·1	1·448	181640
"	"	"	"	35	40	11580	5·1	2·895	363177
710	50	38	618	5	10	12160	40·8	3·040	45661
"	"	"	"	"	15	6059	37·3	1·508	25259
"	"	"	"	"	20	3986	32·61	0·997	18474
"	"	"	"	"	25	2965	26·45	0·741	18147*
"	"	"	"	"	30	2352	18·45	0·588	20997
"	"	"	"	"	35	1943	8·17	0·486	40780
"	"	"	"	10	15	12060	37·3	3·015	50501
"	"	"	"	"	20	5980	32·61	1·495	27601
"	"	"	"	"	25	3953	26·45	0·988	24460*
"	"	"	"	"	30	2940	18·45	0·735	26247
"	"	"	"	"	35	2332	8·17	0·583	48920
"	"	"	"	15	20	11960	32·61	2·990	58375
"	"	"	"	"	25	5930	26·45	1·483	36322
"	"	"	"	"	30	3920	18·45	0·980	35106*
"	"	"	"	"	35	2915	8·17	0·729	51268
"	"	"	"	20	25	11860	26·45	2·965	73013

TABLE 72—(*continued*).

Vacuum.	Absolute pressure.	Steam.		Cooling water.			Air.		
		Temperature.	Total heat.	Initial temperature.	Final temperature.	Weight, *W*.	Pressure.	Weight.	Volume.
mm.	mm.	° C.	*c*.	t_a.	t_e.	kilos.	mm.	kilos.	Litres.
710	50	38	618	20	30	5880	18·45	1·470	52494*
"	"	"	"	"	35	3887	8·17	0·972	81544
"	"	"	"	25	30	11760	18·45	2·940	104587*
"	"	"	"	"	35	5830	6·17	1·458	122341
"	"	"	"	30	35	11660	8·17	2·915	244897
720	40	34·5	617	5	10	12140	30·8	3·035	60457
"	"	"	"	"	15	6020	27·3	1·505	34404
"	"	"	"	"	20	3980	22·61	0·995	27108*
"	"	"	"	"	25	2960	16·45	0·740	28986
"	"	"	"	"	30	2348	8·45	0·587	45937
"	"	"	"	10	15	12040	27·3	3·010	68809
"	"	"	"	"	20	5970	22·61	1·493	42312
"	"	"	"	"	25	3946	16·45	0·987	38641*
"	"	"	"	"	30	2935	8·45	0·734	58690
"	"	"	"	15	20	11940	22·61	2·985	84565
"	"	"	"	"	25	5920	16·45	1·480	58134*
"	"	"	"	"	30	3913	8·45	0·978	79472
"	"	"	"	20	25	11840	16·45	2·960	116269
"	"	"	"	"	30	5870	8·45	1·468	117541
"	"	"	"	25	30	11740	8·45	2·935	234682
730	30	29	615	5	10	12110	20·3	3·028	89599
"	"	"	"	"	15	6000	17·3	1·500	54090
"	"	"	"	"	20	3966	12·61	0·991	50174*
"	"	"	"	"	25	2950	6·45	0·738	123277
"	"	"	"	10	15	12000	17·3	3·000	108180
"	"	"	"	"	20	5950	12·61	1·488	75337*
"	"	"	"	"	25	3933	6·45	0·983	100065
"	"	"	"	15	20	11900	12·61	2·975	147709
"	"	"	"	"	25	5900	6·45	1·475	150553
"	"	"	"	20	25	11800	6·45	2·950	300605
740	20	21	613	5	10	12060	10·8	3·015	172126
"	"	"	"	"	15	5980	7·3	1·495	128929*
"	"	"	"	"	20	3950	2·61	0·985	179950
"	"	"	"	10	15	11960	7·3	2·990	237858
"	"	"	"	"	20	5930	2·61	1·483	270858
"	"	"	"	15	20	11860	2·61	2·965	541676

water, the higher is the vapour pressure over it, and therefore the lower is the pressure of the air and the greater its specific volume.

On the supposition that the weight of air to be exhausted is directly proportional to that of the injected water, this most favourable condition (the exhaustion of the least volume of air), which is indicated in Table 72 by an asterisk (*), also occurs at the same temperatures of the outflow if the cooling water has a proportion of air different to that which we assumed. Unfortunately our supposition of the complete proportionality between air and water is not quite reliable. In reality, therefore, the most favourable condition frequently occurs at another temperature, which cannot be determined beforehand. It must suffice to know that there is a most favourable temperature, which can well be found for apparatus at work.

Since wet air-pumps must carry off the air in addition to the injected water, their dimensions must be so taken that to the volume of air to be exhausted, as given in Table 72, is added the injected water, W.

C. The Volume of Air to be Exhausted from Dry Fall-pipe Jet-condensers.

A dry air-pump is one which exhausts the air and uncondensed gases from the condenser, but *not* the water. It takes the air from the condenser at the place where the cooling water enters, and thus the exhausted air has quite or almost the temperature of this injected water, t_a.

On our assumption, the weight of air taken from the condenser—that to be exhausted by the air-pump—is directly proportional to the quantity of the injected water; therefore equation (255) gives here also the *weight of air*:

$$L = \frac{W0{\cdot}25}{1000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (260)$$

Equation (259) is used to determine the *volume of air*, V_u, which the dry air-pump has to carry away, with the difference, that instead of inserting the temperature of the waste water, t_u, for that of the air, that of the entering water, t_a, is to be used.

$$V_u = \frac{W0{\cdot}25(273 + t_a)29{\cdot}27 \times 760}{1000pb} = 0{\cdot}5385\frac{W(273 + t_a)}{b} \quad (261)$$

Table 73 has been calculated by means of this equation. In this case, as with wet condensers, a larger or smaller proportion of air in the injected water increases or diminishes the volume of air to be exhausted.

The chief *differences* between wet and dry condensers (almost entirely to the advantage of the latter) are the following:—

The temperature of the water from dry (fall-pipe) condensers may be higher than from wet condensers, since, as we know, it may almost attain the temperature of the vapours passing into the condenser. Dry condensers, therefore, require much less water than wet condensers of the same capacity.

The smaller quantity of water brings a correspondingly smaller quantity of air into the apparatus, and, since this air is almost at the temperature of the *entering* cooling water, *i.e.*, much colder than in the wet condenser, the smaller *weight* of air has also a smaller specific *volume*. Also the vapour mixed with the air has a lower temperature, and therefore a lower pressure, and there remains a larger fraction of the total pressure in the condenser for the air. Thus there is almost always a smaller volume of air to be exhausted from a dry condenser.

Dry air-pumps may run at a greater speed than wet, because they have no water to overcome; for the same reason they may always be smaller than wet pumps for the same evaporative capacity.

Comparing the very different volumes of air to be exhausted in the different cases considered in Table 73, the following conclusions may be drawn:—

1. *Even with very warm cooling water fairly good vacua may be reached by means of dry condensation. Such conditions require only much cooling water and large air-pumps. The cooling water is still usable when it is only a few degrees cooler than the temperature of the evaporating liquid.*

2. *The more nearly the temperature of the exhausted air approaches to that of the entering cooling water, and that of the waste water to the temperature of the evaporating liquid, i.e., the more completely the cooling water is utilised, the better is the condensation and the smaller may the air-pump be. When the air-pump is only just large enough under given conditions, the condensation can never be improved, but only made worse, by a larger water supply.*

3. *It is very important to take the air quite cold from the condenser. The colder the air, the better the vacuum.*

TABLE 73.

The consumption of cooling water and volume of air, in litres, to be exhausted, for the condensation of 100 kilos. of steam at vacua of 600-740 mm.

Initial temperature of the cooling water, t_a, = 5° to 50° C.
Final " " " t_e, = 10° to 61·5° C.
in dry, *fall-pipe jet-condensers.*

Vacuum, 600 mm. Temperature, 61·5° C.			Absolute pressure, 160 mm. Total heat, c = 625 cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
5	61·5	997	5	153·5	0·25	978
"	"	"	10	150·8	"	1017
"	"	"	15	147·3	"	1055
"	55	1140	5	153·5	0·285	1114
"	"	"	10	150·8	"	1159
"	"	"	15	147·3	"	1205
"	50	1277	5	153·5	0·319	1247
"	"	"	10	150·8	"	1298
"	"	"	15	147·3	"	1346
10	61·5	1094	10	150·8	0·274	1115
"	"	"	15	147·3	"	1156
"	"	"	20	142·6	"	1210
"	55	1266	10	150·8	0·317	1289
"	"	"	15	147·3	"	1338
"	"	"	20	142·6	"	1400
"	50	1437	10	150·8	0·359	1460
"	"	"	15	147·3	"	1515
"	"	"	20	142·6	"	1586
15	61·5	1212	15	147·3	0·303	1279
"	"	"	20	142·6	"	1338
"	"	"	25	136·5	"	1430
"	55	1425	15	147·3	0·356	1502
"	"	"	20	142·6	"	1572

TABLE 73—(*continued*).

Vacuum, 600 mm. Temperature, 61·5° C.			Absolute pressure, 160 mm. Total heat, c = 625 cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
15	55	1425	25	136·5	0·356	1680
,,	50	1642	15	147·3	0·41	1732
,,	,,	,,	20	142·6	,,	1811
,,	,,	,,	[illegible]	136·5	,,	1938
20	61·5	1385	20	142·6	0·346	1528
,,	,,	,,	25	136·5	,,	1633
,,	,,	,,	30	128·5	,,	1776
,,	55	1629	20	142·6	0·407	1798
,,	,,	,,	25	136·5	,,	1921
,,	,,	,,	30	128·5	,,	2088
,,	50	1917	20	142·6	0·479	2116
,,	,,	,,	25	136·5	,,	2259
,,	,,	,,	30	128·5	,,	2449
25	61·5	1544	25	136·5	0·386	1831
,,	,,	,,	30	128·5	,,	1981
,,	,,	,,	35	118·2	,,	2173
,,	55	1900	25	136·5	0·475	2242
,,	,,	,,	30	128·5	,,	2438
,,	,,	,,	35	118·2	,,	2674
,,	50	2300	25	136·5	0·575	2714
,,	,,	,,	30	128·5	,,	2953
,,	,,	,,	35	118·2	,,	3237
30	61·5	1772	30	128·5	0·443	2274
,,	,,	,,	35	118·2	,,	2494
,,	,,	,,	40	105·1	,,	2856
,,	55	2280	30	128·5	0·570	2926
,,	,,	,,	35	118·2	,,	3209
,,	,,	,,	40	105·1	,,	3675

TABLE 73—(*continued*).

Vacuum, 600 mm. Temperature, 61·5° C.			Absolute pressure, 160 mm Total heat, $c = 625$ cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_w.	Pressure. mm.	Weight. kilos.	Volume. Litres.
30	50	2875	30	128·5	0·719	3691
„	„	„	35	118·2	„	4048
„	„	„	40	105·1	„	4635
35	61·5	2125	35	118·2	0·531	2992
„	„	„	40	105·1	„	3426
„	„	„	45	88·6	„	4128
„	55	2850	35	118·2	0·712	4011
„	„	„	40	105·1	„	4593
„	„	„	45	88·6	„	5524
„	50	3833	35	118·2	0·958	5394
„	„	„	40	105·1	„	6175
„	„	„	45	88·6	„	7427
40	61·5	2626	40	105·1	0·657	4299
„	„	„	45	88·6	„	5094
„	„	„	50	68	„	6747
„	55	3800	40	105·1	0·950	6124
„	„	„	45	88·6	„	7365
„	„	„	50	68	„	9756
„	50	5750	40	105·1	1·437	9263
„	„	„	45	88·6	„	11141
„	„	„	50	68	„	14758
45	61·5	3415	45	88·6	0·854	6621
„	„	„	50	68	„	8770
„	„	„	55	42·5	„	14262
„	55	5700	45	88·6	1·425	11047
„	„	„	50	68	„	14634
„	„	„	55	42·5	„	23798
„	50	11500	45	88·6	2·875	22090

TABLE 73—(*continued*).

Vacuum, 600 mm. Temperature, 61·5° C.			Absolute pressure, 160 mm. Total heat, c = 625 cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
45	50	11500	50	68	2·875	29526
"	"	"	55	42·5	"	58013
50	61·5	4895	50	68	1·224	12450
"	"	"	55	42·2	"	20300
"	"	"	60	12	"	169500
"	55	11300	50	68	2·825	29013
Vacuum, 620 mm. Temperature, 58·5° C.			Absolute pressure, 140 mm. Total heat, c = 624 cals.			
5	58·5	1057	5	133·5	0·260	1185
"	"	"	10	130·8	"	1215
"	"	"	15	127·3	"	1269
"	50	1276	5	133·5	0·319	1454
"	"	"	10	130·8	"	1489
"	"	"	15	127·3	"	1557
"	45	1447	5	133·5	0·362	1650
"	"	"	10	130·8	"	1692
"	"	"	15	127·3	"	1767
10	58·5	1166	10	130·8	0·291	1342
"	"	"	15	127·3	"	1423
"	"	"	20	122·6	"	1505
"	50	1435	10	130·8	0·359	1678
"	"	"	15	127·3	"	1752
"	"	"	20	122·6	"	1856
"	45	1654	10	130·8	0·414	1935
"	"	"	15	127·3	"	2020
"	"	"	20	122·6	"	2140

TABLE 73—(*continued*).

Vacuum, 620 mm. Temperature, 58·8° C.			Absolute pressure, 140 mm. Total heat, c = 624 cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
15	58	1300	15	127·3	0·325	1586
"	"	"	20	122·6	"	1680
"	"	"	25	116·5	"	1797
"	50	1640	15	127·3	0·410	2001
"	"	"	20	122·6	"	2120
"	"	"	25	116·5	"	2267
"	45	1930	15	127·3	0·482	2355
"	"	"	20	122·6	"	2495
"	"	"	25	116·5	"	2668
20	58	1516	20	122·6	0·379	1959
"	"	"	25	116·5	"	2094
"	"	"	30	108·5	"	2310
"	50	1913	20	122·6	0·478	2471
"	"	"	25	116·5	"	2703
"	"	"	30	108·5	"	2913
"	45	2315	20	122·6	0·579	2993
"	"	"	25	116·5	"	3202
"	"	"	30	108·5	"	3529
25	58	1715	25	116·5	0·420	2372
"	"	"	30	108·5	"	2615
"	"	"	35	98·2	"	2913
"	50	2296	25	116·5	0·574	3174
"	"	"	30	108·5	"	3498
"	"	"	35	98·2	"	3892
"	45	2895	25	116·5	0·724	4004
"	"	"	30	108·5	"	4413
"	"	"	35	98·2	"	4908
30	58	2021	30	108·5	0·505	3078

TABLE 73—(*continued*).

Vacuum, 620 mm. Temperature, 58·5° C.			Absolute pressure, 140 mm. Total heat, c = 624 cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
30	58	2021	35	98·2	0·505	3424
,,	,,	,,	40	85·1	,,	4020
,,	50	2870	30	108·5	0·718	4376
,,	,,	,,	35	98·2	,,	4868
,,	,,	,,	40	85·1	,,	5715
,,	45	3860	30	108·5	0·965	5855
,,	,,	,,	35	98·2	,,	6543
,,	,,	,,	40	85·1	,,	7681
35	58	2304	35	98·2	0·576	3905
,,	,,	,,	40	85·1	,,	4585
,,	,,	,,	45	68·6	,,	5727
,,	50	3827	35	98·2	0·957	6488
,,	,,	,,	40	85·1	,,	7618
,,	,,	,,	45	68·6	,,	9599
,,	45	5790	35	98·2	1·448	9817
,,	,,	,,	40	85·1	,,	11526
,,	,,	,,	45	68·6	,,	14523
40	58	3144	40	85·1	0·786	6257
,,	,,	,,	45	68·6	,,	7884
,,	,,	,,	50	48	,,	11444
,,	50	5740	40	85·1	1·435	11022
,,	,,	,,	45	68·6	,,	14393
,,	,,	,,	50	48	,,	20893
,,	45	11580	40	85·1	2·895	28044
,,	,,	,,	45	68·6	,,	29037
,,	,,	,,	50	48	,,	42151
45	58	4354	45	68·6	1·089	10923
,,	,,	,,	50	48	,,	15856

TABLE 73—(*continued*).

Vacuum, 620 mm. Temperature, 58·5° C.			Absolute pressure, 140 mm. Total heat, c = 624 cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
45	58	4354	55	22·5	1·089	34685
,,	50	11480	45	68·6	2·870	28786
,,	,,	,,	50	48	,,	41787
,,	,,	,,	55	22·5	,,	91410
50	58	7075	50	48	1·739	25766
Vacuum, 640 mm. Temperature, 55° C.			Absolute pressure, 120 mm. Total heat, c = 623 cals.			
5	55	1136	5	113·5	0·284	1503
,,	,,	,,	10	110·8	,,	1568
,,	,,	,,	15	107·3	,,	1647
,,	50	1251	5	113·5	0·313	1656
,,	,,	,,	10	110·8	,,	1728
,,	,,	,,	15	107·3	,,	1815
,,	45	1445	5	113·5	0·3615	1924
,,	,,	,,	10	110·8	,,	1995
,,	,,	,,	15	107·3	,,	2096
10	55	1262	10	110·8	0·315	1739
,,	,,	,,	15	107·3	,,	1828
,,	,,	,,	20	102·6	,,	1943
,,	50	1432	10	110·8	0·358	1976
,,	,,	,,	15	107·3	,,	2076
,,	,,	,,	20	102·6	,,	2209
,,	45	1651	10	110·8	0·413	2280
,,	,,	,,	15	107·3	,,	2395
,,	,,	,,	20	102·6	,,	2548

TABLE 73—(*continued*).

Vacuum, 640 mm. Temperature, 55° C.			Absolute pressure, 120 mm. Total heat c = 623 cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
15	55	1420	15	107·3	0·355	2004
"	"	"	20	102·6	"	2190
"	"	"	25	96·5	"	2382
"	50	1637	15	107·2	0·409	2372
"	"	"	20	102·6	"	2524
"	"	"	25	96·5	"	2732
"	45	1927	15	107·2	0·482	2796
"	"	"	20	102·6	"	2974
"	"	"	25	96·5	"	3218
20	55	1625	20	102·6	0·406	2505
"	"	"	25	96·5	"	2712
"	"	"	30	88·5	"	3039
"	50	1910	20	102·6	0·480	2962
"	"	"	25	96·5	"	3206
"	"	"	30	88·5	"	3593
"	45	2312	20	102·6	0·578	3566
"	"	"	25	96·5	"	3861
"	"	"	30	88·5	"	4326
25	55	1893	25	96·5	0·473	3160
"	"	"	30	88·5	"	3540
"	"	"	35	78·2	"	4026
"	50	2292	25	96·5	0·573	3828
"	"	"	30	88·5	"	4289
"	"	"	35	78·2	"	4877
"	45	2890	25	96·5	0·722	4824
"	"	"	30	88·5	"	5408
"	"	"	35	78·2	"	6150
30	55	2272	30	88·5	0·568	4241

TABLE 73—(*continued*).

Vacuum, 640 mm. Temperature, 55° C.			Absolute pressure, 120 mm. Total heat, c = 623 cals.			
Cooling water.			Air.			
Initial temperature.	Final temperature.	Weight.	Temperature.	Pressure.	Weight.	Volume.
t_a.	t_e.	kilos.	t_{la}.	mm.	kilos.	Litres.
30	55	2272	35	78·2	0·568	4766
,,	,,	,,	40	65·1	,,	5927
,,	50	2865	30	88·5	0·716	5359
,,	,,	,,	35	78·2	,,	6094
,,	,,	,,	40	65·1	,,	7471
,,	45	3833	30	88·5	0·956	7156
,,	,,	,,	35	78·2	,,	8137
,,	,,	,,	40	65·1	,,	9976
35	55	2840	35	78·2	0·710	6043
,,	,,	,,	40	65·1	,,	7409
,,	,,	,,	45	48·6	,,	10039
,,	50	3820	35	78·2	0·955	8128
,,	,,	,,	40	65·1	,,	9965
,,	,,	,,	45	48·6	,,	13504
,,	45	5780	35	78·2	1·445	12298
,,	,,	,,	40	65·1	,,	15079
,,	,,	,,	45	48·6	,,	20342
40	55	3787	40	65·1	0·947	9882
,,	,,	,,	45	48·6	,,	13391
,,	,,	,,	50	28	,,	22018
,,	50	5730	40	65·1	1·432	14943
,,	,,	,,	45	48·6	,,	20248
,,	,,	,,	50	28	,,	33294
,,	45	11560	40	65·1	2·89	30157
,,	,,	,,	45	48·6	,,	40685
,,	,,	,,	50	28	,,	67193
45	55	5680	45	48·6	1·420	20779
,,	,,	,,	50	28	,,	35684

TABLE 73—(*continued*).

Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
Vacuum, 640 mm. Temperature, 55° C.			Absolute pressure, 120 mm. Total heat, c = 623 cals.			
Cooling water.			Air.			
45	55	5680	55	2·5	1·420	295360
,,	50	11460	45	48·6	2·865	40511
,,	,,	,,	53	28	,,	71997
,,	,,	,,	55	2·5	,,	595920
50	55	11360	50	28	2·840	71369
Vacuum, 660 mm. Temperature, 52° C.			Absolute pressure, 100 mm. Total heat, c = 622 cals.			
5	52	1213	5	93·5	0·303	1947
,,	,,	,,	10	90·8	,,	1865
,,	,,	,,	15	87·3	,,	2160
,,	45	1440	5	93·5	0·360	2313
,,	,,	,,	10	90·8	,,	2216
,,	,,	,,	15	87·3	,,	2567
,,	40	1660	5	93·5	0·415	2666
,,	,,	,,	10	90·8	,,	2555
,,	,,	,,	15	87·3	,,	2958
10	52	1357	10	90·8	0·339	2087
,,	,,	,,	15	87·3	,,	2417
,,	,,	,,	20	82·6	,,	2600
,,	45	1650	10	90·8	0·412	2539
,,	,,	,,	15	87·3	,,	2941
,,	,,	,,	20	82·6	,,	3164
,,	40	1940	10	90·8	0·485	2986
,,	,,	,,	15	87·3	,,	4458
,,	,,	,,	20	82·6	,,	3720

TABLE 73—(*continued*).

Vacuum, 660 mm. Temperature, 72° C.			Absolute pressure, 100 mm. Total heat, c = 622 cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_w.	Pressure. mm.	Weight. kilos.	Volume. Litres.
15	52	1540	15	87·3	0·385	2745
"	"	"	20	82·6	"	2953
"	"	"	25	76·5	"	3241
"	45	1923	15	87·3	0·481	3429
"	"	"	20	82·6	"	3689
"	"	"	25	76·5	"	4049
"	40	2328	15	87·3	0·582	4149
"	"	"	20	82·6	"	4464
"	"	"	25	76·5	"	4899
20	52	1781	20	82·6	0·445	3413
"	"	"	25	76·5	"	3746
"	"	"	30	68·5	"	4326
"	45	2308	20	82·6	0·577	4426
"	"	"	25	76·5	"	4857
"	"	"	30	68·5	"	5610
"	40	2910	20	82·6	0·782	5584
"	"	"	25	76·5	"	6128
"	"	"	30	68·5	"	7078
25	52	2111	25	76·5	0·528	4445
"	"	"	30	68·5	"	5133
"	"	"	35	58·2	"	6040
"	45	2885	25	76·5	0·721	6069
"	"	"	30	68·5	"	7010
"	"	"	35	58·2	"	8248
"	40	3800	25	76·5	0·950	7997
"	"	"	30	68·5	"	9236
"	"	"	35	58·2	"	10868
30	52	2591	30	68·5	0·648	6300

TABLE 73—(*continued*).

Vacuum, 660 mm.
Temperature, 52° C.

Absolute pressure, 100 mm.
Total heat, c = 622 cals.

Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
30	52	2591	35	58·2	0·648	7413
,,	,,	,,	40	45·1	,,	9662
,,	45	3848	30	68·5	0·962	9353
,,	,,	,,	35	58·2	,,	11005
,,	,,	,,	40	45·1	,,	14478
,,	40	5820	30	68·5	1·455	14146
,,	,,	,,	35	58·2	,,	16645
,,	,,	,,	40	45·1	,,	21898
35	52	3354	35	58·2	0·839	9599
,,	,,	,,	40	45·1	,,	12627
,,	,,	,,	45	28·6	,,	20268
,,	45	5770	35	58·2	1·442	16502
,,	,,	,,	40	45·1	,,	21709
,,	,,	,,	45	28·6	,,	34946
,,	40	11640	35	58·2	2·910	33290
,,	,,	,,	40	45·1	,,	43796
,,	,,	,,	45	28·6	,,	70297
40	52	4750	40	45·1	1·188	17879
,,	,,	,,	45	28·6	,,	28699
,,	,,	,,	50	8	,,	106540
,,	45	11540	40	45·1	2·885	43419
,,	,,	,,	45	28·6	,,	69693
,,	,,	,,	50	8	,,	258727
45	52	8143	45	28·6	2·036	49180
,,	,,	,,	50	8	,,	182108
50	52	—	—	—	—	—

TABLE 73—(*continued*).

Vacuum, 680 mm. Temperature, 48° C.			Absolute pressure, 80 mm. Total heat, $c = 621$ cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
5	48	1356	5	73·5	0·369	2773
,,	,,	,,	10	70·8	,,	2963
,,	,,	,,	15	67·3	,,	3145
,,	40	1718	5	73·5	0·4295	3512
,,	,,	,,	10	70·8	,,	3754
,,	,,	,,	15	67·3	,,	3984
,,	35	1953	5	73·5	0·488	3992
,,	,,	,,	10	70·8	,,	4158
,,	,,	,,	15	67·3	,,	4527
10	48	1509	10	70·8	0·377	3295
,,	,,	,,	15	67·3	,,	3497
,,	,,	,,	20	62·6	,,	3827
,,	40	1937	10	70·8	0·484	4230
,,	,,	,,	15	67·3	,,	4490
,,	,,	,,	20	62·6	,,	4912
,,	35	2344	10	70·8	0·586	5122
,,	,,	,,	15	67·3	,,	5436
,,	,,	,,	20	62·6	,,	5948
15	48	1737	15	67·3	0·434	4026
,,	,,	,,	20	62·3	,,	4405
,,	,,	,,	25	56·5	,,	4958
,,	40	2324	15	67·3	0·581	5389
,,	,,	,,	20	62·6	,,	5897
,,	,,	,,	25	56·5	,,	6638
,,	35	2930	15	67·3	0·732	6790
,,	,,	,,	20	62·6	,,	7435
,,	,,	,,	25	56·5	,,	8369
20	48	2040	20	62·6	0·510	5177

TABLE 73—(*continued*).

Vacuum, 680 mm. Temperature, 48° C.			Absolute pressure, 80 mm. Total heat, c = 621 cals.			
Cooling water.			Air.			
Initial temperature.	Final temperature.	Weight.	Temperature.	Pressure.	Weight.	Volume.
t_a.	t_e.	kilos.	t_{la}.	mm.	kilos.	Litres.
20	48	2040	25	55·5	0·510	5827
„	„	„	30	48·5	„	7043
„	40	2805	20	62·6	0·726	7369
„	„	„	25	55·5	„	8295
„	„	„	30	48·5	„	10026
„	35	3908	20	62·6	0·977	9917
„	„	„	25	55·5	„	11162
„	„	„	30	48·5	„	13492
25	48	2491	25	56·5	0·623	7118
„	„	„	30	48·5	„	8603
„	„	„	35	38·2	„	10870
„	40	3866	25	56·5	0·967	11047
„	„	„	30	48·5	„	13354
„	„	„	35	38·2	„	16903
„	35	5770	25	56·5	1·442	16475
„	„	„	30	48·5	„	19901
„	„	„	35	38·2	„	25215
30	48	3184	30	48·5	0·796	10993
„	„	„	35	38·2	„	13949
„	„	„	40	25·1	„	22246
„	40	5810	30	48·1	1·453	20070
„	„	„	35	38·5	„	25433
„	„	„	40	25·1	„	41059
„	35	11720	30	48·5	2·950	40460
„	„	„	35	38·5	„	51196
„	„	„	40	25·1	„	80780
35	48	4408	35	38·2	1·102	19263
„	„	„	40	25·1	„	30382

TABLE 73—(*continued*).

Vacuum, 680 mm. Temperature, 48° C.			Absolute pressure, 80 mm. Total heat, $c = 621$ cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
35	48	4408	45	8·6	1·102	242247
"	40	11620	35	38·2	2·905	50769
"	"	"	40	25·1	"	80090
"	"	"	45	8·6	"	91895
40	48	7043	40	25·1	1·761	48561
"	"	"	45	8·6	"	146850
45	48	19100	45	8·6	4·775	—
Vacuum, 700 mm. Temperature, 44° C.			Absolute pressure, 60 mm. Total heat, $c = 619$ cals.			
5	44	1474	5	53·5	0·369	4149
"	"	"	10	50·8	"	4446
"	"	"	15	47·3	"	4863
"	35	1945	5	53·5	0·486	5465
"	"	"	10	50·8	"	5816
"	"	"	15	47·3	"	6405
"	30	2356	5	53·5	0·589	6623
"	"	"	10	50·8	"	7097
"	"	"	15	47·3	"	7763
10	44	1691	10	50·8	0·425	5121
"	"	"	15	47·3	"	5502
"	"	"	20	42·6	"	6333
"	35	2335	10	50·8	0·584	7037
"	"	"	15	47·3	"	7697
"	"	"	20	42·6	"	8702
"	30	2945	10	50·8	0·736	8869

TABLE 73—(*continued*).

Vacuum, 700 mm. Temperature, 44° C.			Absolute pressure, 60 mm. Total heat, c = 619 cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_w.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
10	30	2945	15	47·3	0·736	9700
"	"	"	20	42·6	"	10966
15	44	1983	15	47·3	0·496	6537
"	"	"	20	42·6	"	6390
"	"	"	25	36·5	"	8779
"	35	2920	15	47·3	0·730	9621
"	"	"	20	42·6	"	10877
"	"	"	25	36·5	"	12921
"	30	3926	15	47·3	0·981	12936
"	"	"	20	42·6	"	14624
"	"	"	25	36·5	"	17363
20	44	2396	20	42·6	0·599	8925
"	"	"	25	36·5	"	10602
"	"	"	30	28·5	"	14364
"	35	3890	20	42·6	0·972	14483
"	"	"	25	36·5	"	17204
"	"	"	30	28·5	"	23309
"	30	5890	20	42·6	1·472	21933
"	"	"	25	36·5	"	26063
"	"	"	30	28·5	"	35310
25	44	3026	25	36·5	0·757	13399
"	"	"	30	28·5	"	18153
"	"	"	35	18·2	"	27858
"	35	5840	25	36·5	1·460	25842
"	"	"	30	28·5	"	35011
"	"	"	35	18·2	"	53728
"	30	11780	25	36·5	2·945	52126
"	"	"	30	28·5	"	70621
"	"	"	35	18·2	"	108376

TABLE 73—(*continued*).

Vacuum, 700 mm. Temperature, 44° C.			Absolute pressure, 60 mm. Total heat, c = 619 cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
30	44	4108	30	28·5	1·627	24627
"	"	"	35	18·2	"	37794
"	"	"	40	5·1	"	143780
"	35	11680	30	28·5	2·920	70022
"	"	"	35	18·2	"	10746
"	"	"	40	5·1	"	408800
35	44	6410	35	18·2	1·603	58990
"	"	"	46	5·1	"	224420
40	44	14425	40	5·1	3·606	504840
Vacuum, 710 mm. Temperature, 38° C.			Absolute pressure, 50 mm. Total heat, c = 618 cals.			
5	38	1758	5	43·5	0·440	6090
"	"	"	10	40·8	"	7542
"	"	"	15	37·3	"	7366
"	30	2352	5	43·5	0·588	8138
"	"	"	10	40·8	"	10078
"	"	"	15	37·3	"	9843
"	25	2965	5	43·5	0·741	10255
"	"	"	10	40·8	"	12601
"	"	"	15	37·3	"	12404
10	38	2071	10	40·8	0·518	8878
"	"	"	15	37·3	"	8668
"	"	"	20	32·6	"	10117
"	30	2690	10	40·8	0·672	11527
"	"	"	15	37·3	"	11257

TABLE 73—(*continued*).

Vacuum, 710 mm. Temperature, 38° C.			Absolute pressure, 50 mm. Total heat, c = 618 cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
10	30	2690	20	32·6	0·672	13124
,,	25	3953	10	40·8	0·988	16934
,,	,,	,,	15	37·3	,,	16539
,,	,,	,,	20	32·6	,,	19295
15	38	2609	15	37·3	0·652	10914
,,	,,	,,	20	32·6	,,	12732
,,	,,	,,	25	26·5	,,	15935
,,	30	3920	15	37·3	0·980	16405
,,	,,	,,	20	32·6	,,	19239
,,	,,	,,	25	26·5	,,	23951
,,	25	5930	15	37·3	1·482	13849
,,	,,	,,	20	32·6	,,	28943
,,	,,	,,	25	26·5	,,	36220
20	38	3277	20	32·6	0·819	15995
,,	,,	,,	25	26·5	,,	20016
,,	,,	,,	30	18·5	,,	30745
,,	30	5888	20	32·2	1·470	18709
,,	,,	,,	25	26·5	,,	35927
,,	,,	,,	30	18·5	,,	55184
,,	25	11860	20	32·6	2·970	58004
,,	,,	,,	25	26·5	,,	72587
,,	,,	,,	30	18·5	,,	111494
25	38	4530	25	26·5	1·132	27678
,,	,,	,,	30	18·5	,,	42514
,,	,,	,,	35	8·2	,,	96263
,,	30	11760	25	26·5	2·940	71854
,,	,,	,,	30	18·5	,,	110368
,,	,,	,,	35	8·2	,,	249900

TABLE 73—(*continued*).

Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
Vacuum, 710 mm. Temperature, 38° C.			**Absolute pressure, 50 mm. Total heat, $c = 618$ cals.**			
30	38	7250	30	18·5	1·812	68022
,,	,,	,,	35	8·2	,,	154700
35	38	19333	35	8·2	4·833	410805
Vacuum, 720 mm. Temperature, 34·5° C.			**Absolute pressure, 40 mm. Total heat, $c = 617$ cals.**			
5	34·5	1974	5	33·5	0·494	8916
,,	,,	,,	10	30·8	,,	9840
,,	,,	,,	15	27·3	,,	11288
,,	25	2960	5	33·5	0·740	13355
,,	,,	,,	10	30·8	,,	14541
,,	,,	,,	15	27·3	,,	16909
,,	20	3980	5	33·5	0·995	17955
,,	,,	,,	10	30·8	,,	19820
,,	,,	,,	15	27·3	,,	22736
10	34·5	2377	10	30·8	0·594	11832
,,	,,	,,	15	27·3	,,	13573
,,	,,	,,	20	22·6	,,	16846
,,	25	3948	10	30·8	0·987	19651
,,	,,	,,	15	27·3	,,	22533
,,	,,	,,	20	22·6	,,	27991
,,	20	5970	10	30·8	1·493	29740
,,	,,	,,	15	27·3	,,	34121
,,	,,	,,	20	22·6	,,	42741
15	34·5	3000	15	27·3	0·750	17138
,,	,,	,,	20	22·6	,,	21270

TABLE 73—(*continued*).

Vacuum, 720 mm. Temperature, 34·5° C.			Absolute pressure, 40 mm. Total heat, c = 617 cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_s.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
15	34·5	3000	25	16·5	0·750	29108
,,	25	5920	15	27·3	1·480	33818
,,	,,	,,	20	22·6	,,	41973
,,	,,	,,	25	16·5	,,	57439
,,	20	11940	15	27·3	2·985	68207
,,	,,	,,	20	22·6	,,	84654
,,	,,	,,	25	16·5	,,	115850
20	34·5	3949	20	22·6	0·987	27991
,,	,,	,,	25	16·5	,,	38305
,,	,,	,,	30	8·5	,,	87675
,,	25	11840	20	22·6	2·960	85945
,,	,,	,,	25	16·5	,,	114878
,,	,,	,,	30	8·5	,,	262936
25	34·5	6131	25	16·5	1·533	59466
,,	,,	,,	30	8·5	,,	136176
30	34·5	12947	30	8·5	3·236	287494
Vacuum, 730 mm. Temperature, 29° C.			Absolute pressure, 30 mm. Total heat, c = 615 cals.			
5	29	2443	5	23·5	0·611	15782
,,	,,	,,	10	20·8	,,	18087
,,	,,	,,	15	17·3	,,	21972
,,	20	3966	5	23·5	0·991	25697
,,	,,	,,	10	20·8	,,	29440
,,	,,	,,	15	17·3	,,	35636
,,	15	6000	5	23·5	1·500	38740

TABLE 73—(*continued*).

Cooling water.			Air.			
Vacuum, 730 mm. Temperature, 29° C,			Absolute pressure, 30 mm. Total heat, c = 615 cals.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
5	15	6000	10	20·8	1·500	44382
,,	,,	,,	15	17·3	,,	53940
10	29	3084	10	20·8	0·771	20612
,,	,,	,,	15	17·3	,,	27725
,,	,,	,,	20	12·6	,,	39051
,,	20	5950	10	20·8	1·488	44027
,,	,,	,,	15	17·3	,,	53508
,,	,,	,,	20	12·6	,,	75367
,,	15	12000	10	20·8	3·000	88764
,,	,,	,,	15	17·3	,,	106788
,,	,,	,,	20	12·6	,,	151950
15	29	4185	15	17·3	1·046	37494
,,	,,	,,	20	12·6	,,	52980
,,	,,	,,	25	6·5	,,	101012
,,	20	11900	15	17·3	2·975	86981
,,	,,	,,	20	12·6	,,	150684
,,	,,	,,	25	6·5	,,	287296
20	29	6511	20	12·6	1·628	82458
,,	,,	,,	25	6·5	,,	157916
25	29	14650	25	6·5	3·660	353446
Vacuum, 740 mm. Temperature, 21° C.			Absolute pressure, 20 mm. Total heat, c = 613 cals.			
5	21	3694	5	13·5	0·924	41626
,,	,,	,,	10	10·8	,,	52742

TABLE 73—(*continued*).

Vacuum, 740 mm. Temperature, 21° C.			Absolute pressure, 20 mm. Total heat, $c = 613$ cals.			
Cooling water.			Air.			
Initial temperature. t_a.	Final temperature. t_e.	Weight. kilos.	Temperature. t_{la}.	Pressure. mm.	Weight. kilos.	Volume. Litres.
5	21	3694	15	7·3	0·924	79679
,,	15	5980	5	13·5	1·495	67350
,,	,,	,,	10	10·8	,,	85835
,,	,,	,,	15	7·3	,,	128718
,,	10	12060	5	13·5	3·015	135600
,,	,,	,,	10	10·8	,,	171280
,,	,,	,,	15	7·3	,,	258699
10	21	5382	10	10·8	1·345	76773
,,	,,	,,	15	7·3	,,	115983
,,	,,	,,	20	2·6	,,	245718
,,	15	11960	10	10·8	2·990	170670
,,	,,	,,	15	7·3	,,	257836
,,	,,	,,	20	2·6	,,	566243
15	21	9867	15	7·3	2·467	212737
,,	,,	,,	20	2·6	,,	450696
20	,,	59200	20	2·6	14·800	2703812

D. The Volume of Air to be Exhausted from Surface-condensers.

The cooling water does not come in contact with the interior of surface-condensers, from which the air-pump exhausts; hence the air carried by this water has not in this case to be taken away by the pump. In surface-condensers the air-pumps have only to extract the air introduced from the liquid to be evaporated or distilled and

by leakages in the apparatus. The pumps may, therefore, be smaller for surface- than for jet-condensers.

Since there is no experimental guide to the quantity of air introduced by these means, we can only rely on the general experience that the volume of air to be exhausted from surface-condensers is about 0·6 of that from jet-condensers. The temperature of this air is that of the condensed liquid after it has been cooled. If the condensed liquid has the temperature t_{wc}, which is a few degrees higher than that of the entering cooling water, then the volume of air to be exhausted per 100 kilos. of condensed liquid is:

$$V_{lo} = 0{\cdot}6 \frac{L_l(273 + t_{wc})29{\cdot}27 \times 760}{pb} \quad . \quad . \quad . \quad . \quad (262)$$

These volumes of air may be found by multiplying by 0·6 those given in Table 73 for dry jet-condensers.

Both wet and dry air-pumps may be used in connection with surface-condensers,—the former when the condensed liquid is to be taken *together with* the air, the latter when the distillate is caught and carried away separately.

The wet air-pump of a surface-condenser has to exhaust, per 100 kilos. of distillate, the volume:

$$V_{ln} = 100 + V_{lo} \text{ litres} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (263)$$

The dry air-pump has to exhaust the volume:

$$V_{ln} = V_{lo} \text{ litres} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (264)$$

CHAPTER XXIV.

A FEW REMARKS ON AIR-PUMPS AND THE VACUA THEY PRODUCE.

THERE are two chief forms of air-pump used in connection with evaporating apparatus—(A) air-pumps with flap-valves; (B) with slide-valves.

A. Air-pumps with Flap-valves.

The valves of these pumps are sheets of rubber or metal, which are opened and closed by the pressure of the air without mechanical aid. They are called "wet" air-pumps if they are to exhaust the warm (condensed) water together with the air. Since the water can never be given as high a velocity in the pump as the air, these pumps must possess much larger valves if they are to exhaust water than when they extract air only. The speed also should not be very high in the former case—about 30-50 revolutions per minute. There is another reason why the speed of wet air-pumps should not be too high—it is desirable to expel at each stroke the *whole* quantity of air brought in during that stroke, which can only be accomplished when the air is *first* expelled through the water, which must be as quiescent as possible, and which is *then* itself expelled. If the air and water are mixed, which is the case when the water is in too violent motion in the pump, they are both expelled *together* through the valve, but only a portion of each, and there remains much air in the cylinder, which condition diminishes tho efficiency of the next stroke. The larger valves and passages of the wet pumps cause them to have as a rule greater dead spaces than the slide-valve pumps described later. We shall at once see what influence this has upon the action of the pump.

When a pump with flap-valves is used as a dry pump, *i.e.*, when, along with the air, it does not take in water which would fill the dead space and to a great extent neutralise its effect, it is advisable to allow a

small regulated quantity of cold water or glycerin to enter the pump at each stroke and be expelled, in order to overcome the dead space (German Pat. No. 24,092 of C. Heckman, Berlin).

If the water which is sucked in is cold and the pump does not work too rapidly, very good results can be obtained with wet air-pumps. Vacua of 700-720, or even 730 mm., can be permanently maintained in the evaporating apparatus.

Generally speaking, the flap-valve pumps are less sensitive and less exposed to slight accidents than slide-valve pumps, so that they are suitable for small and medium capacities. They have the further advantage, that they can themselves pump from the well the water for the condenser, which it is convenient to attach directly to the pump. Thus no special water pump is required, which is necessary with dry condensers in the great majority of cases. This suction of the water from a tank or well at a lower level is always permissible if the water level is not more than 5 m. below the middle of the pump. It is, however, advisable to arrange, for starting and special requirements, a small cold water supply-pipe, which can be used for a short time to commence the condensation when the apparatus is first set in motion.

B. Slide-valve Air-pumps.

In these pumps the ports by which the air enters and leaves are mechanically opened. As a rule they should exhaust no water with the air, and are, therefore, called "dry" pumps. Their dead spaces are smaller, their speed can be greater (60-200 revolutions per minute), and they are specially suitable for large capacities. They require a surface- or a dry-condenser (if possible counter-current), and they use less power than wet pumps. But since the dry (fall-pipe) condensers must lie at least 10·2 m. above the water level, they almost always require a special water pump to remove the injected water.

In order to remove the diminution in efficiency produced by the dead spaces, Weiner proposed many years ago to equalise the pressure at the dead-point, and now almost all air-pumps are provided with arrangements of this kind.

When the piston of the air-pump has nearly reached the dead-point, in the small space, V_s, in front of the piston there is air at the atmospheric pressure, p_a, and in the large space behind the piston,

$J + V_s$, there is air at a very much lower pressure. At this moment, the entrance and exit to the cylinder being closed, the two ends of the cylinder are put in communication. After the equalisation there is on both sides of the piston the same pressure:

$$p = \frac{p_a V_s}{J + 2V_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (265)$$

The communication between the two ends of the cylinder is then shut off, the new stroke begins, and almost at once the suction commences.

The details of the arrangements for equalising the pressure are different with different makers, and will not be further considered here.

The question, *to what vacuum* (to what lowest absolute pressure, p_∞) *a vessel can be exhausted*, is answered in the following manner:—

A vessel of the volume V_g is to be exhausted by a double-action pump, *without* equalisation of pressure, with a cylinder of volume J; let the ratio, $\frac{J}{V_g} = \beta$, the original pressure in the vessel $= p$, and the pressure after n half-strokes $= p_n$.

This pressure is (after A. v. Ihering, *Die Gebläse*):

$$p_n = p\left[\frac{1}{b^n} + \frac{\epsilon\beta}{b-1}\left(1 - \frac{1}{b^n}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (266)$$

in which the ratio of the dead spaces to the volume traversed by the piston, $\frac{V_s}{J} = \epsilon$ and $b = 1 + \alpha(1 + \epsilon)$.

After an infinite number of strokes the pressure in the vessel is, therefore:

$$p_\infty = \frac{p\epsilon}{1 + \epsilon} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (267)$$

If the pump is provided with a complete equalisation of pressure, then the pressure in the vessel after n half-strokes is:

$$p_n = p\left[\frac{1}{b^n} + \frac{\epsilon\beta}{b_n} + \frac{\epsilon\beta}{ac}\left\{\frac{\epsilon\beta}{b-1}\left(1 - \frac{b}{b^n}\right) + \frac{p_n}{p}\left(1 - \frac{b^{n-1}}{b^n}\right)\right\}\right] \quad (268)$$

in which $c = 1 + 2\epsilon + \epsilon_1$. After an infinite number of strokes the pressure is very nearly

$$p_\infty = \frac{p\epsilon^2}{(1 + \epsilon)(1 + 2\epsilon + \epsilon_1)} = \frac{p\epsilon}{1 + \epsilon} \cdot \frac{\epsilon}{1 + 2\epsilon + \epsilon_1} \quad . \quad (269)$$

TABLE 74.

The lowest pressures, p_∞, which can be reached by air-pumps, with and without complete equalisation of pressure, at proportions of the dead space, $\epsilon = \frac{V_\epsilon}{J}$, from 0·01 – 0·20.

Ratio of the dead space to the volume of the pump.	Lowest pressure reached after an infinite number of strokes.						Ratio
	Pumps without equalisation of pressure.			Pumps with complete equalisation of pressure.			
ϵ.	Kilos. per sq. cm. p_∞.	Millimetres of Mercury b_0.	Measured as Vacuum. $760 - b_0$	Kilos. per sq. cm. p_∞.	Millimetres of Mercury. b_m.	Measured as Vacuum. $760 - b_m$.	$\frac{\epsilon}{c}$.
0·01	0·010233	7·52	752·5	0·0001003	0·074	759·9	0·0098
0·02	0·020266	14·91	745·1	0·000388	0·285	759·7	0·0191
0·03	0·030105	22·15	727·9	0·000626	0·620	759·38	0·0280
0·04	0·03975	29·23	730·8	0·00143	1·050	759	0·0360
0·05	0·04904	36·2	723·8	0·00216	1·622	758·38	0·0448
0·06	0·05851	43·2	716·8	0·00309	2·281	757·72	0·0528
0·07	0·06761	49·72	710·3	0·00409	3·013	757	0·0606
0·08	0·07655	56·3	703·7	0·00521	3·834	756·17	0·0681
0·09	0·08534	62·75	697·2	0·00643	4·722	755·28	0·0750
0·10	0·0939	69·0	691	0·00773	5·678	754·43	0·0823
0·11	0·1024	75·3	684·7	0·00912	6·707	753·3	0·0891
0·125	0·1148	84·4	675·6	0·01133	8·33	751·67	0·0987
0·135	0·1229	91·2	668·8	0·01290	9·576	750·42	0·1051
0·150	0·1348	100	660	0·01537	11·4	748·2	0·1140
0·165	0·1464	107·6	652·4	0·01796	13·20	746·8	0·1227
0·175	0·1539	113·2	646·8	0·01985	14·60	745·2	0·1290
0·185	0·1614	118·6	641·4	0·02156	15·84	744·2	0·1336
0·200	0·1723	127	633	0·02435	17·95	742·05	0·1413

In order to obtain a representation of the effect of the dead spaces and of the equalisation of pressure, Table 74 has been drawn up. It gives, by means of equation (269), the final pressure obtained after an infinite number of strokes in a vessel, in which the pressure was originally p, for pumps with and without the equalisation of pressure.

Various dimensions are assumed for the dead spaces ($\epsilon = 0{\cdot}01 - 0{\cdot}20$) and for the ratio of the volume of the equalising channel to the volume traversed by the piston—$\epsilon_a = \frac{V_a}{J} = 0{\cdot}015$.

This Table 74 shows the great extent to which the injurious action of the dead spaces is reduced by the equalisation of pressure, even when it is not quite complete, which would be the case in practice. It also shows what vacua can theoretically be obtained with dry air-pumps under various conditions.

CHAPTER XXV.

THE VOLUMETRIC EFFICIENCY OF AIR-PUMPS.

(See A. v. Ihering, *Die Gebläse.*)

A. Air-pumps without Equalisation of Pressure.

WHEN the piston reaches the end of its stroke, after the air has been expelled there remains in a small portion of the cylinder—the dead space—the volume, V_s, at the pressure of the atmosphere, p. As soon as the piston recedes, this volume, V_s, expands, and continues to expand until its pressure is equal to that in the vessel to be evacuated, p_0. Let the space through which the piston has then travelled $= V_x$. (These conditions are *the same* both for air-pumps, which are to create or maintain the very small pressure, p_0, in a vessel and which expel the exhausted air into the atmosphere at the pressure, p, *and also* for compressors, which press the air from the atmosphere, where the pressure is p_0, into a vessel, in which the pressure, p, is to be maintained.)

Air is warmed by compression; this is the case when air at a very small absolute pressure (a partial vacuum) is brought to the pressure of the atmosphere, just as when air at atmospheric pressure is compressed.

Let the temperature of the compressed air be T, its temperature after expansion to the pressure, p_0, be T_0, then by Mariotte's law

$$\frac{V_s p}{T} = \frac{V_s + V_x}{T_0} p_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (270)$$

whence

$$V_x = \frac{\dfrac{V_s p}{T} - \dfrac{V_s p_0}{T_0}}{p_0} T_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (271)$$

If V_e is the volume through which the piston travels whilst exhausting, and J the total volume it describes, then

$$J - V_x = V_e.$$

Therefore

$$V_e = J - \frac{\left(\frac{V_e p}{T} - \frac{V_e p_0}{T_0}\right)T_0}{p_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (272)$$

and since $V_e = \epsilon J$

$$V_e = J - \frac{\left(\frac{\epsilon J p}{T} - \frac{\epsilon J p_0}{T_0}\right)T_0}{p_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (273)$$

The ratio of the volume during exhaustion, V_e (the useful work), to the whole volume of the stroke, J, *i.e.*, the volumetric efficiency, χ_{vol}, is, therefore,

$$\chi_{vol} = \frac{V_e}{J} = 1 - \frac{\left(\frac{\epsilon p}{T} - \frac{\epsilon p_0}{T_0}\right)T_0}{p_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad (274)$$

$$\chi_{vol} = 1 - \epsilon\left(\frac{p}{p_0}\frac{T_0}{T} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (275)$$

This is the volumetric efficiency for the condition that the heat produced in compression is in no way lost. This is called *adiabatic* compression.

From this equation we see that the volumetric efficiency is greater:—

1. The smaller the dead space, ϵ.

2. The lower the ratio of the pressure of compression to the pressure of the exhausted air (*i.e.*, in compressors, the lower the air pressure to be attained; in vacuum pumps, the smaller is the vacuum to be produced).

3. The higher the temperature of the compressed air and the lower that of the exhausted air (*i.e.*, the greater the difference in temperature between exhausted and compressed air).

Thus in order to obtain high volumetric efficiency artificial cooling during compression is not advantageous, but is advantageous during the period of expansion.

The cooling may be effected by means of a jacket or by injecting water; the latter is more effective, but necessitates a slower speed and readily causes fouling.

If complete cooling were attained, so that the air was at a constant temperature during the whole operation, then $T = T_0$, and the efficiency equation would be

$$\chi_{ri} = 1 - \epsilon\left(\frac{p}{p_0} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad (276)$$

Compression under these conditions is called *isothermal*.

Generally complete cooling is not obtained, although attempts are made; a condition occurs which is a mean between complete cooling and absence of cooling, which is known as *polytropic* compression. The useful work may then be expressed as the mean of the results of equations (275) and (276):—

$$\chi_{ra} = 1 - \epsilon\left(\frac{p}{p_0}\frac{T_0}{T} - 1\right) \text{ and } \chi_{ri} = 1 - \epsilon\left(\frac{p}{p_0} - 1\right) . \quad . \quad (277)$$

Now in determining the useful work in adiabatic compression the temperatures T and T_0 are not known; if the useful work is to be calculated these factors must be replaced by others which are known. This is effected by means of Poisson's law (the so-called involuted Mariotte's law), by which the pressures may be put in place of the temperatures:—

$$\frac{T_0}{T} = \left(\frac{p_0}{p}\right)^{\frac{k-1}{k}} = \frac{p_0}{p}\left(\frac{p}{p_0}\right)^{\frac{1}{k}} . \quad . \quad . \quad . \quad . \quad (278)$$

in which

$$k = \frac{\sigma_t}{\sigma_v} = \frac{0{\cdot}23751}{0{\cdot}16847} = 1{\cdot}41 \quad . \quad . \quad . \quad . \quad . \quad (279)$$

or

$$\frac{1}{k} = 0{\cdot}7092 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (280)$$

σ_t is the specific heat of air at constant pressure $= 0{\cdot}2375$.

σ_v is the specific heat of air at constant volume $= 0{\cdot}16847$.

If these values be inserted in equation (275), we obtain an equation for the *adiabatic* efficiency, from which numerical results can be obtained:—

$$\chi_{ra} = 1 - \epsilon\left[\left(\frac{p}{p_0}\right)^{\frac{1}{k}} - 1\right] = 1 - \epsilon\left[\left(\frac{p}{p_0}\right)^{0{\cdot}7092} - 1\right] . \quad (281)$$

B. Air-pumps, with Equalisation of Pressure.

When the piston reaches the end of its stroke, the condition of the air in the dead space before the equalisation of pressure, assuming

that the equalising channel, V_a, is always in communication with the compressed air, is :—

$$\frac{V_s + V_a}{T} p \quad \ldots \ldots \ldots \quad (282)$$

in the other and larger space the condition is :—

$$\frac{J + V_s}{T_0} p_0 \quad \ldots \ldots \ldots \quad (283)$$

After the equalisation of pressure has taken place the condition is :—

$$\frac{J + 2V_s + V_a}{T_a} p_s \quad \ldots \ldots \ldots \quad (284)$$

and since the conditions before and after equalisation must be equal :—

$$\frac{V_s + V_a}{T} p + \frac{J + V_s}{T_0} p_0 = \frac{J + 2V_s + V_a}{T_a} p_s \quad \ldots \quad (285)$$

or

$$p_s = \frac{\left(\frac{V_s + V_a}{T} p + \frac{J + V_s}{T_0} p_0\right) T_a}{J + 2V_s + V_a} \quad \ldots \quad (286)$$

If we put $V_s = \epsilon J$ and $V_a = \epsilon_a J$ and eliminate J, then

$$p_s = \frac{\left(\frac{(\epsilon + \epsilon_a)p}{T} + \frac{(1 + \epsilon)p_0}{T_0}\right) T_a}{1 + 2\epsilon + \epsilon_a} \quad \ldots \quad (287)$$

or

$$\frac{p_s}{p_0} = \frac{\left(\frac{(\epsilon + \epsilon_a)}{T} \frac{p}{p_0} + \frac{1 + \epsilon}{T_0}\right) T_a}{1 + 2\epsilon + \epsilon_a} \quad \ldots \quad (288)$$

In *isothermal* compression, in which all the temperatures remain constant, $T = T_a = T_0$, and

$$\frac{p_s}{p_0} = \frac{(\epsilon + \epsilon_a)\frac{p}{p_0} + (1 + \epsilon)}{1 + 2\epsilon + \epsilon_a} \quad \ldots \ldots \quad (289)$$

In finding the equation for the *adiabatic* compression (291) it is permissible to put $T_a = T_0$, which is not correct, but causes only an inconsiderable error. Equation (288) then becomes

$$\frac{p_s}{p_0} = \frac{(\epsilon + \epsilon_a)\frac{p}{p_0}\frac{T_0}{T} + (1 + \epsilon)}{1 + 2\epsilon + \epsilon_a} \quad \ldots \ldots \quad (290)$$

TABLE 75. PART I.

The isothermal and adiabatic values of $\frac{p_\epsilon}{p_0} = \frac{\text{pressure after equalisation}}{\text{pressure in empty vessel}}$, 0·01-0·20, and for isothermal and adia-

Dead space. $\frac{V_\epsilon}{J} = \epsilon$.	Isothermal, i. Adiabatic, a.	Isothermal and adiabatic values of $\frac{p}{p_0} = \frac{\text{pressure of the atmosphere}}{\text{pressure in evacuated vessel}}$ or						
		1·1	1 5	2	2·5	3	3·5	4·11
0·01	i	1·001	1·011	1·024	1·036	1·048	1·060	1·075
	a	1·005	1·012	1·019	1·026	1·032	1·038	1·046
0·02	i	1·002	1·016	1·033	1·049	1·060	1·083	1·106
	a	1·000	1·016	1·018	1·025	1·034	1·041	1·052
0·03	i	1·003	1·020	1·042	1·063	1·083	1·105	1·130
	a	0·988	1·000	1·012	1·023	1·035	1·046	1·058
0·04	i	1·004	1·025	1·050	1·075	1·100	1·125	1·165
	a	0·980	0·999	1·009	1·023	1·036	1·048	1·063
0·05	i	1·005	1·029	1·058	1·087	1·116	1·143	1·181
	a	0·972	0·985	1·005	1·020	1·037	1·051	1·068
0·06	i	1·006	1·033	1·066	1·099	1·132	1·165	1·209
	a	0·965	0·985	1·005	1·025	1·038	1·054	1·074
0·07	i	1·007	1·037	1·075	1·111	1·144	1·174	1·237
	a	0·955	0·960	0·999	1·019	1·039	1·065	1·077
0·08	i	1·008	1·045	1·088	1·121	1·160	1·200	1·259
	a	0·950	0·971	0·993	1·017	1·040	1·059	1·085
0·09	i	0·940	1·044	1·091	1·140	1·176	1·230	1·273
	a	1·099	0·963	0·990	1·017	1·040	1·062	1·096
0·10	i	1·010	1·048	1·095	1·155	1·189	1·260	1·337
	a	0·936	0·960	0·975	1·015	1·042	1·065	1·093
0·125	i	1·012	1·053	1·115	1·169	1·230	1·280	1·370
	a	0·920	0·945	0·982	1·015	1·046	1·073	1·103
0·150	i	1·015	1·062	1·126	1·188	1·256	1·313	1·400
	a	0·903	0·9[illegible]2	0·979	1·011	1·046	1·077	1·112
0·175	i	1·017	1·070	1·139	1·200	1·286	1·350	1·433
	a	0·892	0·928	0·970	1·009	1·047	1·080	1·113
0·200	i	1·090	1·079	1·152	1·228	1·300	1·380	1·472
	a	0·879	0·925	0·972	1·007	1·048	1·085	1·125

TABLE 75. PART I.

and the volumetric efficiency, χ_v, for air-pumps and compressors, *with and without* equalisation of pressure, with dead spaces, ϵ, from batic compression. ϵ_a is taken at 0·015.

$\frac{p'}{p_0} = \frac{\text{pressure after equalisation}}{\text{pressure in evacuated vessel}}$								
$\frac{\text{pressure in compression vessel}}{\text{pressure of the atmosphere}} = \frac{p}{p_0}$								
4·74	5·38	6·33	7·6	9·5	12·67	19	36	76·0
1·090	1·105	1·128	1·150	1·203	1·280	1·434	1·845	2·84
1·053	1·060	1·063	1·082	1·100	1·125	1·174	1·285	1·48
1·135	1·150	1·182	1·226	1·281	1·395	1·615	2·164	3·50
1·061	1·071	1·084	1·101	1·124	1·161	1·237	1·392	1·68
1·156	1·186	1·222	1·274	1·355	1·487	1·752	2·464	4·14
1·070	1·084	1·095	1·120	1·153	1·195	1·280	1·475	1·86
1·187	1·220	1·267	1·331	1·447	1·585	1·904	2·758	4·78
1·070	1·092	1·112	1·138	1·178	1·219	1·330	1·564	2·03
1·918	1·255	1·310	1·375	1·485	1·675	2·050	3·044	5·40
1·085	1·102	1·117	1·155	1·201	1·260	1·377	1·650	2·20
1·246	1·290	1·351	1·436	1·540	1·770	2·292	3·314	5·95
1·092	1·112	1·138	1·172	1·225	1·280	1·428	1·733	2·36
1·275	1·323	1·390	1·486	1·625	1·859	2·325	3·576	6·55
1·100	1·121	1·155	1·185	1·247	1·322	1·465	1·813	2·51
1·302	1·358	1·430	1·533	1·690	1·950	2·440	3·825	7·06
1·106	1·130	1·163	1·213	1·260	1·384	1·510	1·895	2·66
1·327	1·377	1·470	1·580	1·747	2·025	2·590	4·075	7·55
1·112	1·139	1·174	1·218	1·285	1·375	1·553	1·900	2·82
1·354	1·414	1·504	1·625	1·805	2·137	2·704	4·313	8·10
1·119	1·145	1·185	1·232	1·309	1·895	1·590	2·015	2·95
1·471	1·484	1·590	1·670	1·940	2·300	2·990	4·842	9·33
1·134	1·165	1·212	1·283	1·356	1·466	1·685	2·206	3·28
1·485	1·514	1·668	1·750	2·061	2·464	3·180	5·392	11·17
1·147	1·178	1·227	1·291	1·403	1·529	1·790	2·365	3·58
1·520	1·534	1·741	1·917	2·183	2·660	3·560	5·768	11·80
1·161	1·210	1·251	1·325	1·439	1·575	1·935	2·511	3·87
1·561	1·665	1·810	2·010	2·292	2·775	3·733	6·320	12·55
1·166	1·219	1·275	1·350	1·477	1·625	1·940	2·647	4·14

TABLE 75. PART II.

Dead space. $\frac{V_s}{V} = \epsilon$	Isothermal, *i*. Adiabatic, *a*.	*o*	*m*	*o*	*m*	*o*	*m*
		o = without equalisation of pressure. *m* = with equalisation of pressure.					
		Vacuum in mm. of mercury.					
		70		260		380	
		$\frac{p}{p_0} = \frac{\text{pressure of the atmosphere}}{\text{pressure in evacuated vessel}}$					
		1·1	1·1	1·5	1·5	2	2
		Volumetric efficiency, χ_v, of air-pumps and com-					
0·01	*i*	0·999	0·999	0·995	0·999	0·990	0·999
	a	0·999	0·999	0·997	0·999	0·993	0·999
0·02	*i*	0·998	0·999	0·990	0·999	0·980	0·999
	a	0·998	0·999	0·994	0·999	0·987	0·999
0·03	*i*	0·997	0·999	0·995	0·999	0·970	0·999
	a	0·997	0·997	0·990	0·999	0·981	0·999
0·04	*i*	0·996	0·999	0·980	0·999	0·960	0·998
	a	0·997	0·999	0·987	1·012	0·975	0·999
0·05	*i*	0·995	0·999	0·975	0·999	0·950	0·997
	a	0·996	0·999	0·984	0·999	0·967	0·999
0·06	*i*	0·994	0·999	0·970	0·998	0·940	0·996
	a	0·995	0·999	0·980	0·999	0·962	0·999
0·07	*i*	0·993	0·999	0·965	0·998	0·930	0·995
	a	0·995	0·999	0·977	0·999	0·955	0·999
0·08	*i*	0·992	0·999	0·960	0·997	0·920	0·993
	a	0·994	0·999	0·973	0·999	0·950	0·999
0·09	*i*	0·991	0·999	0·955	0·996	0·910	0·992
	a	0·994	0·999	0·970	0·999	0·943	0·999
0·10	*i*	0·990	0·999	0·950	0·995	0·900	0·991
	a	0·993	0·999	0·967	0·999	0·937	0·999
1·125	*i*	0·988	0·998	0·937	0·993	0·875	0·986
	a	0·991	0·999	0·959	0·999	0·916	0·999
0·150	*i*	0·985	0·998	0·925	0·991	0·850	0·981
	a	0·990	0·999	0·930	0·999	0·905	0·999
0·175	*i*	0·983	0·997	0·912	0·988	0·825	0·977
	a	0·987	0·999	0·942	0·999	0·880	0·999
0·200	*i*	0·980	0·996	0·900	0·999	0·820	0·999
	a	0·986	0·999	0·934	0·985	0·874	0·970

TABLE 75. PART II.

o = without equalisation of pressure.
m = with equalisation of pressure.

	m	*o*	*m*	*o*	*m*	*o*	*m*
Vacuum in mm. of mercury.							
456		507		543		580	
or $\frac{\text{pressure in compression vessel}}{\text{pressure of the atmosphere}}$							
2·5	2·5	3	3	3·5	3·5	4·11	4·11
pressors with and without equalisation of pressure.							
0·985	0·999	0·980	0·999	0·975	0·999	0·969	0·999
0·991	0·999	0·989	0·999	0·986	0·999	0·983	0·999
0·970	0·999	0·960	0·998	0·950	0·998	0·938	0·998
0·982	0·999	0·977	0·999	0·972	0·999	0·066	0·999
0·955	0·998	0·940	0·998	0·925	0·997	0·907	0·996
0·973	0·999	0·965	0·999	0·958	0·899	0·949	0·998
0·940	0·997	0·920	0·99	0·900	0·995	0·876	0·994
0·964	0·999	0·953	0·999	0·944	0·999	0·032	0·998
0·925	0·996	0·900	0·994	0·875	0·993	0·844	0·991
0·954	0·999	0·941	0·999	0·929	0·999	0·915	0·998
0·910	0·994	0·883	0·992	0·850	0·991	0·814	0·988
0·945	0·999	0·930	0·999	0·915	0·998	0·893	0·997
0·895	0·992	0·860	0·991	0·825	0·989	0·783	0·983
0·936	0·999	0·912	0·997	0·900	0·997	0·881	0·996
0·880	0·991	0·840	0·988	0·780	0·984	0·751	0·980
0·927	0·999	0·906	0·998	0·886	0·997	0·863	0·996
0·865	0·998	0·820	0·985	0·775	0·980	0·720	0·976
0·917	0·999	0·894	0·998	0·872	0·997	0·847	0·995
0·850	0·985	0·800	0·981	0·750	0·974	0·689	0·966
0·909	0·999	0·882	0·998	0·857	0·996	0·828	0·994
0·812	0·980	0·750	0·971	0·688	0·965	0·612	0·954
0·884	0·999	0·853	0·996	0·822	0·995	0·827	0·992
0·775	0·973	0·700	0·962	0·625	0·953	0·533	0·940
0·860	0·999	0·823	0·996	0·786	0·991	0·785	0·989
0·738	0·965	0·650	0·951	0·563	0·938	0·456	0·926
0·838	0·999	0·794	0·968	0·750	0·958	0·742	0·985
0·700	0·999	0·600	0·940	0·500	0·924	0·378	0·983
0·814	0·955	0·765	0·994	0·714	0·989	0·655	0·906

TABLE 75. PART II.—(*continued*).

Dead space. $\frac{V_s}{J} = \epsilon$.	Isothermal, *i*. Adiabatic, *a*.	*o* = without equalisation of pressure. *m* = with equalisation of pressure.							
		o	*m*	*o*	*m*	*o*	*m*	*o*	*m*
		Vacuum in mm. of mercury.							
		600		620		640		660	
		$\frac{p}{p_0} = \frac{\text{pressure of the atmosphere}}{\text{pressure in evacuated vessel}}$							
		4·74	4·74	5·38	5·38	6·33	6·33	7·6	7·6
		Volumetric efficiency, χ_v, of air-pumps and com-							
0·01	*i*	0·963	0·999	0·956	0·999	0·947	0·999	0·934	0·998
	a	0·980	0·999	0·977	0·999	0·973	0·999	0·968	0·999
0·02	*i*	0·925	0·998	0·912	0·997	0·893	0·997	0·868	0·996
	a	0·960	0·999	0·954	0·999	0·947	0·999	0·936	0·999
0·03	*i*	0·888	0·995	0·878	0·994	0·840	0·993	0·802	0·992
	a	0·940	0·998	0·931	0·998	0·920	0·998	0·904	0·997
0·04	*i*	0·851	0·993	0·825	0·991	0·787	0·990	0·736	0·987
	a	0·920	0·998	0·908	0·997	0·883	0·997	0·872	0·996
0·05	*i*	0·813	0·990	0·781	0·983	0·734	0·984	0·670	0·987
	a	0·900	0·998	0·885	0·997	0·866	0·996	0·840	0·995
0·06	*i*	0·776	0·986	0·738	0·983	0·680	0·879	0·604	0·975
	a	0·880	0·997	0·862	0·996	0·839	0·994	0·808	0·992
0·07	*i*	0·738	0·982	0·694	0·978	0·627	0·973	0·538	0·966
	a	0·860	0·995	0·839	0·993	0·812	0·992	0·776	0·989
0·08	*i*	0·701	0·976	0·650	0·972	0·574	0·968	0·472	0·958
	d	0·840	0·995	0·816	0·993	0·785	0·992	0·744	0·989
0·09	*i*	0·664	0·972	0·606	0·967	0·520	0·960	0·406	0·948
	a	0·820	0·994	0·793	0·992	0·760	0·990	0·712	0·987
0·10	*i*	0·620	0·965	0·562	0·959	0·467	0·950	0·340	0·938
	a	0·800	0·963	0·770	0·990	0·731	0·988	0·680	0·985
0·125	*i*	0·533	0·941	0·463	0·949	0·334	0·926	0·175	0·916
	a	0·748	0·989	0·715	0·986	0·663	0·983	0·600	0·976
0·150	*i*	0·439	0·928	0·343	0·923	0·201	0·900	0·010	0·887
	a	0·698	0·985	0·655	0·982	0·600	0·978	0·520	0·971
0·175	*i*	0·344	0·909	0·234	0·906	0·063	0·871	—	0·840
	a	0·650	0·981	0·600	0·976	0·500	0·971	0·440	0·962
0·200	*i*	0·252	0·978	0·124	0·971	—	0·963	—	0·954
	a	0·598	0·888	0·540	0·868	0·460	0·838	0·360	0·598

TABLE 75. PART II.—(*continued*).

o = without equalisation of pressure.
m = with equalisation of pressure.

Vacuum in mm. of mercury.

or $\frac{\text{pressure in compression vessel}}{\text{pressure of the atmosphere}}$.

pressors with and without equalisation of pressure.

680		700		720		740		750	
o	*m*	*o*	*m*	*o*	*m*	*o*	*m*	*o*	*m*
9·5	9·5	12·67	12·67	19	19	36	36	75·0	75·0
0·915	0·998	0·883	0·997	0·820	0·996	0·650	0·992	0·26	0·982
0·961	0·999	0·953	0·999	0·930	0·999	0·883	0·998	—	0·997
0·830	0·994	0·767	0·993	0·640	0·987	0·300	0·977	—	0·950
0·922	0·999	0·900	0·999	0·860	0·998	0·767	0·995	—	0·991
0·745	0·989	0·640	0·987	0·460	0·978	—	0·957	—	0·936
0·882	0·997	0·850	0·996	0·790	0·996	0·650	0·991	—	0·984
0·660	0·983	0·534	0·970	0·280	0·964	—	0·932	—	0·849
0·853	0·996	0·800	0·994	0·720	0·993	0·533	0·980	—	0·974
0·575	0·976	0·417	0·967	0·100	0·953	—	0·890	—	0·780
0·804	0·993	0·750	0·991	0·650	0·989	0·416	0·979	—	0·963
0·490	0·968	0·300	0·954	—	0·941	—	0·862	—	0·703
0·765	0·997	0·700	0·988	0·580	0·985	0·299	0·977	—	0·951
0·405	0·957	0·183	0·941	—	0·928	—	0·821	—	0·612
0·725	0·988	0·650	0·985	0·510	0·981	0·182	0·962	—	0·937
0·310	0·944	0·068	0·924	—	0·917	—	0·776	—	0·516
0·686	0·986	0·600	0·981	0·440	0·976	0·045	0·955	—	0·923
0·235	0·934	—	0·909	—	0·859	—	0·784	—	0·411
0·647	0·983	0·550	0·967	0·370	0·970	—	0·949	—	0·903
0·150	0·920	—	0·886	—	0·830	—	0·669	—	0·290
0·607	0·980	0·500	0·970	0·300	0·963	—	0·937	—	0·885
—	0·883	—	0·838	—	0·750	—	0·520	—	—
0·509	0·971	0·377	0·968	0·118	0·945	—	0·908	—	0·835
—	0·841	—	0·771	—	0·673	—	0·338	—	—
0·410	0·960	0·246	0·948	—	0·925	—	0·976	—	0·780
—	0·792	—	0·712	—	0·552	—	0·167	—	—
0·330	0·940	0·130	0·935	—	0·898	—	0·848	—	0·720
—	0·934	—	0·909	—	0·860	—	—	—	—
0·214	0·542	—	0·445	—	0·259	—	0·805	—	0·652

or, applying Poisson's law,

$$\frac{p_s}{p_0} = \frac{(\epsilon + \epsilon_a)\left(\frac{p}{p_0}\right)^{\frac{1}{k}} + (1 + \epsilon)}{1 + 2\epsilon + \epsilon_a} \quad . \quad . \quad . \quad . \quad . \quad (291)$$

After equalisation has taken place, the equalising channel at the piston end of the cylinder is closed, and the piston in returning must pass through the space, V_x, in order to reduce the pressure, p_s, existing after the equalisation to that to be attained, p_0. When this is the case, the exhaustion begins, therefore,

$$\frac{V_s p_s}{T_a} = \frac{V_s + V_x}{T_0} p_0 = \frac{V_s p_0}{T_0} + \frac{V_x p_0}{T_0}$$

or

$$V_x = \left(\frac{V_s p_s}{T_a} - \frac{V_s p_0}{T_0}\right)\frac{T_0}{p_0}$$

$$V_x = V_s\left(\frac{p_s}{p_0}\frac{T_0}{T_a} - 1\right).$$

The *isothermal* volumetric efficiency is, since $T_a = T_0$,

$$\chi_{vi} = 1 - \frac{V_x}{J} = 1 - \epsilon\left(\frac{p_s}{p_0} - 1\right) \quad . \quad . \quad . \quad . \quad (292)$$

or, inserting the value of $\frac{p_s}{p_0}$ from equation (289),

$$\chi_{vi} = 1 - \epsilon\left[\frac{(\epsilon + \epsilon_a)\frac{p}{p_0} + (1 + \epsilon)}{1 + 2\epsilon + \epsilon_a} - 1\right] \quad . \quad . \quad . \quad (293)$$

The *adiabatic* volumetric efficiency is

$$\chi_{va} = 1 - \frac{V_x}{J} = 1 - \epsilon\left(\frac{p_s}{p_0}\frac{T_0}{T_a} - 1\right) \quad . \quad . \quad . \quad (294)$$

$$= 1 - \epsilon\left\{\left(\frac{p'}{p_0}\right)^{\frac{1}{k}} - 1\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (295)$$

or, inserting the value of $\frac{p_s}{p_0}$ from equation (291),

$$\chi_{va} = 1 - \epsilon\left[\left(\frac{(\epsilon + \epsilon_a)\left(\frac{p}{p_0}\right)^{\frac{1}{k}} + (1 + \epsilon)}{1 + 2\epsilon + \epsilon_a}\right)^{\frac{1}{k}} - 1\right] \quad . \quad . \quad (296)$$

All these equations, which appear more unwieldy than they really are, are calculated out in Table 76 for many cases, indeed for most ordinary cases.

In the first place will be found the values of $\frac{p}{p_0}$, calculated by means of equations (289) and (291) for most degrees of evacuation and compression. The isothermal and adiabatic volumetric efficiencies can then readily be determined by the aid of equations (293) and (296). The calculated values of these efficiencies are given in the second part of Table 75, together with those for pumps without equalisation of pressure (equations (276) and (281)), so that all calculable efficiencies may be examined together, which was the purpose of this table. *From this comparison it may be seen that the volumetric efficiency is the greatest when no heat is taken from the air-pump, and that the cooling of the cylinder of the air-pump, when only the volumetric effect is in contemplation, is rather injurious than useful.* But all these figures do not quite represent actual practice, for, whether artificial cooling is applied or not, a certain and not inappreciable cooling takes place through the metal walls. The so-called *polytropic* compression then occurs, which is approximately represented by taking for each case the mean between completely cooled and uncooled air-pumps. This assumption corresponds best to the reality, and in most ordinary cases the difference is not very great.

CHAPTER XXVI.

DETERMINATION OF THE VOLUME OF AIR, V_l, WHICH MUST BE EXHAUSTED FROM A VESSEL CONTAINING THE VOLUME V_g, AT THE PRESSURE p_a, IN ORDER TO REACH THE LOWER PRESSURE, p_e.

(After F. J. Weiss, *Zeits. d. V. d. Ing.*, 1886, 646.)

SOMETIMES it is required to know how large an air-pump must be in order to exhaust a vessel of known capacity in a definite time down to a certain degree of vacuum, or the reverse: in what time a certain vessel can be exhausted down to a certain vacuum by means of the pump provided.

Let V_g = the volume of the vessel in litres.

J = the useful volume of the air-pump in litres.

p_a = the initial pressure in the vessel in atmos.

p_e = the final pressure in the vessel in atmos.

V_l = the volume in litres which must be exhausted in order to reduce the pressure from p_a to p_e.

If the pressure in the vessel after the

	1st	2nd	3rd	nth single stroke
is	p_1	p_2	p_3	p_e atmos.

then

$$p_1(V_g + J) = p_a V_g, \text{ therefore } p_1 = p_a \frac{V_g}{V_g + J} \quad \ldots \quad (297)$$

$$p_2(V_g + J) = p_1 V_g \quad ,, \quad p_2 = p_1 \frac{V_g}{V_g + J} = p_a \left(\frac{V_g}{V_g + J}\right)^2 \quad (298)$$

$$p_3(V_g + J) = p_2 V_g \quad ,, \quad p_3 = p_2 \frac{V_g}{V_g + J} = p_a \left(\frac{V_g}{V_g + J}\right)^3 \quad (299)$$

$$p_e = p_a \left(\frac{V_g}{V_g + J}\right)^n \quad \ldots \quad (300)$$

or

$$\frac{p_e}{p_a} = \left(\frac{V_g}{V_g + J}\right)^n \quad . \quad . \quad . \quad . \quad (301)$$

whence

$$n = \frac{\log \frac{p_e}{p_a}}{\log \frac{V_g}{V_g + J}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (302)$$

If $\frac{V_g}{V_g + J}$ be expanded in a binomial series and the higher powers of $\frac{J}{V_g}$ neglected because of their smallness, then

$$\frac{V_g}{V_g + J} = 1 - \frac{J}{V_g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (303)$$

or:

$$\log \frac{V_g}{V_g + J} = \log\left(1 - \frac{J}{V_g}\right) \quad . \quad . \quad . \quad . \quad (304)$$

If now $\log\left(1 - \frac{J}{V_g}\right)$ be expanded in a series and higher powers neglected, we obtain

$$\log\left(1 - \frac{J}{V_g}\right) = -\frac{J}{V_g} \quad . \quad . \quad . \quad . \quad . \quad . \quad (305)$$

When this value is inserted in equation (302) we have:

$$n = \frac{\log \frac{p_e}{p_a}}{-\frac{J}{V_g}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (306)$$

or

$$nJ = V_g\left(-\log \frac{p_e}{p_a}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (307)$$

Now nJ is the total volume, which is to be exhausted from the vessel, *i.e.*, through which the piston has to run, in order to reduce the contents from the pressure p_a to the pressure p_e, therefore

$$nJ = V_1 = V_g\left(-\log \frac{p_e}{p_a}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (308)$$

p_e is always less than p_a, therefore $\log \frac{p_e}{p_a}$ is always negative, and consequently $-\log \frac{p_e}{p_a}$ always positive.

TABLE 76.

Examples of the volume, V_l, in litres, which must be exhausted from vessels containing V_g = 500 to 4,500 litres of air, in order to reduce the original internal pressure p_a = 1 atmos. abs. (760 mm. of mercury) to 0·9-0·01 atmos. abs. (vacua of 76 to 754·4 mm.).

1	2	3	4	5	6	7	8	9	10	11	12
The pressure in the vessel is to be diminished from the atmospheric pressure p_a to		Log p_e	If the original pressure of the atmos. abs. in a vessel of the capacity V_g is to be brought to the lower pressure p_e atmos., the air-pump has to exhaust the following volumes, V_l, in litres.								
			Capacity of the vessel, V_g, in litres.								
the abs. pressure p_e	i.e., to a vacuum of		500	1000	1500	2000	2500	3000	3500	4000	4500
			Volume to be exhausted, V_l, in litres.								
atmos.	mm.										
0·9	76	0·105	53	105	158	210	263	315	368	420	473
0·8	152	0·223	112	223	335	446	558	669	781	892	1004
0·7	228	0·357	176	351	527	702	878	1053	1229	1404	1760
0·6	334	0·511	256	511	767	1022	1288	1535	1789	2044	2310
0·5	380	0·693	347	693	1040	1386	1733	2079	2426	2762	3119
0·4	456	0·916	458	916	1374	1832	2290	2748	3206	3664	4122
0·3	532	1·204	602	1204	1806	2408	3010	3612	4214	4816	5418
0·25	570	1·385	693	1385	2078	2770	3463	4155	4848	5540	6233
0·2	608	1·61	810	1610	2415	3220	4025	4830	5635	6440	7245
0·15	646	1·90	950	1900	2850	3800	4750	5700	6650	7600	8550
0·1	684	2·30	1150	2300	3450	4600	5750	6900	8050	9200	10550
0·09	691·6	2·41	1205	2410	3615	4820	6025	7230	8435	9640	10845
0·08	699·2	2·53	1265	2530	3795	5060	6325	7590	8855	10120	11385
0·07	706·8	2·66	1330	2660	3990	5320	6650	7980	9310	10640	11970
0·06	717·4	2·81	1405	2810	4215	5620	7025	8430	9835	11240	12645
0·05	722	3·00	1500	3000	4500	6000	7500	9000	10500	12000	13500
0·04	729·6	3·22	1610	3220	4830	6440	8050	9660	11270	12880	14490
0·03	737·2	3·51	1755	3510	5265	7020	8775	10530	12285	14040	15795
0·02	751·1	3·91	1950	3910	5865	7820	9775	11730	13685	15640	17595
0·01	753·4	4·61	2305	4610	6915	9220	11525	13830	16135	18440	20745

If $p_a = 1$, *i.e.*, if the absolute pressure in the vessel at the beginning is 1 atmos., then $\log p_a = 0$, and the expression becomes $V_t = V_g\,(-\log p_e)$, which is always positive since p_e must be less than 1.

Table 76 has been calculated by means of this formula. It gives immediately the volume, V_t, which must be exhausted from vessels of V_g = 500 to 4,500 litres capacity, in order to reduce the contents from the absolute pressure of 1 atmos. to the desired lower pressure, p_e. The number of strokes required for this purpose is obtained from the dimensions of the pump. If the time be given in which the desired effect is to be produced, the dimensions can readily be found. The table shows at once that almost as many strokes (or as much time) are required to reduce the pressure of 1 atmos. down to 0·1 atmos., as 0·1 to 0·01 atmos.

If it is required to reduce the pressure in a vessel from p_m, which is lower than 1 atmos., to the still lower pressure p_e, in order to find the volume of air to be exhausted in that case, it is only necessary to subtract the volume, which must be exhausted in order to reduce the pressure from 1 to p_m, from that required to reduce the pressure from 1 to p_e.

Examples.—(*a*) A vessel of the capacity of V_g = 2,000 litres, in which the absolute pressure p_a = 1 atmos., is to be evacuated down to 0·2 atmos.

Table 76, column 7, line 9 shows that 3,220 litres must be exhausted for this purpose.

(*b*) The pressure in a vessel of the capacity, V_g = 2,000 litres is 0·5 atmos.; it is to be reduced to 0·2 atmos. What volume must be exhausted?

From Table 76, column 7, line 9 it is seen that, in order to reduce the pressure in the vessel from 1 atmos. to 0·2 atmos., 3,220 litres must be exhausted, and column 7, line 5, shows that 1,386 litres must be exhausted in order to reduce the pressure in the vessel from 1 atmos. to 0·5 atmos.

Thus, to reduce the pressure in the vessel from 0·5 to 0·2 atmos., 3,220 − 1,386 = 1,834 litres must be pumped out, whence the dimensions of the air-pump can be determined.

APPENDIX.

METRIC CONVERSION DIAGRAMS.

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To facilitate the use of this book by designers working in British units, the following diagrams have been prepared.

Taking the first three diagrams, we read the metric unit on the bottom or top scale as the case may be, and run up vertically until the diagonal line is reached; we then run horizontally to the right or left and read off the result on the British scale.

To convert 5 kilos, for example, into lb, we find 5 on the bottom scale of diagram 1, run up vertically to the diagonal line and then move horizontally to the right, reading 11·1 lb.

To convert 6 sq. metres to sq. feet, we take the point 6 on the top scale of diagram 2, run down vertically until we meet the diagonal and then run out horizontally to the left, reading 64·5 sq. feet.

In diagram 4 we read litres on the bottom scale and run up to the diagonal, then moving horizontally to either side according as cub. feet or gallons are required. This diagram may be used to convert cub. feet to gallons direct by running straight across.

In diagram 5, we read mm. of mercury on the left and run right across to convert to lb. per sq. inch and down vertically from the intersection with the diagonal to read atmospheres. For higher pressures the same diagram may be used by shifting the decimal point.

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